# SOLUTIONS & ANSWERS FOR KERALA ENGINEERING ENTRANCE EXAMINATION-2017 – PAPER II VERSION – B1

## [MATHEMATICS]

**1.** Ans: $q - p$

Sol: $C_2 \Rightarrow C_2 - C_1$
$C_3 \Rightarrow C_3 - C_1$

$$\begin{vmatrix} 1 & 0 & 0 \\ p & q-p & r-p \\ p & q-p & r-p+1 \end{vmatrix}$$

$(q - p)(r - p + 1) - (r - p)(q - p)$
$= (q - p)(r - p)(r - p + 1 - 1)$
$= (q - p)(r - p)$

$$(q-p)\begin{vmatrix} 1 & 0 & 0 \\ p & 1 & r-p \\ p & 1 & r-p+1 \end{vmatrix}$$

$(q - p)(r - p + 1 - r + p) = q - p.$

**2.** Ans: $\begin{bmatrix} 20 & 5 \\ -1 & 0 \end{bmatrix}$

Sol: $\Rightarrow \begin{bmatrix} 20 & 0 \\ 4 & 0 \end{bmatrix} + \begin{bmatrix} 0 & 5 \\ -5 & 0 \end{bmatrix} - C = 0$

$C = \begin{bmatrix} 20 & 5 \\ -1 & 0 \end{bmatrix}.$

**3.** Ans: $U^T$

Sol: $U^{-1} = \begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{bmatrix} = U^T.$

**4.** Ans: $A^T$

Sol: $\text{adj}(A) = \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$|A| = -1$
$\therefore A^{-1} = A^T.$

**5.** Ans: 0, 0, 1

Sol: $x + y = 0 = x - y$
$x - y \Rightarrow x = 0, y = 0$
$\Rightarrow z = 1.$

**6.** Ans: $\begin{pmatrix} 27 \\ 16 \end{pmatrix}$

Sol: It is $\begin{pmatrix} 22 \\ 16 \end{pmatrix} + \begin{pmatrix} 5 \\ 0 \end{pmatrix} = \begin{pmatrix} 27 \\ 16 \end{pmatrix}.$

**7.** Ans: $a = 6$

Sol: The det = 0

$$\Rightarrow \begin{vmatrix} 1 & 2 & 4 \\ 0 & 1 & 1 \\ 0 & 1 & a-5 \end{vmatrix} = 0$$

$\Rightarrow a - 5 - 1 = 0 \Rightarrow a = 6.$

**8.** Ans: (6, –1, 1)

Sol: The equation gives $z = 1$
$4y + 5 = 1 \Rightarrow 4y = -4 \Rightarrow y = -1$
$x - 2 - 3 = 1 \Rightarrow x = 6$
solution is (6, –1, 1).

**9.** Ans: $A^2 - 3A + 2I = 0$

Sol: A "satisfies" $\begin{vmatrix} 1-\lambda & 5 \\ 0 & 2-\lambda \end{vmatrix} = 0$

i.e $\lambda^2 - 3\lambda + 2 = 0$
$\therefore A^2 - 3A + 2I = 0.$

**10.** Ans: 0, 1, 0, 0

Sol: $p - q = 0 = p + q \Rightarrow p = 0 = q$
$2x + y = 1 = x + y \Rightarrow x = 0, y = 1$

**11.** Ans: 2

Sol: It is $\left|\sqrt{3}+1\right| - \left|\sqrt{3}-1\right| = \sqrt{3}+1-\sqrt{3}+1 = 2$.

**12.** Ans: –1

Sol: It is $2^2 - 2 - 3 = -1$

**13.** Ans: $(x - 2)(4x - 6)$

Sol: $16 - 28 + q = 0 \Rightarrow q = 12$
$y = 4x^2 - 14x + 12$
$= 2(2x^2 - 7x + 6)$
$= 2(x - 2)(2x - 3)$
$= (x - 2)(4x - 6).$

**14.** Ans: $\frac{40}{9}$

Sol: $x_1^2 + x_2^2 = (x_1 + x_2)^2 - 2x_1x_2$

$= \left(\frac{2}{3}\right)^2 - 2\left(\frac{-6}{3}\right)$

$= \frac{4}{9} + 4 = \frac{40}{9}$.

**15.** Ans: $-2$ or $2$

Sol: $10 = x_1^2 + x_2^2 = (x_1 + x_2)^2 - 2x_1x_2$

$= p^2 + 6 \Rightarrow p^2 = 4$

$\Rightarrow p = \pm 2$

**16.** Ans: $\frac{1}{3}$

Sol: $\frac{2m-1}{m} = -1 \Rightarrow 3m = 1 \Rightarrow m = \frac{1}{3}$.

**17.** Ans: 4

Sol: $f(x) = \frac{1}{(x+2)^2} - \frac{4}{x^2(x+2)^2} + \frac{4}{x^2(x+2)}$

$= \frac{x^2 - 4 + 4(x+2)}{x^2(x+2)^2}$

$= \frac{x^2 + 4x + 4}{x^2(x+2)^2}$

$= \frac{1}{x^2}$

$f\left(\frac{1}{2}\right) = 4$.

**18.** Ans: b

Sol: x, y are roots of $x^2 + bx + 1 = 0$

$\Rightarrow$ x + b, y + b are roots of

$(x - b)^2 + b(x - b) + 1 = 0$

i.e., x + b, y + b are roots of

$x^2 - bx + 1 = 0$

$\therefore \frac{1}{x+b} + \frac{1}{y+b} = b$.

**19.** Ans: $-2$

Sol: If $\alpha$ is the common root,

$\alpha^5 + a\alpha + 1 = 0$

$\alpha^6 + a\alpha^2 + 1 = 0$

$\therefore \alpha^6 + a\alpha^2 + \alpha = 0 = \alpha^6 + a\alpha^2 + 1$

$\Rightarrow \alpha = 1$

$\therefore 1 + a + 1 = 0 \Rightarrow a = -2$.

**20.** Ans: $\frac{1}{4}$

Sol: The roots are equal

$\therefore 1 - 4a = 0$

$\Rightarrow a = \frac{1}{4}$.

**21.** Ans: 6

Sol: $z^2 + z + 1 = 0$

$z = \omega, \omega^2$

$\therefore z + \frac{1}{z} = \omega + \omega^2 = -1$

$z^2 + \frac{1}{z^2} = \omega + \omega^2 = -1$

$z^3 + \frac{1}{z^3} = 1 + 1 = 2$

$\therefore$ Exp = 1 + 1 + 4 = 6.

**22.** Ans: $3w^2$

Sol: $\begin{vmatrix} 1 & 1 & 1 \\ 1 & w & w^2 \\ 1 & w & w \end{vmatrix}$

$C_2 \to C_2 - C_1$

$C_3 \to C_3 - C_1$

$\begin{vmatrix} 1 & 0 & 0 \\ 1 & w-1 & w^2-1 \\ 1 & w-1 & w-1 \end{vmatrix}$

$\begin{vmatrix} 1 & 0 & 0 \\ 0 & w-1 & w^2-1 \\ 1 & w-1 & w-1 \end{vmatrix}$

$(w - 1)^2 - (w^2 - 1)(w - 1)$

$= (w - 1)(w - 1 - w^2 + 1)$

$= (w - 1)(w - w^2)$

$= w^2 - 1 - w + w^2$

$= 2w^2 + w^2 = 3w^2$

**23.** Ans: x = 0, y = 0

Sol: $9i \times I \begin{vmatrix} 3i & -1 & -i \\ 2 & 1 & i \\ 10 & -i & -i \end{vmatrix} = 0 = x + iy$

$\therefore x = 0, \quad y = 0$.

**24.** Ans: 0

Sol: $z = \frac{1}{2} - i\frac{\sqrt{3}}{2} = -\left(-\frac{1}{2} + i\frac{\sqrt{3}}{2}\right) = -\omega$

$\therefore z^7 - z + 1$

$\omega^7 + \omega + 1 = 0$.

**25.** Ans: 1

Sol: $\left(\frac{2\cos^2\frac{\pi}{24} + 2i\sin\frac{\pi}{24}\cos\frac{\pi}{24}}{2\cos^2\frac{\pi}{24} - 2i\sin\frac{\pi}{24}\cos\frac{\pi}{24}}\right)^{72}$

$= \left(\dfrac{\cos\frac{\pi}{24} + i\sin\frac{\pi}{24}}{\cos\frac{\pi}{24} - i\sin\frac{\pi}{24}}\right)^{72}$

$= \left(\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}\right)\left(\cos\left(\frac{-\pi}{3}\right) - i\sin\left(\frac{\pi}{3}\right)\right)$

$= \cos\left(\frac{2\pi}{3}\right) + i\sin\left(\frac{2\pi}{3}\right)$

= 1.

**26.** Ans: 8

Sol: $4K^2 = 256$
$K^2 = 64$
$\therefore |K| = 8.$

**27.** Ans: $I + nA$

Sol: $A^2 = \begin{bmatrix}1 & 0\\ 1 & 1\end{bmatrix}\begin{bmatrix}1 & 0\\ 1 & 1\end{bmatrix}$

$= \begin{bmatrix}1 & 0\\ 2 & 1\end{bmatrix}$

$A^3 = \begin{bmatrix}1 & 0\\ 2 & 1\end{bmatrix}\begin{bmatrix}1 & 0\\ 1 & 1\end{bmatrix}$

$= \begin{bmatrix}1 & 0\\ 3 & 1\end{bmatrix}$

$\therefore A^n = \begin{bmatrix}1 & 0\\ n & 1\end{bmatrix}$

$\therefore A^n + nI = \begin{bmatrix}1 & 0\\ n & 1\end{bmatrix} + \begin{bmatrix}n & 0\\ 0 & n\end{bmatrix}$

$= \begin{bmatrix}I+n & 0\\ n & n+1\end{bmatrix}$

$= I + nA.$

**28.** Ans: 0

Sol: $z = 5e^{i\theta}$

$\therefore w = \dfrac{5(e^{i\theta} - 1)}{5(e^{i\theta} + 1)} = \dfrac{(e^{i\theta} - 1)(e^{-i\theta} + 1)}{2(1 + \cos\theta)}$

$= \dfrac{e^{i\theta} - e^{-i\theta}}{2(1 + \cos\theta)} = \dfrac{i\sin\theta}{1 + \cos\theta}$

Re w = 0.

**29.** Ans: $2^{2016}A$.

Sol: $A = \begin{pmatrix}1 & 1\\ 1 & 1\end{pmatrix}$

$\Rightarrow A^2 = \begin{pmatrix}1 & 1\\ 1 & 1\end{pmatrix}\begin{pmatrix}1 & 1\\ 1 & 1\end{pmatrix} = \begin{pmatrix}2 & 2\\ 2 & 2\end{pmatrix}$

$\therefore A^2 = 2A$

$A^3 = \begin{pmatrix}2 & 2\\ 2 & 2\end{pmatrix}\begin{pmatrix}1 & 1\\ 1 & 1\end{pmatrix} = \begin{pmatrix}4 & 4\\ 4 & 4\end{pmatrix} = 2^2A$

$\therefore A^n = 2^{n-1}A$
$\Leftrightarrow A^{2017} = 2^{2016}A.$

**30.** Ans: $i\cot\frac{\theta}{2}$

Sol: $a = \cos\theta + i\sin\theta$

$\therefore \dfrac{1+a}{1-a} = \dfrac{1 + \cos\theta + i\sin\theta}{1 - \cos\theta - i\sin\theta}$

$= \dfrac{2\cos^2\theta/2 + i2\sin\theta/2\cos\theta/2}{2\sin^2\theta/2 - i2\sin\theta/2\cos\theta/2}$

$= \dfrac{2\cos\theta/2\left[\cos\theta/2 + i\sin\theta/2\right]}{2\sin\theta/2\left[\sin\theta/2 - i\cos\theta/2\right]}$

$= \dfrac{\cot\theta/2\left(\cos\theta/2 + i\sin\theta/2\right)}{-i\left[\cos\theta/2 + i\sin\theta/2\right]}$

$= -\dfrac{1}{i}\cot\theta/2$

$= i\cot\theta/2.$

**31.** Ans: 21

Sol: $x + y + z = -3$
In AP $\Rightarrow y = -1 \Rightarrow x + z = -2$
$xz = -8$
$x^2 + y^2 + z^2 = x^2 + z^2 + 1$
$\therefore x^2 + z^2 = (x + z)^2 - 2xz$
$= 4 - 2(-8) = 20$
$\therefore x^2 + y^2 + z^2 = 21.$

**32.** Ans: –77

Sol: 10, 7, 4
$d = -3$
$t_{30} = 10 + 29(-3) = 10 - 87 = -77.$

**33.** Ans: 34

Sol: $\dfrac{x+y}{2} = 3$

$\Rightarrow x + y = 6$

$\sqrt{xy} = 1$

$\Rightarrow xy = 1$

$\therefore x^2 + y^2 = (x + y)^2 - 2xy$
$= 36 - 2 = 34.$

**34.** Ans: $\dfrac{5}{6}$

Sol: $3^{2x-1} = 81^{1-x}$

$3^{2x-1} = 3^{4(1-x)} = \dfrac{3^4}{3^x}$

$\dfrac{3^{2x}}{3} = \dfrac{3^4}{3^{4x}} \Leftrightarrow 3^{6x} = 3^5$

$6x = 5 \Rightarrow x = \dfrac{5}{6}.$

**35.** Ans: $\frac{1}{81}$

Sol: $3, 1, \frac{1}{3}$..... GP

ratio = $\frac{1}{3}$

$t_6 = ar^5 = 3 \times \frac{1}{3^5} = \frac{1}{81}$.

**36.** Ans: 315

Sol: x, y, z in AP

$x + y + z = 21 \Rightarrow y = 7$

Also $xz = 45$

$\therefore xyz = 315$.

**37.** Ans: 2

Sol: $(2a + 1) – (a + 1) = (4a – 1) – (2a + 1)$

$a = 2a – 2 \Rightarrow a = 2$.

**38.** Ans: $x^2 – 18x + 16 = 0$

Sol: $\frac{x+y}{2} = 9$  $\sqrt{xy} = 4$

$\therefore x + y = 18$  $xy = 16$

Equation whose roots are x and y can be written as

$x^2 – (18)x + 16 = 0$.

**39.** Ans: $\frac{1}{2}$

Sol: P(at least 2 tails)

= 1 – (P (exactly one tail) + P(no tails))

$= 1 - \left[{}^3C_1 \times \left(\frac{1}{2}\right)^3 + \left(\frac{1}{2}\right)^3\right]$

$= 1 - \frac{1}{2} = \frac{1}{2}$.

**40.** Ans: $\frac{1}{3}$

Sol: $P(T) = \frac{1}{6}$; $P(R) = \frac{1}{6}$

P(T or R is selected) = $\frac{1}{6} + \frac{1}{6} = \frac{1}{3}$.

**41.** Ans: $\frac{9}{21}$

Sol: 4R, 2W, 4B

four balls out of 10

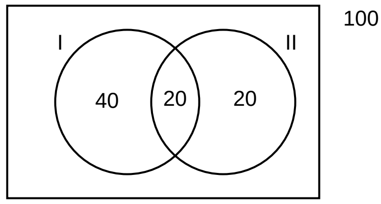
$\Rightarrow {}^{10}C_4 = 210$

2 Red balls $\Rightarrow {}^4C_2 \times {}^6C_2 = 6 \times 15 = 90$

Required probability = $\frac{9}{21}$.

**42.** Ans: $\frac{1}{5}$

Sol:




From the Venn diagram it is clear that required answer is $\frac{20}{100}$

i.e $\frac{1}{5}$

Aliter : $P(I \cup II) = P(I) + P(II) – P(I \cap II)$

$= \frac{60}{100} + \frac{40}{100} - \frac{20}{100} = \frac{80}{100}$

$\therefore P\left[(I \cup II)'\right] = \frac{20}{100} = \frac{1}{5}$.

**43.** Ans: $\frac{2}{5}$

Sol: 1, 2, 3, 4, 5

A.M is an integer $\Rightarrow \frac{x+y}{2} \in Z$

$\Rightarrow$ x + y divisible by 2

$\Rightarrow$ both odd or both even

$\Rightarrow$ 4 cases

$5C_2 = 10$

$\therefore$ Required probability $\frac{4}{10} = \frac{2}{5}$.

**44.** Ans: $2^9$

Sol: 9 elements are three in a $3 \times 3$ matrix,

Entries can be –1 or +1 $\Rightarrow$ 2 ways

$\therefore$ each element can be entered in 2 ways and 9 elements $\Rightarrow 2^9$.

**45.** Ans: $\frac{1}{2}$

Sol: Symmetric matrix can be of the form

$\begin{pmatrix} x & 0 \\ 0 & y \end{pmatrix}$ OR $\begin{pmatrix} x & 1 \\ 1 & y \end{pmatrix}$

Number of symmetric matrices using 0, 1is

4 + 4 = 8

From the first type $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ is the only non singular matrix.

From the second type

$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \begin{pmatrix} 1 & 1 \\ 1 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 1 \\ 1 & 1 \end{pmatrix}$ are the required ones

$\therefore$ Probability = $\frac{4}{8} = \frac{1}{2}$.

**46.** Ans: 7!

Sol: Seven distinct letters to be permitted
$\Rightarrow$ 7!.

**47.** Ans: 9

Sol: Number of diagonals = $^nC_2 - n$
Hexagon $\Rightarrow n = 6$
$\therefore$ Required answer = $^6C_2 - 6 = 9$.

**48.** Ans: $1001^2$

Sol: Sum of n consecutive odd integers = $n^2$
$2001 = 2(1001) - 1$
$\therefore 1 + 3 + 5 + \ldots\ldots + 2001 = 1001^2$.

**49.** Ans: $\frac{1}{6}$

Sol: $^4C_2 = 6$
P(no black ball) = P(2 red balls)
$= \frac{1}{6}$.

**50.** Ans: 0 + 0i

Sol: $z = i^9 + i^{19}$
$= (i^4)^2 i + (i^4)^4 . i^3$
$= i + i^3$
$= i - i = 0$
$\Rightarrow$ 0 + 0i is the correct option.

**51.** Ans: 9

Sol: Mean = $\frac{\text{sum}}{8} = \frac{72}{8} = 9$.

**52.** Ans: $(-\infty, 3)$

Sol: $x - 2 < 1 \Rightarrow x < 3$
Required option is $(-\infty, 3)$.

**53.** Ans: $x \in (3, \infty)$

Sol: clearly $x > 3$.

**54.** Ans: 8

Sol: mode = 8.

**55.** Ans: 246
Sol: $\sum x = n \times \bar{x} = 6 \times 41 = 246$

**56.** Ans: 2 + e

Sol: $\int_0^x f(t)dt = x^2 + e^x$
$f(x) = 2x + e^x$
$\therefore f(1) = 2 + e$.

**57.** Ans: $\frac{2}{3}x^{3/2} + 2x^{1/2} + C$.

Sol: $\int \frac{x+1}{x^{1/2}} dx = \int \left(\sqrt{x} + x^{-1/2}\right) dx$
$= \frac{2}{3}x^{3/2} + 2x^{1/2} + C$.

**58.** Ans: 60

Sol: $n(A \cup B) = 50 + 20 - 10 = 60$

**59.** Ans: x

Sol: $f(f(x)) = \frac{\left(\frac{x+1}{x-1}\right) + 1}{\left(\frac{x+1}{x-1}\right) - 1}$
$= \frac{(x+1) + (x-1)}{(x+1) - (x-1)}$
$= \frac{2x}{2}$
$= x$.

**60.** Ans: $\frac{3}{4}$

Sol: P = 1 – P(product odd)
$= 1 - \frac{3.3}{6.6} = 1 - \frac{1}{4} = \frac{3}{4}$.

**61.** Ans: $\frac{1}{\sqrt{2}}$

Sol: $\lim_{x \to 0} \left[\frac{\sqrt{2+x} - \sqrt{2-x}}{x}\right]$
$\lim_{x \to 0} \left[\frac{\frac{1}{2\sqrt{2+x}} + \frac{1}{2\sqrt{2-x}}}{1}\right]$
$= \frac{1}{2\sqrt{2}} + \frac{1}{2\sqrt{2}} = \frac{1}{\sqrt{2}}$

**62.** Ans: $\frac{-1}{\left(e^x+1\right)}+C$

Sol: $\int \frac{dx}{e^x+e^{-x}+2} = \int \frac{e^x dx}{\left(e^x\right)^2+2e^x+1}$

$= \int \frac{e^x}{\left(e^x+1\right)^2} dx = \int \frac{dt}{t^2} = \frac{-1}{t}+C$

$= \frac{-1}{\left(e^x+1\right)}+C$

**63.** Ans: $2\sec\theta$

Sol: $S = \frac{1+\tan\frac{\theta}{2}}{1-\tan\frac{\theta}{2}} + \frac{1-\tan\frac{\theta}{2}}{1+\tan\frac{\theta}{2}}$

$= \frac{\left(1+\tan\frac{\theta}{2}\right)^2+\left(1-\tan\frac{\theta}{2}\right)^2}{1-\tan^2\frac{\theta}{2}}$

$= \frac{2\left(1+\tan^2\frac{\theta}{2}\right)}{\left(1-\tan^2\frac{\theta}{2}\right)} = \frac{2}{\cos\theta} = 2\sec\theta$

**64.** Ans: No answer

Sol: $\int_{-1}^{0} \frac{dx}{x^2+x+2}$

$\int_{-1}^{0} \frac{1}{x^2+x+2} dx = \int_{-1}^{0} \frac{1}{\left(x+\frac{1}{2}\right)^2-\frac{1}{4}+2} dx$

$= \int_{-1}^{0} \frac{dx}{\left(x+\frac{1}{2}\right)^2+\left(\frac{\sqrt{7}}{2}\right)^2} = \frac{2}{\sqrt{7}}\left[\tan^{-1}\left(\frac{x+\frac{1}{2}}{\frac{\sqrt{7}}{2}}\right)\right]_{-1}^{0}$

$= \frac{2}{\sqrt{7}}\left[\tan^{-1}\left(\frac{2x+1}{\sqrt{7}}\right)\right]_{-1}^{0}$

$= \frac{2}{\sqrt{7}}\left[\tan^{-1}\left(\frac{1}{\sqrt{7}}\right)-\tan^{-1}\left(\frac{-1}{\sqrt{7}}\right)\right]$

$= \frac{4}{\sqrt{7}}\tan^{-1}\left(\frac{1}{\sqrt{7}}\right)$

No correct option is given

**65.** Ans: $\frac{\pi}{4}$

Sol: $I = \int_0^{\frac{\pi}{2}} \frac{\sqrt{\sin x}}{\sqrt{\sin x}+\sqrt{\cos x}} dx$

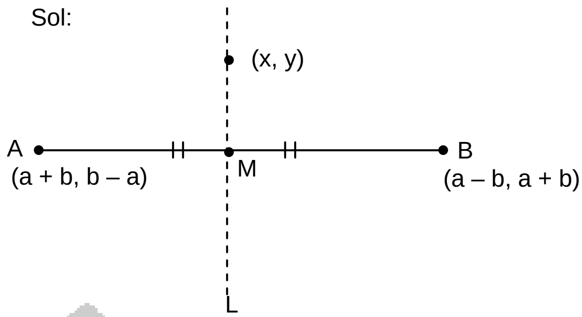
$= \int_0^{\frac{\pi}{2}} \frac{\sqrt{\cos x}}{\sqrt{\cos x}+\sqrt{\sin x}}$

$2I = \int_0^{\frac{\pi}{2}} 1 dx = \frac{\pi}{2}$

$I = \frac{\pi}{4}$

**66.** Ans: bx – ay = 0

Sol:




Slope of AB $= \frac{(a+b)-(b-a)}{(a-b)-(a+b)} = \frac{2a}{-2b} = \frac{-a}{b}$

$\therefore$ slope of L $= \frac{b}{a}$

$\therefore$ Equal point M

$= \left(\frac{a+b+a-b}{2}, \frac{b-a+a+b}{2}\right)$

= (a, b)

$\therefore$ Equation of L is $(y-b) = \frac{b}{a}(x-a)$

$\Rightarrow ay - ab = bx - ab$

$\Rightarrow bx - ay = 0$

**67.** Ans: –7

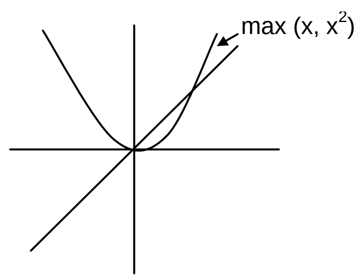
Sol: Equating slopes,

$\frac{1-0}{0-1} = \frac{8-1}{x-0}$

$-1 = \frac{7}{x} \Rightarrow x = -7$

**68.** Ans: 0

Sol:



Clearly minimum value of maximum $(x, x^2)$ $= 0$

**69.** Ans: 33

Sol: $f'(3) = \lim_{h\to 0} \frac{f(3+h)-f(3)}{h}$

$= \lim_{h\to 0} \left[\frac{f(3)\,f(h) - f(3)}{h}\right]$

$= \lim_{h\to 0} f(3)\times\left[\frac{f(h)-1}{h}\right]$

$= 3 \times f'(0) = 3 \times 11 = 33$

**70.** Ans: No answer

Sol: Question is wrong

**71.** Ans: $\sqrt{2}\cos x$

Sol: $\cos\left(\frac{\pi}{4}+x\right)+\cos\left(\frac{\pi}{4}-x\right)$

$= 2\cos\left(\frac{\frac{\pi}{4}+x+\frac{\pi}{4}-x}{2}\right)\cos\left(\frac{\frac{\pi}{4}+x-\frac{\pi}{4}+x}{2}\right)$

$= 2\cos\frac{\pi}{4}\cos x = \sqrt{2}\cos x$

**72.** Ans: $\frac{33}{2}$

Sol: $A = \left|\frac{1}{2}\begin{vmatrix} x_1-x_2 & x_2-x_3 \\ y_1-y_2 & y_2-y_3 \end{vmatrix}\right|$

$= \left|\frac{1}{2}\begin{vmatrix} -3 & -5 \\ -3 & 6 \end{vmatrix}\right| = \left|\frac{1}{2}(-18-15)\right| = \frac{33}{2}$

**73.** Ans: $5x - y + 20 = 0$

Sol: Slope of the given line $= \frac{1-0}{-4-1} = \frac{-1}{5}$

Slope of the required line = 5

$\therefore$ Required line is $(y - 5) = 5(x + 3)$

$\Rightarrow 5x + 15 = y - 5$

$\Rightarrow 5x - y + 20 = 0$

**74.** Ans: 60

Sol: $(1 + x^2)^5\ (1 + x)^4$

$= \left(1+{}^5C_1x^2+{}^5C_2x^4+{}^5C_3x^8+....\right)$

$\left(1+{}^4C_1x+{}^4C_2x^2+{}^4C_3x^3+{}^4C_4x^4\right)$

Coefficient of $x^5 = ({}^5C_1)\ ({}^4C_3) + ({}^5C_2)\ ({}^4C_1)$

$= (5)\ (4) + (10)\ (4) = 60$

**75.** Ans: 80

Sol: $T_{r+1} = {}^5C_r\ (-2x)^r = {}^5C_r(-2)^r\ x^r$

$\therefore$ Coefficient of $x^4 = {}^5C_4(-2)^4$

$= 5 \times 16 = 80$

**76.** Ans: an ellipse

Sol: $5x^2 + y^2 + y + \frac{1}{4} - \frac{1}{4} = 8$

$\Rightarrow 5x^2 + \left(y+\frac{1}{2}\right)^2 = \frac{33}{4}$

$\Rightarrow \frac{5x^2}{\left(\frac{33}{4}\right)} + \frac{\left(y+\frac{1}{2}\right)^2}{\left(\frac{33}{4}\right)} = 1$

$\therefore$ Ellipse

**77.** Ans: $(1, -2)$

Sol: $(4x^2 - 8x) + (y^2 + 4y) = 8$

$\Rightarrow 4(x^2 - 2x + 1 - 1) + y^2 + 4y + 4 - 4 = 8$

$\Rightarrow 4(x - 1)^2 - 4 + (y + 2)^2 - 4 = 8$

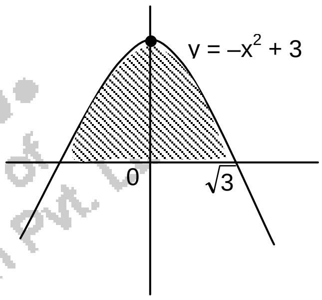
$\Rightarrow 4(x - 1)^2 + (y + 2)^2 = 16$

$\Rightarrow \frac{(x-1)^2}{4} + \frac{(y+2)^2}{16} = 1$

$\therefore$ centre $(1, -2)$

**78.** Ans: $4\sqrt{3}$

Sol:




$A = 2\int_0^{\sqrt{3}} (3 - x^2)\,dx = 2\left[3x - \frac{x^3}{3}\right]_0^{\sqrt{3}}$

$= 2\left(3\sqrt{3} - \frac{3\sqrt{3}}{3}\right) = 2\times\frac{2}{3}\times 3\sqrt{3}$

$= 4\sqrt{3}$

**79.** Ans: 3

Sol: order 3

**80.** Ans: 0

Sol: $f'(x) = \sqrt{2}\ \frac{1}{2\sqrt{x}} + 2\sqrt{2}\left(\frac{-1}{2x\sqrt{x}}\right)$

$= \frac{1}{\sqrt{2x}} - \frac{\sqrt{2}}{x\sqrt{x}}$

$f'(2) = \frac{1}{\sqrt{4}} - \frac{\sqrt{2}}{2\sqrt{2}} = \frac{1}{2} - \frac{1}{2} = 0$

**81.** Ans: 1

Sol: $x^2 + y^2 - 2x - 10y + k = 0$

$r = \sqrt{g^2 + f^2 - c} = \sqrt{1 + 25 - k}$

$A = \pi r^2 = \pi(26 - k) = 25\pi$

$\Rightarrow k = 1$

**82.** Ans: $\frac{1}{2}$

Sol: $I = \int_{2016}^{2017} \frac{\sqrt{x}}{\sqrt{x} + \sqrt{4033 - x}}$

$= \int_{2016}^{2017} \frac{\sqrt{4033 - x}}{\sqrt{x} + \sqrt{4033 - x}}$

$2I = \int_{2016}^{2017} 1dx = 1 \Rightarrow I = \frac{1}{2}$

**83.** Ans: $\Rightarrow$ ysecx = tanx

Sol: If = $e^{\int \tan x\, dx} = \sec x$

$\therefore$ ysecx = $\int \sec^2 x dx$

$\Rightarrow$ ysecx = tanx + C

put x = 0 and y = 0,

0 = 0 + C $\Rightarrow$ C = 0

$\Rightarrow$ ysecx = tanx

**84.** Ans: 2

Sol: $\begin{vmatrix} 2 & 2 & 6 \\ 2 & \lambda & 6 \\ 2 & -3 & 1 \end{vmatrix} = 0$

$2(\lambda + 18) - 2(2 - 12) + 6(-6 - 2\lambda) = 0$

$\lambda = 2$

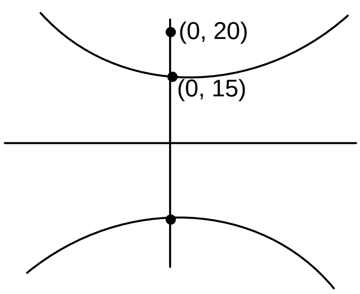
**85.** Ans: $\frac{10}{3}$

Sol: $d = \left|\frac{2(2) + 1 + 2(0) + 5}{\sqrt{2^2 + 1^2 + 2^2}}\right|$

$= \frac{10}{3}$

**86.** Ans: $\frac{y^2}{225} - \frac{x^2}{175} = 1$

Sol:




$\frac{y^2}{a^2} - \frac{x^2}{b^2} = 1$

a = 15

c = 20 = $\sqrt{225 + b^2}$

$400 = 225 + b^2$

$b^2 = 175$

$\therefore \frac{y^2}{225} - \frac{x^2}{175} = 1$

**87.** Ans: $\frac{27}{7}$

Sol: Let 15 = a and 6 = b

$\Rightarrow \frac{a^3 + b^3 + 3ab(a + b)}{1 + {}^4C_1 b + {}^4C_2 b^2 + {}^4C_3 b^3 + {}^4C_4 b^4}$

$= \frac{(a + b)^3}{(1 + b)^4} = \frac{21^3}{7^4} = \left(\frac{21}{7}\right)^3 \times \frac{1}{7}$

$= \frac{27}{7}$

**88.** Ans: x + 2y – 4z + 7 = 0

Sol: $\begin{vmatrix} x - 1 & y - 0 & z - 2 \\ -2 & 1 & 0 \\ 4 & 0 & 1 \end{vmatrix} = 0$

$\Rightarrow (x - 1)(1 - 0) - y(-2 - 0) + (z - 2)(0 - 4) = 0$

$\Rightarrow x - 1 + 2y - 4z + 8 = 0$

$\Rightarrow x + 2y - 4z + 7 = 0$

**89.** Ans: (–1, 2)

Sol: $(y - 2)^2 - 4 - x + 3 = 0$

$\Rightarrow (y - 2)^2 = x + 1$

$\Rightarrow (y - 2)^2 = 4\left(\frac{1}{4}\right)(x + 1)$

vertex = (–1, 2)

**90.** Ans: $\frac{\pi}{3}$

Sol: $\bar{b} + \bar{c} = -\bar{a}$

$(\bar{b} + \bar{c})^2 = (-\bar{a})^2$

$\bar{b}^2 + \bar{c}^2 + 2\bar{b}.\bar{c} = \bar{a}^2$

$25 + 9 + 2(5)(3)\cos\theta = 49$

$30\cos\theta = 15$

$\cos\theta = \frac{1}{2}$

$\theta = \frac{\pi}{3}$

**91.** Ans: $\sqrt{2}$

Sol: $y = 2x^3 - 9ax^2 + 12a^2x + 1$

$$\frac{dy}{dx} = 6x^2 - 18ax + 12a^2$$
$= 6(x^2 - 3ax + 2a^2)$
$= 6(x - 2a)(x - a) = 0$
$\Rightarrow x = a$ or $x = 2a$

$$\frac{d^2y}{dx^2} = 12x - 18a$$

If x = a, $\frac{d^2y}{dx^2} = 12a - 18a < 0$ max

If x = 2a, $\frac{d^2y}{dx^2} = 12(2a) - 18a > 0$ min

$\therefore$ 2a = q and a = p

$p^3 = q \Rightarrow a^3 = 2a \Rightarrow a^2 = 2 \Rightarrow a = \sqrt{2}$

**92.** Ans: $7 \times 50 \times 101$

Sol: f(1) = 7
f(2) = f(1 + 1) = f(1) + f(1) = 2(7)
f(3) = f(2 + 1) = f(2) + f(1) = 3(7)
etc

$$\therefore \sum_{r=1}^{100} f(r) = f(1) + f(2) + f(3) + ..... + f(100)$$

$= (1 + 2 + 3 .... + 100)7 = \frac{100 \times 101}{2} \times 7$
$= 7 \times 50 \times 101$

**93.** Ans: $\frac{1}{\sqrt{2}}$

Sol: $\frac{(x-1)^2}{2} + \frac{(4y+3)^2}{16} = \frac{1}{16}$

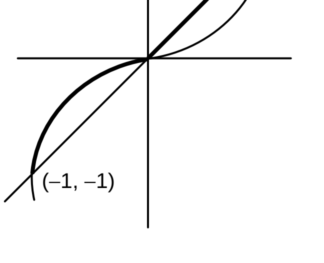
$\Rightarrow 8(x - 1)^2 + \frac{(4y+3)^2}{1} = 1$

$$\Rightarrow \frac{(x-1)^2}{\left(\frac{1}{8}\right)} + \frac{\left(y + \frac{3}{4}\right)^2}{\left(\frac{1}{16}\right)} = 1$$

$$e = \sqrt{1 - \frac{b^2}{a^2}} = \sqrt{1 - \frac{\frac{1}{16}}{\frac{1}{8}}} = \frac{1}{\sqrt{2}}$$

**94.** Ans: $\frac{1}{4}$

Sol:




$$\int_{-1}^{1} \max(x, x^3) = \int_{-1}^{0} x^3 dx + \int_{0}^{1} x dx$$

$$= \left[\frac{x^4}{4}\right]_{-1}^{0} + \left[\frac{x^2}{2}\right]_{0}^{1} = \frac{1}{4}(0 - 1) + \frac{1}{2} = \frac{1}{4}$$

**95.** Ans: $\frac{\pi}{2}$

Sol: x = 90°, y = 0°
x + y = $\frac{\pi}{2}$

**96.** Ans: 4

Sol: a(a + r) (a + 2r) = 64; a constant
$\therefore$ sum = a + (a + r) + (a + 2r)
= 3a + 3r is minimum
when a = a + r = a + 2r ($\because$ numbers are +ve reals)
$\Rightarrow$ r = 0
$\Rightarrow$ a = 4
$\therefore$ minimum value of a + 2r is 4

**97.** Ans: $\frac{1}{10!} 2^9$

Sol: $S = \frac{1}{10!}\left[{}^{10}C_1 + {}^{10}C_3 + {}^{10}C_5 + {}^{10}C_7 + {}^{10}C_9\right]$

$= \frac{1}{10!} 2^9$

**98.** Ans: $6x^2$

Sol: $f(x) = x(12x^2 - 6x^2) - 1(6x^3 - 2x^3)$
$= 12x^3 - 6x^3 - 6x^3 + 2x^3$
$= 2x^3$
$f'(x) = 6x^2$

**99.** Ans: $\frac{1}{3}\tan^{-1}(x^3) + C$

Sol: $\int \frac{x^2}{1 + (x^3)^2} dx$ $u = x^3$

$du = 3x^2 dx$

$$= \int \frac{\frac{1}{3} du}{1 + u^2} = \frac{1}{3}\tan^{-1}(u) + C$$

$$= \frac{1}{3}\tan^{-1}(x^3) + C$$

**100.** Ans: 3

Sol: $f(x) = x^2 + e^x$
$f_1(x) = 2x + e^x$
$f_2(x) = 2 + e^x$
$f_3(x) = e^x$
$f_4(x) = e^x$
$\therefore f_n = f_{n+1}$
least value of n is 3

**101.** Ans: $\frac{1}{\sqrt{2}}$

Sol: $\sin 765° = \sin(4\pi + 45°)$

$= \sin 45° = \frac{1}{\sqrt{2}}$

**102.** Ans: $\frac{3}{5}$

Sol: distance $\frac{\left|3\times3-4\times^{-}5-26\right|}{\sqrt{3^2+(-4)^2}}$

$= \frac{9+20-26}{\sqrt{9+16}} = \frac{3}{5}$

**103.** Ans: $\frac{59}{6}$

Sol: $f'(x) = x^2 + x + 1 = x^2 + x + \frac{1}{4} + \frac{3}{4}$

$= \left(x+\frac{1}{2}\right)^2 + \frac{3}{4} > 0$

f(x) is increasing in [2, 3]

∴ minimum is at x = 2

maximum is at x = 3

Difference between maximum and minimum is

$\int_2^3 (x^2+x+1)dx = \left[\frac{x^3}{3}+\frac{x^2}{2}+x\right]_2^3$

$= \left(9+\frac{9}{2}+3\right)-\left(\frac{8}{3}+2+2\right)$

$= \left(\frac{18+9+6}{2}\right)-\left(\frac{8+6+6}{3}\right)$

$= \frac{33}{2}-\frac{20}{3} = \frac{99-40}{6} = \frac{59}{6}$

**104.** Ans: $\frac{-9}{4}$

Sol: $\alpha + \beta = a + b = -a$

$\Rightarrow b = -2a$

'$\alpha\beta$' = $a \times b = b$

$ab = b$

$b(a-1) = 0 \Rightarrow a = 1$ & $b = -2$

Equation is $x^2 + x - 2 = 0$

$f(x) = x^2 + x - 2$

$= x^2 + x + \frac{1}{4} - 2 - \frac{1}{4}$

$= \left(x+\frac{1}{2}\right)^2 - \frac{9}{4}$

∴ minimum value is $\frac{-9}{4}$

**105.** Ans: $\pm\sqrt{65}$

Sol: $y = 4x + c$; $\frac{x^2}{4}+\frac{y^2}{1}=1$

$y = mx + c$ is a tangent to $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$

$\Rightarrow c^2 = a^2m^2 + b^2$

$c^2 = 4 \times 4^2 + 1 = 65$

$c = \pm\sqrt{65}$

**106.** Ans: $\lambda = -1, 4$

Sol: $\lambda x - y - 2 = 0$

$2x - 3y + \lambda = 0$

$3x - 2y + 1 = 0$ are consistent

$$\begin{vmatrix} \lambda & -1 & -2 \\ 2 & -3 & \lambda \\ 3 & -2 & 1 \end{vmatrix} = 0$$

$\Rightarrow \lambda(-3 + 2\lambda) + 1(2 - 3\lambda) - 2(-4 + 9) = 0$

$-3\lambda + 2\lambda^2 + 2 - 3\lambda - 10 = 0$

$2\lambda^2 - 6\lambda - 8 = 0$

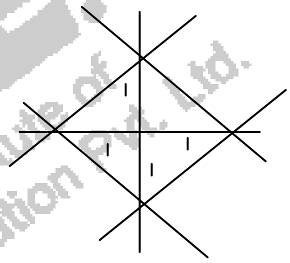
$\lambda^2 - 3\lambda - 4 = 0$

$(\lambda + 1)(\lambda - 4) = 0$

$\lambda = -1, 4$

**107.** Ans: a square

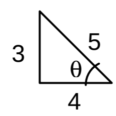
Sol:



set {(x, y) : |x| + |y| = 1] in the xy – plane represent the lines x + y = 1; x – y = 1; –x + y = 1 and –x – y = 1 which is a square

**108.** Ans: $\frac{4}{5}$

Sol: $\cos\left(\tan^{-1}\left(\frac{3}{4}\right)\right)$




$\tan\theta = \frac{3}{4}$

$\therefore \cos\theta = \frac{4}{5}$

**109.** Ans: –3, 5

Sol: A(6, –1) B(1, 3) C(x, 8)

AB = BC

$\sqrt{(6-1)^2+(-1-3)^2} = \sqrt{(x-1)^2+(8-3)^2}$

$\sqrt{25+16} = \sqrt{(x-1)^2 + 25}$

$25 + 16 = (x-1)^2 + 25$

$(x-1)^2 = 16$

$x - 1 = \pm 4$

$\therefore x = 4 + 1; -4 + 1$

$x = 5, -3$

**110.** Ans: 78

Sol: $\sum x^2 = 2830; \sum x = 170, \ n = 15$

Correct sum = 170 – 20 + 30 = 180

Correct $\sum x^2 = 2830 - 20^2 + 30^2$

= 2830 – 400 + 900

= 2830 + 500

= 3330

New variance = $\dfrac{\sum x^2}{n} - \left(\dfrac{\sum x}{n}\right)^2$

$= \dfrac{3330}{15} - \left(\dfrac{182}{15}\right)^2$

= 222 – 144 = 78

**111.** Ans: $\cos^{-1}\left(\dfrac{26}{9\sqrt{38}}\right)$

Sol: lines are $\dfrac{x-2}{2} = \dfrac{y-1}{5} = \dfrac{z+3}{-3}$

$\dfrac{x+2}{-1} = \dfrac{y-4}{8} = \dfrac{z-5}{4}$

$\cos\theta = \dfrac{2 \times {}^{-}1 + 5 \times 8 + {}^{-}3 \times 4}{\sqrt{2^2 + 5^2 + (^{-}3)^2}\sqrt{(-1)^2 + 8^2 + 4^2}}$

$= \dfrac{-2 + 40 - 12}{\sqrt{38}\ \sqrt{81}} = \dfrac{26}{\sqrt{38}\ 9}$

$\theta = \cos^{-1}\left(\dfrac{26}{9\sqrt{38}}\right)$

**112.** Ans: $\sqrt{13}$

Sol: $|\bar{a}| = 1 \ (\bar{x} - \bar{a}) \bullet (\bar{x} + \bar{a}) = 12$

$x^2 - a^2 = 12$

$\therefore x^2 = 12 + 1 = 13 \quad x = \sqrt{13}$

**113.** Ans: 8

Sol: y = 2x + 1; y = 3x + 1 ; x = 4

Solving when x = 4

y = 9 and y = 13

solving y = 2x + 1 &

<u>y = 3x + 1</u>

x = 0; y = 1

$\therefore$ vertices are (0, 1) (4, 9) and (4, 13)

Area = $\dfrac{1}{2}\begin{vmatrix} 0 & 1 & 1 \\ 4 & 9 & 1 \\ 4 & 13 & 1 \end{vmatrix}$

$= \dfrac{1}{2}[0 - 1(4-4) + 1(52-36)]$

$= \dfrac{1}{2} \times {}^{-}16$

= –8

Area = 8 square units

**114.** Ans: 3

Sol: ${}^nC_{r-1} = 36$ —— (1)

${}^nC_r = 84$ —— (2)

${}^nC_{r+1} = 126$ —— (3)

$\dfrac{(1)}{(2)} \Rightarrow \dfrac{r!\ (n-r)!}{(r-1)\ (n-r+1)!} = \dfrac{36}{84}$

$\Rightarrow \dfrac{r}{n-r+1} = \dfrac{3}{7}$

7r = 3n – 3r + 3

3n – 10r = –3 ———— (4)

$\dfrac{(2)}{(3)} \Rightarrow \dfrac{r!\ (n+r)!\ (n-r-1)!}{r!(n-r)!} = \dfrac{84}{126}$

$\dfrac{r+1}{n-r} = \dfrac{2}{3}$

3r + 3 = 2n – 2r

2n – 5r = 3 ——— (5)

(5) × 2 ⇒ 4n – 10r = 6 —— (6)

<u>–3n +10r = 3 — (4)</u>

n = 9

substituting in (4)

27 –10r = –3

10r = 30

r = 3

**115.** Ans: 3f(x) g(0)

Sol: $f'(x) = \lim_{h\to 0}\left(\dfrac{f(x+h) - f(x)}{h}\right)$

$= \lim_{h\to 0} f(x) \lim_{h\to 0}\left(\dfrac{f(h) - 1}{h}\right)$

$f(x) \lim_{h\to 0}\left(\dfrac{f(h) - f(0)}{h}\right)$

= f(x) f'(0)

= f(x) × 3g(0)

[since f(0) = 1

f(x) = 1+ sin(3x) g(x)

f '(x) = sin3x g '(x) + 3cos3x g(x)

f '(0) = 3g(0)]

**116.** Ans: 1, 2

Sol: $\begin{vmatrix} x-1 & 1 & 1 \\ 1 & x-1 & 1 \\ 1 & 1 & x-1 \end{vmatrix} = 0$

$R_1 \to R_1 + R_2 + R_3$

$\Rightarrow \begin{vmatrix} x+1 & x+1 & x+1 \\ 1 & x-1 & 1 \\ 1 & 1 & x-1 \end{vmatrix} = 0$

$$\Rightarrow (x+1)\begin{vmatrix}1 & 1 & 1\\ 1 & x-1 & 1\\ 1 & 1 & x-1\end{vmatrix}=0$$

$c_2 \to c_2 - c_1 \quad c_3 \to c_3 - c_1$

$$\Rightarrow (x+1)\begin{vmatrix}1 & 0 & 0\\ 1 & x-2 & 0\\ 1 & 0 & x-2\end{vmatrix}=0$$

$\Rightarrow (x + 1)\,(x - 2)^2 = 0$

$x = -1, 2$

**117.** Ans: No answer

Sol: $T_{r+1} = (-1)^r\ {}^nC_r\,(2a)^{n-r}\,(3b)^r$

$T_7 = {}^nC_6\,(2a)^{n-6}\,(3b)^6$

$T_8 = -{}^nC_7\,(2a)^{n-7}\,(3b)^7$

$$\frac{T_7}{T_8}=\frac{{}^nC_6(2a)^{n-6}(3b)^6}{-{}^nC_7(2a)^{n-7}(3b)^7}=1$$

$$\Rightarrow \frac{\frac{n!}{6!\,(n-6)!}}{\frac{-n!}{7!\,(n-7)!}}\ \frac{2a}{3b}=1$$

$$\Rightarrow \frac{-7}{n-6}\times\frac{2a}{3b}=1$$

$$\Rightarrow \frac{2a}{3b}=\frac{6-n}{7}$$

$$\Rightarrow \frac{2a+3b}{2a-3b}=\frac{6-n+7}{6-n-7}$$

$\therefore\ \dfrac{2a+3b}{2a-3b}=\dfrac{n-13}{n+1}$ if $T_7$ and $T_8$ are equal

This answer is NOT in the given options.

Even when we assume $T_7$ and $T_8$ to be numerically equal,

$$\frac{2a}{3b}=\frac{n-6}{7}\Rightarrow\frac{2a+3b}{2a-3b}=\frac{n+1}{n-13}$$

This answer is also NOT in the given option.

Question is ambiguous. No correct option.

**118.** Ans: $\sqrt{\dfrac{n^2-1}{3}}$

Sol: 1, 3, 5 ........, 2n – 1

$$\sigma=\sqrt{\frac{\sum x_i^2}{n}-\left(\frac{\sum x_i}{n}\right)^2}$$

$$\sum x_i^2=\sum(2n-1)^2=\sum\left(4n^2-4n+1\right)$$

$$=\frac{4\times n(n+1)(2n+1)}{6}-4\times\frac{n(n+1)}{2}+n$$

$$=\frac{n}{3}\left(4n^2-1\right)$$

$$\sum x_i=\sum(2n-1)=n^2$$

$$\therefore\ \sigma=\sqrt{\frac{4n^2-1}{3}-n^2}$$

$$=\sqrt{\frac{n^2-1}{3}}$$

**119.** Ans: 31

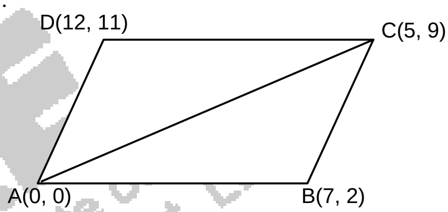
Sol: odd no: 's are 1, 3, 5, 7, 9

No. of subsets with only odd no:'s is

$2^5 - 1 = 31$

**120.** Ans: 53

Sol:




Area of the parallelogram

$= 2 \times$ Area of $\Delta ABC$

$$=2\times\frac{1}{2}\begin{vmatrix}0 & 0 & 1\\ 7 & 2 & 1\\ 5 & 9 & 1\end{vmatrix}$$

= 53