SOLUTIONS & ANSWERS FOR JEE MAINS-2014
VERSION – E

[PHYSICS, CHEMISTRY & MATHEMATICS]

PART – A – PHYSICS

1. The current voltage relation of diode is given by I = ($e^{1000\,V/T}$ − 1) mA, where the applied voltage V is in volts ----

Ans: 0.2 mA

Sol: di = $\frac{1000}{300}e^{\frac{10}{3T}V}$ dVmA —(1)

$5 = e^{\frac{10\,V}{3T}} - 1$ —(2)

(2) in (1)

di = $\frac{10}{3} \times 6 \times 0.01$

= 0.2 mA

2. From a tower of height H, a particle is thrown vertically upwards with a speed u. The time taken ----

Ans: $2gH = nu^2(n-2)$

Sol: $0 = u - gt \Rightarrow t = \frac{u}{g}$

$-H = u.\frac{n.u}{g} - \frac{1}{2}g\left(\frac{nu}{g}\right)^2$

$-gH = n^2u - \frac{1}{2}n^2u^2 = u^2\left(n - \frac{1}{2}n^2\right)$

$gH = u^2n\left(\frac{1}{2}n - 1\right)$

$2gH = u^2n(n-2)$

3. A mass `m' is supported by a massless string wound around a uniform ----

Ans: $\frac{g}{2}$

Sol: $\tau = RT = I\alpha = mR^2.\alpha$

$mg - T = ma$

Solving a = $\frac{g}{2}$

4 A block of mass m is placed on a surface with a vertical cross section given by $y = \frac{x^3}{6}$. If the coefficient of friction ----

Ans: $\frac{1}{6}$ m

Sol: $\frac{dy}{dx} = \tan\theta = \frac{x^2}{2} = 0.1 = \mu$

$x^2 = 1,\ x = 1$

$y = \frac{1}{6}$

5. When a rubber-band is stretched by a distance x, it exerts a restoring force of magnitude $F = ax + bx^2$ where a and b ----

Ans: $\frac{aL^2}{2} + \frac{bL^3}{3}$

Sol: $F = ax + bx^2$

$\int_0^L F dx = \int ax dx + \int bx^2 dx$

$= \left[\frac{ax^2}{2} + \frac{bx^3}{3}\right]_0^L$

$= \frac{aL^2}{2} + \frac{bL^3}{3}$

6. A bob of mass m attached to an inextensible string of length ℓ is suspended from a vertical support. The bob rotates in a ----

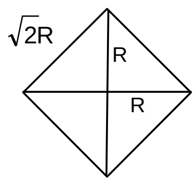
Ans: Angular momentum changes in direction but not in magnitude.

Sol: Direction of L changes but the magnitude remains to be constant.

7. Four particles, each of mass M and equidistant from each other, move along a circle of radius R ----

Ans: $\frac{1}{2}\sqrt{\frac{GM}{R}\left(1+2\sqrt{2}\right)}$

Sol: $\frac{GM^2}{4R^2} + 2.\frac{GM^2}{2R^2}.\frac{1}{\sqrt{2}} = \frac{Mv^2}{R}$



$\frac{GM^2}{R^2}\left[\frac{1}{4} + \frac{1}{\sqrt{2}}\right] = \frac{Mv^2}{R}$

$$\frac{1}{4}\frac{GM^2}{R^2}\left[1+2\sqrt{2}\right]=\frac{Mv^2}{R}$$

$$v = \frac{1}{2}\sqrt{\frac{GM}{R}\left(1+2\sqrt{2}\right)}$$

8. The pressure that has to be applied to the ends of a steel wire of length 10 cm to keep ----

Ans: 2.2×10^8 Pa

Sol: $\frac{F}{A} = Y\alpha\Delta t$

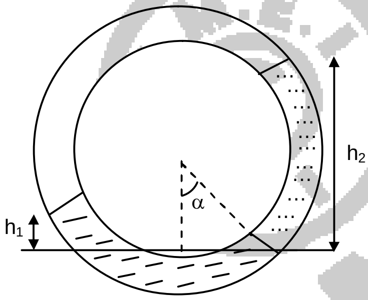
$= 2 \times 10^{11} \times 1.1 \times 10^{-5} \times 100$

$= 2.2 \times 10^8$ Pa

9. There is a circular tube in a vertical plane. Two liquids which do no mix and of densities d_1 and d_2 ----

Ans: $\frac{1+\tan\alpha}{1-\tan\alpha}$

Sol:



$$\frac{d_1}{d_2} = \frac{h_1}{h_2} = \frac{\cos(90-\alpha)-\cos\alpha}{\sin(90-\alpha)+\sin\alpha}$$

$$= \frac{\sin\alpha + \cos\alpha}{\sin\alpha - \cos\alpha}$$

$$= \frac{1+\tan\alpha}{1-\tan\alpha}$$

10. On heating water, bubbles being formed at the bottom of the vessel detach and rise. Take the bubbles ----

Ans: $R^2\sqrt{\frac{\rho_W g}{3T}}$

Sol: $2\pi r \times T \sin\theta = \frac{4}{3}\pi R^3 \rho_W g$

$2\pi r.T.\frac{r}{R} = \frac{4}{3}\pi R^3 \rho_W g$

$r^2 = \frac{\frac{4}{3}\pi R^4 \rho_W g}{2\pi T}$

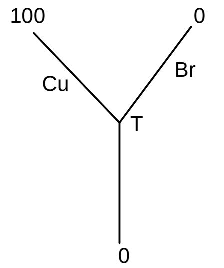
$r^2 = \frac{2}{3T}R^4 \rho_W g$

$r = R^2\sqrt{\frac{2}{3}\frac{\rho_W g}{T}}$

11. Three rods of Copper, Brass and Steel are welded together to form a Y-shaped structure. Are of cross-section ----

Ans: 4.8 cal s^{-1}

Sol:



$$\frac{(100-T)0.92}{46} = \frac{(T-0)0.26}{13} + (T-0)\frac{0.12}{13}$$

T = 40 °C

$$\left(\frac{d\theta}{dt}\right)_{Cu} = 0.92 \times 4 \times \frac{100-40}{46}$$

= 4.8 cal s^{-1}

12. One mole of diatomic ideal gas undergoes a cyclic process ABC as shown in figure. The process BC is adiabatic ----

Ans: The change in internal energy in the process BC is –500 R

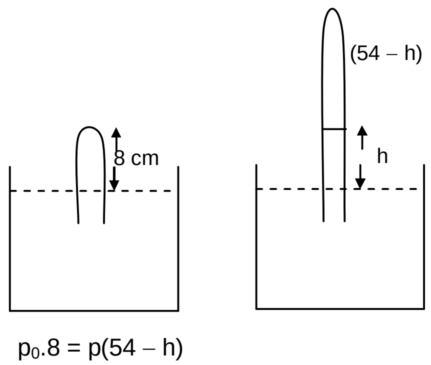
Sol: $dU_{BC} = 1 \times \frac{5R}{2} \times (600 - 800)$

= –500 R

13. An open glass tube is immersed in mercury in such a way that a length of 8 cm extends above the mercury level. The open end of the tube ----

Ans: +16 cm

Sol:



$p_0.8 = p(54 - h)$

$76 \times 8 = p(54 - h)$

$p + \rho gh = p_0$

$\frac{76 \times 8}{54-h} + h = 76$

Solving h = +38 cm

Air column = 54 – 38 = +16 cm

14. A particles moves with simple harmonic motion in a straight line. In first τ s, after starting ----

Ans: time period of oscillations is 6T

Sol: Considering the corresponding equivalent uniform circular motion 3T $\rightarrow$ half the time period $\rightarrow$ time period = 6T

15. A pipe of length 85 cm is closed from one end. Find the number of possible natural oscillations ----

Ans: 6

Sol: $\lambda = \frac{v}{f} = \frac{340}{1250} = \frac{34}{125}$

$\frac{\lambda}{4} = \frac{34}{500} = 0.068$

$\frac{3\lambda}{4} = 19.4$

$\frac{5\lambda}{4} = 32.0$

$\frac{7\lambda}{4} = 47.6$

$\frac{9\lambda}{4} = 62.2$

$\frac{11\lambda}{4} = 74.8$

n = 6

16. Assume that an electric field $\bar{E} = 30x^2\hat{i}$ exists in space. Then the potential ----

Ans: -80

Sol: $E = 30x^2\hat{i}$

$V = \int dV = \int -E.dx$

$= -\int 30x^2.dx$

$= -30\frac{x^3}{x} = -10x^3$

$V_A = -10.2^3 = -80$

$V_O = 0$

$V_A - V_O = -80$

17. A parallel plate capacitor is made of two circular plates separated by a distance of 5 mm and with a dielectric ----

Ans: 6.6×10^4 C m^{-2}

Sol: $\frac{\sigma}{\varepsilon_0 \varepsilon_r} = 3 \times 10^4$

$\sigma = \varepsilon_0\, \varepsilon_r \times 3 \times 10^4$

$= \frac{1}{4\pi} \times \frac{2.2}{9 \times 10^9} \times 3 \times 10^4$

$\cong 6.6 \times 10^4$ C m^{-2}

18. In a large building, there are 15 bulbs of 40 W, 5 bulbs of 100 W, 5 fans of 80 W and 1 heater of 1 kW. The voltage ----

Ans: 12 A

Sol: Bulbs (40 W) $I_1 = \frac{W.n}{V} = \frac{40}{220} \times 15 = \frac{600}{220}$

Bulbs (100 W) $I_2 = \frac{W.n}{V} = \frac{100 \times 5}{220} = \frac{500}{220}$

Fans (80 W) $I_3 = \frac{80 \times 5}{220} = \frac{400}{220}$

Heater (1000 W) $I_4 = \frac{1000 \times 1}{220} = \frac{1000}{220}$

$I = \frac{2500}{220} \cong 12$ A

19. A conductor lies along the z-axis at $-1.5 \le z < 1.5$ m and carries a fixed current of 10.0 A in ----

Ans: 2.97 W

Sol: $P = \frac{\text{work done}}{\text{time}}$

$= \frac{30 \times 10^{-4} \times 3 \times 10}{5 \times 10^{-3}} \int_0^2 e^{-0.2x} dx$

$e^{-0.4} \cong \frac{2}{3}$

Solving P = 2.97 W

20. The coercivity of a small magnet where the ferromagnet gets demagnetized is $e \times 10^3$ A m^{-1}. The current required ----

Ans: 3 A

Sol: $\frac{n}{\ell} i = 3 \times 10^3$

i = 3 A

21. In the circuit shown here, the point `C' is kept connected to point `A' till the current flowing through the ----

Ans: -1

Sol: $i = i_0 e^{-t/\tau}$ $\qquad t = \tau = \frac{L}{R}$

$i = i_0 e^{-1} = \frac{E}{R}.\frac{1}{e}$

$V_R = iR = \frac{E}{Re} \times R = \frac{E}{e}$ —(1)

$V_L = -L\frac{di}{dt}$

$\frac{di}{dt} = i_0\left[-\frac{t}{\tau} e^{\frac{-t}{\tau}}\right]$

$\frac{di}{dt} = -i_0\left(\frac{R}{L}.\frac{1}{e}\right)$

$$\therefore\ V_L = -L\left(-i_0\frac{R}{L}.\frac{1}{e}\right)$$

$$= \frac{E}{e}$$

$$\frac{V_R}{V_L} = 1$$

22. During the propagation of electromagnetic waves ----

Ans: Both electric and magnetic energy densities are zero.

Sol: Knowledge based.

23. A thin convex lens made from crown glass $\left[\mu = \frac{3}{2}\right]$ has coal length f. When it is measured in two different ----

Ans: $f_1 > f$ and f_2 becomes negative

Sol: $\mu_{lens} > \frac{4}{3}$, so $f_1 > f$

$\mu_{lens} < \frac{5}{3}$, so $f_2 \to -ve$

24. A green light is incident from the water to the air – water interface at the ----

Ans: The spectrum of visible light whose frequency is less than that of green light will come out to the air medium.

Sol: $\sin\theta_c = \frac{1}{n}$

When θ_c increases for smaller frequencies.

25. Two beams, A and B, of plane polarized light with mutually perpendicular planes of polarization are seen through a Polaroid. From the position ----

Ans: $\frac{1}{3}$

Sol: $I_A\cos^2 30 = I_B\cos^2 60$

$$\frac{I_A}{I_B} = \frac{\cos^2 60}{\cos^2 30} = \frac{1}{3}$$

26. The radiation corresponding to $3 \to 2$ transition of hydrogen atom falls on a metal surface to produce photoelectrons. These electrons ----

Ans: 1.1 eV

Sol: Energy of the transition

$$= 13.6\left[\frac{1}{4} - \frac{1}{9}\right]$$

$= 1.89$ eV

$$KE = \frac{1}{2}\frac{B^2q^2r^2}{m} = 0.8\ \text{eV}$$

$\therefore\ \phi = 1.89 - 0.8 = 1.1$ eV

27. Hydrogen ($_1H^1$), Deuterium ($_1H^2$), singly ionized Helium $(_2He^4)^+$ and doubly ionised lithium ----

Ans: $\lambda_1 = \lambda_2 = \lambda_3 = 9\lambda_4$

Sol: $\frac{1}{\lambda} \propto Z^2 \quad \therefore \lambda = \frac{1}{Z^2}$

$\lambda_1 \propto 1$

$\lambda_2 \propto 1$

$\lambda_3 \propto \frac{1}{4}$

$\lambda_4 \propto \frac{1}{9}$

$\frac{\lambda_1}{\lambda_3} = 4$

$\frac{\lambda_1}{\lambda_4} = 9$

$\therefore\ \lambda_1 = \lambda_2 = \lambda_3 = 9\lambda_4$

28. The forward biased diode connection is ----

Ans: 1

29. Match List - I (Electromagnetic wave type) with List - II (Its association / application) and select ----

Ans: (i) (ii) (iii) (iv)

30. A student measured the length of a rod and wrote it as 3.50 cm. Which instrument ----

Ans: A vernier caliper where the 10 divisions in the vernier scale matches with 9 division in main scale and main scale has 10 divisions in 1 cm.

Sol: LC for vernier = $\frac{1\ \text{MSD}}{\text{Number of VSD}}$

1 MSD = $\frac{1\ \text{cm}}{10}$ = 0.1 cm

LC = $\frac{0.1\ \text{cm}}{10}$ = 0.01 cm

Hence (2) is the answer

PART – B –CHEMISTRY

31. The correct set of four quantum numbers for the valence ----

Ans: 5, 0, 0, $+\frac{1}{2}$

Sol: Valence electron is $5s^1$

32. If Z is a compressibility factor, van der Waal's equation at low pressure ----

Ans: $Z = 1 - \frac{a}{VRT}$

Sol: At low pressures, $V - b \cong V$

$$\left(P + \frac{a}{V^2}\right)V = RT$$

$$Z = 1 - \frac{a}{RTV}$$

33. CsCl crystallises in body centred cubic lattice. ----

Ans: $r_{Cs} + r_{Cl^-} = \frac{\sqrt{3}\,a}{2}$

Sol: $2(r_{Cs} + r_{Cl^-}) = \sqrt{3}\,a$

$$r_{Cs} + r_{Cl^-} = \frac{\sqrt{3}\,a}{2}$$

34. For the estimation of nitrogen, 1.4 g of an organic compound was digested ----

Ans: 10%

Sol: % of N = $\frac{1.4 \times M \times 2(V - V_1/2)}{m}$

$$= \frac{1.4 \times 0.1 \times 2\left(60 - \frac{20}{2}\right)}{1.4}$$

$$= \frac{1.4 \times 0.1 \times 2 \times 50}{1.4}$$

= 10%

35. Resistance of 0.2 M solution of an electrolyte is 50 Ω ----

Ans: 5×10^{-4}

Sol: $\frac{\ell}{a} = \kappa \times R$

$= 1.4 \times 50 = 70\ m^{-1}$

$$\kappa = C \times \frac{\ell}{a} = \frac{1}{280} \times 70 = \frac{1}{4}\,S\,m^{-1}$$

$$\Lambda_m = \frac{\kappa}{1000M} = \frac{1}{4 \times 10^3 \times 0.5}$$

$= 5 \times 10^{-4}\ S\ m^2\ mol^{-1}$

36. For complete combustion of ethanol, ----

Ans: $-$ 1366.95 kJ mol^{-1}

Sol: $\Delta_c H = \Delta_c U + \Delta n_g RT$

$= -1364.47 - 1 \times 8.314 \times 10^{-3} \times 298$

$= -$ 1366.95 kJ mol^{-1}

37. The equivalent conductance of NaCl at concentration C and at infinte dilution ----

Ans: $\lambda_C = \lambda_\infty - B\sqrt{C}$

Sol: $\lambda_C = \lambda_\infty - B\sqrt{C}$

38. Consider separate solution of 0.500 M $C_2H_5OH(aq)$, 0.100 M $Mg_3(PO_4)_2(aq)$,----

Ans: They all have the same osmotic pressure

Sol: All the solution have the same effective concentration

39. For the reaction

$SO_{2(g)} + \frac{1}{2} O_{2(g)} \rightleftharpoons SO_{3(g)}$ ----

Ans: $-\frac{1}{2}$

Sol: $x = \Delta n_g$

$= 1 - 1\frac{1}{2}$

$= -\frac{1}{2}$

40. For the non-stoichiometre reaction $2A + B \rightarrow C + D$, the following kinetic data ----

Ans: $\frac{dc}{dt} = k[A]$

Sol: $\frac{dc}{dt} = k[A]$

[B] has no contribution to the rate of the reaction

41. Among the following oxoacids, the correct decreasing order of acid strength is:

Ans: $HClO_4 > HClO_3 > HClO_2 > HOCl$

Sol: The stability of the anions decreases in the order

$ClO_4^- > ClO_3^- > ClO_2^- > ClO^-$

$\therefore$ The acid strength decreases in the same order

42. The metal that cannot be obtained by electrolysis of an aqueous solution ----

Ans: Ca

Sol: Calcium can be obtained by the electrolysis of its fused salt

43. The octahedral complex of a metal ion M^{3+} with four monodentate ligands ----

Ans: $L_1 < L_3 < L_2 < L_4$

Sol: The wavelength absorbed will be in the opposite order of excitation energy as $E = \frac{hc}{\lambda}$. Greater the excitation energy, greater will be the crystal field splitting of the ligands. Hence the ligand strength order is
$L_1 < L_3 < L_2 < L_4$

44. Which one of the following properties is **not** shown by NO?

Ans: It is diamagnetic in gaseous state

Sol: It is an odd electron molecule (11electrons) and hence is paramagnetic in the gaseous state

45. In which of the following reactions H_2O_2 acts as a reducing agent?

Ans: (b), (d)

Sol: H_2O_2 is oxidised to O_2

46. The correct statement for the molecule, CsI_3 is:

Ans: It contains Cs^+ and I_3^- ions

Sol: CsI_3 exists as Cs^+ and I_3^-

47. The ratio of masses of oxygen and nitrogen in a particular gaseous mixture is 1 : 4. ----

Ans: 7 : 32

Sol: Mass ratio = 1 : 4
Mole ratio = $\frac{1}{32} : \frac{4}{28}$ = 7 : 32

48. Given below are the half-cell reaction:
$Mn^{2+} + 2e^- \rightarrow Mn$; $E° = -1.18$ V ----

Ans: −2.69V; the reaction will not occur

Sol: Cell is
$Mn^{2+} / Mn^{3+} // Mn^{2+} / Mn$
$E_{cell} = -1.51 - 1.18$
$= -2.69$ V
Since E_{cell} is negative, the reaction will not occur

49. Which series of reaction correctly represents chemical relations related to iron and its compound?

Ans: $Fe \xrightarrow{O_2, heat} Fe_3O_4 \xrightarrow{CO, 600°C} FeO \xrightarrow{CO, 700°C} Fe$

Sol: Finely divided pure iron burns in air or oxygen forming Fe_3O_4
$3Fe + 2O_2 \rightarrow F_3O_4$
At lower temperature range in the blast furnace,
$Fe_3O_4 + 4CO \rightarrow 3FeO + CO_2$
At higher temperature range,
$FeO + CO \rightarrow Fe + CO_2$

50. The equation which is balanced and represents the correct product(s) is:

Ans: $[CoCl(NH_3)_5]^+ + 5H^+ \rightarrow Co^{2+} + 5NH_4^+ + Cl^-$

Sol: This is the only reaction that represents the correct products as well as balanced in terms of atom and charge.

51. In S_N2 reactions, the correct order of reactivity for the following compounds is:

Ans: $CH_3Cl > CH_3CH_2Cl > (CH_3)_2CHCl > (CH_3)_3CCl$

Sol: Order of reactivity of alkyl halides in S_N2 reaction is:
methyl halide > 1° halide > 2° halide > 3° halide

52. On heating an aliphatic primary amine with chloroform and ethanolic potassium hydroxide----

Ans: an alkyl isocyanide

Sol: It is carbylamine reaction. Aliphatic primary amine gives alkyl isocyanide

53. The most suitable reagent for the conversion of $R-CH_2-OH \rightarrow R-CHO$ is: ----

Ans: PCC(Pyridinium chlorochromate)

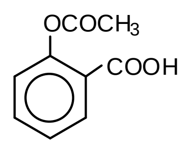
Sol: PCC is a mild oxidising agent which will oxidise 1° alcohols to aldehydes

54. The major organic compound formed by the reaction 1, 1, 1-trichloroethane with silver powder is ----

Ans: 2-Butyne

Sol: $2CH_3CCl_3 + 6Ag \rightarrow CH_3C{\equiv}CCH_3 + 6AgCl$

55. Sodium phenoxide when heated with CO_2 under pressure at 125°C yields ----

Ans:


Sol: Final product is aspirin (acetyl salicylic acid)

56. Considering the basic strength of amines in aqueous solution, which one has the smallest ----

Ans: $(CH_3)_2NH$

Sol: $(CH_3)_2NH$ is the strongest base among the given amines. It will have the smallest pK_b value

57. For which of the following molecules significant $\mu = 0$?

Ans: (c) and (d)

Sol: C–O–H and C–S–H bonds are angular and hence the given molecules have resultant dipole moment

58. Which one is classified as a condensation polymer?

Ans: Dacron

Sol: Dacron (nylon) is a condensation polymer

59. Which one of the following bases in **not** present in DNA? ----

Ans: Quinoline

Sol: Bases present in DNA are guanine, adenine, thymine and cytosine

60. In the reaction,

$CH_3COOH \xrightarrow{LiAlH_4} A \xrightarrow{PCl_5}$

$B \xrightarrow{Alc.KOH} C$, the product is: ----

Ans: Ethylene

Sol: $CH_3COOH \xrightarrow{LiAlH_4} \underset{(A)}{CH_3CH_2OH}$

$\xrightarrow{PCl_5} \underset{(B)}{CH_3CH_2Cl} \xrightarrow{alc.KOH}$

$\underset{(C)}{CH_2 = CH_2}$

PART – C – MATHEMATICS

61. If $X = \{4^n - 3n - 1 : n \in N\}$ and Y = ----

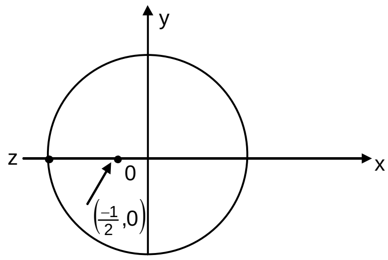
Ans: Y

Sol: $X = \{0, 9, 54, 243\text{-------}\}$
$Y = \{0, 9, 18, 27 \text{--------}\}$
$X \cup Y = Y$

62. If z is a complex numbers such that $|z| \geq 2$, ----

Ans: lies in the interval (1, 2)

Sol:



Minimum of $\left|z + \frac{1}{2}\right|$

= distance between $\frac{-1}{2}$ and z

= $\frac{3}{2}$ which lies between 1 and 2

63. If $a \in \mathbf{R}$ and the equation ----

Ans: $(-1, 0) \cup (0, 1)$.

Sol: $-3(x - [x])^2 + 2(x - [x]) + a^2 = 0$
$x - [x] = \{x\}$
$-3(\{x\})^2 + 2\{x\} + a^2 = 0$
$3\{x\}^2 - 2\{x\} - a^2 = 0$
$\{x\} = \frac{2 \pm \sqrt{4 + 12a^2}}{6}$
$0 < \{x\} < 1$
$\frac{1 + 3\sqrt{1 + 3a^2}}{3} < 1$
$1 + \sqrt{1 + 3a^2} < 3$
$1 + 3a^2 < 4$
$a^2 < 1$
$a \neq 0$
$a \in (-1, 0) \cup (0, 1)$.

64 Let α and β be the roots of equation ----

Ans: $\frac{2\sqrt{13}}{9}$

Sol: $2q = p + r \quad \frac{\alpha + \beta}{\alpha\beta} = 4 \Rightarrow \alpha + \beta = 4\alpha\beta$
$\frac{-q}{p} = 4.\frac{r}{p} \Rightarrow q = 4r$
$\therefore p = -9r$
$\therefore px^2 + qx + r = 0 \Rightarrow 9x^2 + 4x - 1 = 0$
$\therefore (\alpha - \beta)^2 = \frac{52}{81} \Rightarrow |\alpha - \beta| = \frac{2\sqrt{13}}{9}$

65. If $\alpha, \beta \neq 0$, and $f(n) = \alpha^n + \beta^n$ and ----

Ans: 1

Sol:

$$\alpha = \begin{vmatrix} 1+\alpha^\circ+\beta^\circ & 1+\alpha^1+\beta^1 & 1+\alpha^2+\beta^2 \\ 1+\alpha^1+\beta^1 & 1+\alpha^2+\beta^2 & 1+\alpha^3+\beta^3 \\ 1+\alpha^2+\beta^2 & 1+\alpha^3+\beta^3 & 1+\alpha^4+\beta^4 \end{vmatrix}$$

$$= \begin{vmatrix} 1 & \alpha & 1 \\ 1 & \alpha & \alpha^2 \\ 1 & \beta & \beta^2 \end{vmatrix}^2$$

$= [(1-\alpha)(1-\beta)(\alpha-\beta)]^2$

Then k = 1

66. If A is an 3×3 non- singular matrix such that AA' ----

Ans: I

Sol: $AA' = A^{-1}A'$, $B = A^{-1}A'$

$BB' = (A^{-1}A')(A^{-1}A')'$

$= (A^{-1}A')(A(A^{-1})')$

$= A^{-1}(A'A)(A^{-1})'$

$= (A^{-1}A)A'(A^{-1})'$

$= I\,A'(A')^{-1}$

$= I.\ \ I = I$

67. If the coefficients of x^3 and x^4 in the expansion ----

Ans: $\left(16, \dfrac{272}{3}\right)$

Sol: $(1+ ax + bx^2)\ (1-2x)^{18}$

$= (1+ ax + bx^2)\ (1- {}^{18}C_1(2x) + {}^{18}C_2 4x^2 - {}^{18}C_3\ 8x^3 + {}^{18}C_4\ (8x^4)$ -----)

Coefficient of $x^3 = 0 \Rightarrow$

$4a.\ {}^{18}C_2 - {}^{18}C_1\ 2b = {}^{18}C_3\ 8$

$\Rightarrow 51a - 3b = 544$ ———(1)

Coefficient of $x^4 = 0 \Rightarrow$

$2a\ {}^{18}C_3 - {}^{18}C_2\ b = 4 \times {}^{18}C_4$

$\Rightarrow 32a - 3b = 240$ ———(2)

Solving (1) and (2) we get

$a = 16 \quad b = \dfrac{272}{3}$

68. If $(10)^9 + 2(11)^1\ (10)^8 + 3(11)^2\ (10)^7 +$ ----

Ans: 100

Sol: a =10 b =11

The equation become

$9^9 + 2a^8 b + 3a^7 b^2 + \ldots\ldots + 10 b^9 = k a^9$

$\Rightarrow 1 + 2x + 3x^2 + \ldots. + 10x^9 = k,\ \dfrac{b}{a} = x$

$\Rightarrow k(1-x) = \dfrac{1-x^{10}}{1-x} = 10x^{10}$

i.e $\dfrac{-1}{10}k = \dfrac{1-x^{10}}{\frac{-1}{10}} - 10x^{10}$

$= -10 + 10x^{10} - 10x^{10}$

k = 100

69. Three positive numbers form an increasing G.P ----

Ans: $2+\sqrt{3}$

Sol: a, 2ar, ar^2 are in A. P

$\therefore 4ar = ar^2 + a$

$\Rightarrow r^2 - 4r + 1 = 0$

$\therefore (r-2)^2 = 3$

$r = 2+\sqrt{3}$

70. $\lim\limits_{x\to 0} \dfrac{\sin(\pi\cos^2 x)}{x^2}$ is equal to----

Ans: π

Sol: $\lim\limits_{x\to 0} \dfrac{\sin(\pi\cos^2 x)}{x^2}$

$= \lim\limits_{x\to 0} \dfrac{\sin(\pi(1-\sin^2 x))}{x^2}$

$= \lim\limits_{x\to 0} \dfrac{\sin(\pi - \pi\sin^2 x)}{x^2}$

$= \lim\limits_{x\to 0} \dfrac{\sin(\pi\sin^2 x)}{x^2}$

$= \lim\limits_{x\to 0} \dfrac{\sin(\pi\sin^2 x)}{\pi\sin^2 x} . \dfrac{\pi\sin^2 x}{\sin}$

$= \pi$

71. If g is the inverse of a function f and f'(x) ----

Ans: $1+\{g(x)\}^5$

Sol: $g(f(x)) = x \Rightarrow g^{-1}(x = f(x)$

$g'(f(x))\ f'(x) = 1$

$g'(f(x)) = \dfrac{1}{f'(x)}$

$g'(y) = \dfrac{1}{f'(x)}$ at $y = f(x)$

$x = f^{-1}(y)$

$= g(x)$

$= \dfrac{1}{f'(g(x_1))}$

$= 1 + (g(x))^5$

72. If f and g are differentiable functions in [0, 1] ----

Ans: $f'(c) = 2g'(c)$

Sol: $f(0) = 2 = g(1)\ g(0) = 0\ f(1) = 6$

$f'(c) = \dfrac{f(1)-f(0)}{1-0} = \dfrac{6-2}{1} = 4$

$$g(c) = \frac{g(1)-g(0)}{1-0} = \frac{2-1}{1} = 1$$

$$f'(c) = \frac{f(1)-f(0)}{1-0} = \frac{6-2}{1-0} = 4$$

$$g'(c) = \frac{g(c)-g(1)}{1-0} = \frac{2-0}{1-0} = 2$$

$$2g'(c) = f'(c)$$

73. If x = –1 and x = 2 are extreme points of ----

Ans: $\alpha = 2, \beta = \frac{-1}{2}$

Sol: $f'(x) = \frac{\alpha}{x} + 2\beta x + 1$

$$f''(-1) = \frac{-\alpha}{1} - 2\beta x + 1 = 0$$

$$f'(2) = \frac{\alpha}{2} + 4\beta + 1 = 0$$

$$-2\alpha - 4\beta + 2 = 0$$

$$\frac{\alpha}{2} + 4\beta + 1 = 0$$

$$-\frac{3\alpha}{2} + 3 = 0 \Rightarrow \alpha = 2$$

$$4\beta = -1 - \frac{\alpha}{2} = -2$$

$$\beta = \frac{-1}{2}$$

74. The integral $\int \left[1 + x - \frac{1}{x}\right] e^{x+\frac{1}{x}} dx$ is ----

Ans: $xe^{x+1/x} + c$

Sol: $\int \left(1 + X - \frac{1}{X}\right) E^{X+1/X} dn$

$$= \int 1.e^{x+1/x} dn + \int \left(x - \frac{1}{x}\right) e^{\left(x+1/x\right)} dn$$

$$= x e^{x+1/x} - \int x.e^{\left(x+1/x\right)} \left(1 - \frac{1}{x}\right) dn$$

$$+ \int \left(x - \frac{1}{x}\right) e^{x+1/x} dn$$

$$= xe^{x+1/x} + c$$

75. The integral ----

Ans: $4\sqrt{3} - 4 - \frac{\pi}{3}$

Sol: $\int_0^{\pi} \sqrt{\left(1 - 2\sin\frac{x}{2}\right)^2} dx$

$$= \int_0^{\frac{\pi}{3}} \left(1 - 2\sin\frac{x}{2}\right) dx$$

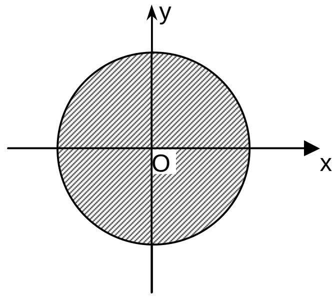
$$+ \int_{\frac{\pi}{3}}^{\pi} \left(2\sin\frac{x}{2} - 1\right) dx$$

$$= 4\sqrt{3} - 4 - \frac{\pi}{3}$$

76. The area of the region described by ----

Ans: $\frac{\pi}{2} + \frac{4}{3}$

Sol:



Required area

$$= \frac{\pi}{2} + 2 \int_{y=0}^{1} \left(1 - y^2\right) dy$$

$$= \frac{\pi}{2} + \frac{4}{3}$$

77. Let the population of rabbits surviving at a time t be ----

Ans: $400 - 300e^{\frac{t}{2}}$

Sol: $\frac{dp(t)}{dt} = \frac{1}{2} P^{(t)-200}, P^{(0)-100}$

$$\text{I.f} = e^{\int \frac{-1}{2} dt} = e^{\frac{-1}{2}t}$$

$$P^{e^{\frac{-1}{2}t}} = \int -200\, e^{\frac{-t}{2}} dt$$

$$P^{e^{\frac{-t}{2}}} = -200e^{\frac{-t}{2}} \times -2 + c$$

$$P(0) = 100$$

$$\Rightarrow 100 = 400 + c$$

$$\Rightarrow c = -300$$

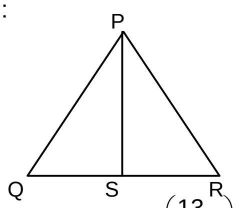
$$\therefore P^{e^{\frac{-t}{2}}} = 400e^{\frac{-t}{2}} - 300$$

$$\Rightarrow P = 400 - 300e^{\frac{t}{2}}$$

78. Let PS be the median of the triangle with vertices P(2,2) ----

Ans: $2x + 9y + 7 = 0$

Sol:



P(2, 2) S $\left(\frac{13}{2}, 1\right)$

Slope of PS = $\frac{-2}{9}$

$\therefore$ Equation of the passing through (1, –1) have slope = $\frac{-2}{9}$

$\therefore y + 1 = \frac{-2}{9}(x - 1)$

$\Rightarrow 2x + 9y + 7 = 0$

79. Let a, b, c and d be non- zero numbers. If the point of intersection ----

Ans: $3bc - 2ad = 0$

Sol: The intersecting point lies on the fourth quadrant it is of the form $(\alpha_1 - \alpha)$ where $\alpha > 0$. Substituting in the equation

$$\begin{cases} 4a\alpha - 2a\alpha + c = 0 \\ 5b\alpha - 2b\alpha + d = 0 \end{cases}$$

$$\begin{cases} 2a\alpha = c \\ 3b\alpha = d \end{cases}$$

Eliminating $\alpha \Rightarrow 2a\left(\frac{d}{3b}\right) = c$

$\Rightarrow 2ad = 3bc$

80. The locus of the foot of perpendicular drawn from the centre of the ellipse ----

Ans: $(x^2 + y^2) = 6x^2 + 2y^2$

Sol: $\frac{x^2}{6} + \frac{y^2}{2} = 1$

$\frac{x\cos\theta}{\sqrt{6}} + \frac{y\sin\theta}{\sqrt{2}} = 1 \Rightarrow$ tangent at 'θ'

Slope of the perpendicular from the origin

$$= \frac{\sqrt{3}\sin\theta}{\cos\theta}$$

$$\frac{-\cos\theta}{\sqrt{6}} \times \frac{\sqrt{2}}{\sin\theta}$$

$$= -\frac{\cos\theta}{\sqrt{3}\sin\theta}$$

Equation of the line through the origin $\perp$ to the tangent, is

$$y = \left(\frac{\sqrt{3}\sin\theta}{\cos\theta}\right)x \text{ ——— (2)}$$

Solve (1) and (2)

$$\frac{x\cos\theta}{\sqrt{6}} + \left(\frac{\sin\theta}{\sqrt{2}}\right)\frac{\sqrt{3}\sin\theta}{\cos\theta}x = 1$$

$$\left[\frac{\cos\theta}{\sqrt{6}} + \frac{\sqrt{3}\sin^2\theta}{\sqrt{2}\cos\theta}\right]x = 1$$

$$\left[\frac{\cos^2\theta + 3\sin^2\theta}{\sqrt{6}\cos\theta}\right]x = 1$$

$$x = \frac{\sqrt{6}\cos\theta}{\cos^2\theta + 3\sin^2\theta} = \frac{\sqrt{6}\cos\theta}{3 - 2\cos^2\theta}$$

$$y = \left(\frac{\sqrt{3}\sin\theta}{\cos\theta}\right)\frac{\sqrt{6}\cos\theta}{\cos^2\theta + 3\sin^2\theta}$$

$$= \frac{3\sqrt{2}\sin\theta}{3 - 2\cos^2\theta}$$

$$x^2 + y^2 = \frac{6\cos^2\theta + 18\sin^2\theta}{(3 - 2\cos^2\theta)^2}$$

$$= \frac{6(\cos^2\theta + 3\sin^2\theta)}{(3 - 2\cos^2\theta)^2}$$

$$= \frac{6}{3 - 2\cos^2\theta}$$

$$6x^2 = \frac{36}{(3 - 2\cos^2\theta)^2}$$

$$2y^2 = \frac{36\sin^2\theta}{(3 - 2\cos^2\theta)^2}$$

$$6x^2 + 2y^2 = \frac{36}{(3 - 2\cos^2\theta)^2}$$

focus is $(x^2 + y^2) = 6x^2 + 2y^2$

81. Let C be the circle with centre at (1, 1) and radius 1. If T is the circle centred at ----

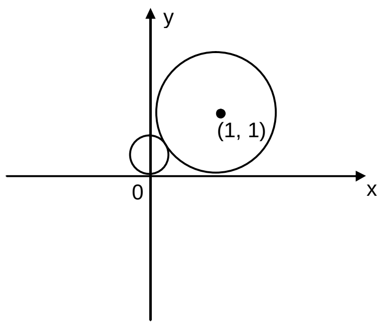
Ans: $\frac{1}{4}$

Sol: C: $(x - 1)^2 + (y - 1)^2 = 1$

$x^2 + y^2 - 2x - 2y + 1 = 0$

Let the centre of T be (0, k) since it passes through the origin, equation of T

$x^2 (y - k)^2 = k^2$



$$\left.\begin{array}{l}\text{Centre of T: } (0, k \\ \text{Centre of C } (1,1)\end{array}\right\}$$

$(x-1)^2 + (y-1)^2 = 1$
$x^2 + (y-k)^2 = k^2$

Since T and C touch externally, $C_1C_2 = r_1 + r_2$ where C_1, C_2 denote the centre of the circles

$(1 + k)^2 = (1 - 0)^2 + (1 - k)^2$
$1 + k^2 + 2k = 1 + 1 + k^2 - 2k$

$4k = 1,\ k = \frac{1}{4}$

Radius of the circle T = $\frac{1}{4}$

82. The slope of the line touching both the parabolas $y^2 = 4x$ ----

Ans: $\frac{1}{2}$

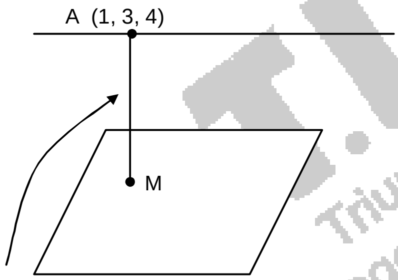
Sol: Equation of common tangent to
$y^2 = 4a^3 x$ and $x^2 = -4 b^3 y$ is
$ax - by + a^2b^2 = 0$
$4a^3 = 4 \qquad 4b^3 = 32$
$a = 1 \qquad b = 2$

Slope = $\frac{a}{b} = \frac{1}{2}$

83. The image of the line ----

Ans: $\frac{x+3}{3} = \frac{y-5}{1} = \frac{z-2}{-5}$

Sol: Line is parallel to the plane



$\frac{x-1}{2} = \frac{y-3}{-1} = \frac{z-4}{1} = R$

$(2R + 1, -R + 3, R + 4)$
$2(2R + 1) - (-R + 3) +$
$R + 4 - 3 = 0$
$\Rightarrow R = -1$
M is $(-1, 4, 3)$
Let the image of A is the plane be $A'(x, y, z)$

$\frac{x+1}{2} = -1, \frac{y+3}{2} = 4, \frac{z+4}{2} = 3$

$\Rightarrow$ A' is $(-3, 5, 2)$
Image is

$\frac{x+3}{3} = \frac{y-5}{1} = \frac{z-2}{-5}$

84. The angle between the lines whose direction cosines satisfy ----

Ans: $\frac{\pi}{3}$

Sol: $\ell + m + n = 0$ ——————(1)
$\ell^2 = m^2 + n^2$ ————————(2)
(2) is
$m^2 + n^2 + 2m\,n = m^2 + n^2$
$2\,m\,n = 0$
$m = 0$ or $n = 0$
$m = 0 \quad \ell^2 = n^2, \ell = \pm n$
D R S of the lines are
$[n, 0, n]$ or $[-n, 0, -n]$
If θ is the angle between the lines

$\cos\theta = \left|\frac{-n^2 - n^2}{\sqrt{(2n^2)(2n^2)}}\right|$

$= \left|\frac{-2n^2}{4n^2}\right| = \left|\frac{-1}{2}\right| = \frac{1}{2}$

$\theta = \frac{\pi}{3}$

85. If $\left[\bar{a}\times\bar{b}\ \ \bar{b}\times\bar{c}\ \ \bar{c}\times\bar{a}\right]^2$ then λ is ----

Ans: 1

Sol: $\left[a\times b,\ \ \bar{b}\times\bar{c}\ \ \ \bar{c}\times a\right] = \left[\bar{a}\ \ \ \bar{b}\ \ \ \bar{c}\right]^2$
$\Rightarrow \lambda = 1$

86. Let A and B be two events such that ----

Ans: Independent but not equally likely

Sol: $P(\overline{A\cup B}) = 1/6$ P(AB), $1/4 \quad P(\bar{A}) = 1/4$

$P(\bar{A}) = \frac{1}{4} \Rightarrow P(A) = \frac{3}{4}$

$P(\overline{A\cup B}) = \frac{1}{6} \Rightarrow P(A\cup B) = \frac{5}{6}$ let P(B) = x

$\frac{5}{6} = \frac{3}{4} + x - \frac{1}{4}$

$\Rightarrow x = \frac{1}{3}$

$\Rightarrow P(A) = \frac{1}{4} \qquad P(B) = \frac{1}{3}$

$P(A\cap B) = \frac{3}{4}$

A and B are independent but not equal likely

87. The variance of first 50 even natural numbers is : ----

Ans: 833

Sol: $S.D^2 = \frac{n^2-1}{3} = \frac{50^2-1}{3}$

$= \frac{2500-1}{3} = \frac{2499}{3}$

$= 833$

88. Let $f_k(x) = \frac{1}{k}\left(\sin^k x + \cos^k x\right)$ where----

Ans: $\frac{1}{12}$

Sol: $f_4(x) = \frac{1}{4}\left(\sin^4 x + \cos^4 x\right)$

$= \frac{1}{4}\left\{1 - 2\sin^2 x \cos^2 x\right\}$

$= \frac{1}{4}\left\{1 - \frac{1}{2}\sin^2 2x\right\}$

$f_6(x) = \frac{1}{6}\left(\sin^6 x + \cos^6 x\right)$

$= \frac{1}{6}\left\{1 - 3\sin^2 x \cos^2 x\right\}$

$= \frac{1}{6}\left\{1 - \frac{3}{4}\sin^2 2x\right\}$

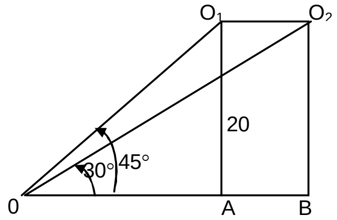
$f_4(x) - f_6(x) = \frac{1}{4} - \frac{1}{6} - \frac{1}{8}\sin^2 2x + \frac{1}{8}\sin^2 x$

$= \frac{1}{12}$

89. A bird is sitting on the top of a vertical pole 20 m high ----

Ans: $20\left(\sqrt{3}-1\right)$

Sol:



O_2 is the position of the bird after 1 second. We have OA = 20

$\frac{20}{OB} = \tan 30° = \frac{1}{\sqrt{3}}$

$OB = 20\sqrt{3}$

Speed = AB = $20\left(\sqrt{3}-1\right)$

90. The statement $\sim(p \leftrightarrow \sim q)$ is: ----

Ans: equivalent to $\sim p \leftrightarrow q$

Sol:

p q	~ q	$p \leftrightarrow \sim q$	$\sim(p \leftrightarrow \sim q)$	$p \leftrightarrow q$
T T	F	F	T	T
T F	T	T	F	F
F T	F	T	F	F
F F	T	F	T	T