T.I.M.E.
Triumphant Institute of
Management Education Pvt. Ltd.

MODEL SOLUTIONS TO IIT JEE ADVANCED 2015

Paper II – Code 0

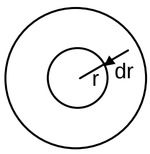
PART I

1	2	3	4	5	6	7	8
6	**3**	**2**	**2**	**1**	**2**	**7**	**4**

9	10	11	12
B	**D**	**B, C**	**A, B**
13	14	15	16
D	**A, C**	**B, D**	**A**
17	18	19	20
A, C	**D**	**A, D**	**A, C**

Section I

1.



Sphere A

$dm = 4\pi r^2 dr\ \rho(r)$

$= 4\pi r^2 \frac{kr}{R} dr = \frac{4\pi k r^3 dr}{R}$

$dI = \frac{2}{3}(dm) r^2 = \frac{8\pi}{3}\frac{k}{R}.r^5 dr$

$I_A = \int dI = \frac{8\pi}{3}\frac{k}{R}\int_0^R r^5 dr = \frac{8\pi k}{18}.R^5$

Sphere B

$dm = (4\pi r^2 dr)\ \rho(r)$

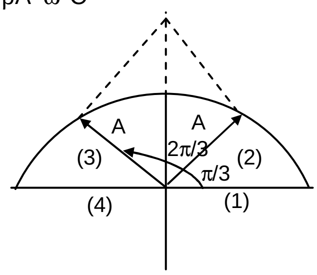
$= 4\pi r^2 \frac{kr^5}{R^5} dr = \frac{4\pi k}{R^5}.r^7 dr$

$dI = \frac{2}{3}(dm) r^2 = \frac{8\pi}{3}\frac{k}{R^5}.r^9 dr$

$I_B = \int dI = \frac{8\pi k}{3R^5}.\left.\frac{r^{10}}{10}\right]_0^R = \frac{8\pi k}{30R^5}.R^{10} = \frac{8\pi k}{30}.R^5$

$\frac{I_A}{I_B} = \frac{8\pi k R^5}{30} \times \frac{18}{8\pi k.R^5} = \frac{18}{30} = \frac{n}{10} \Rightarrow n = 6$

2. $I = \frac{1}{2}\rho A^2\omega^2 C$



Phasor diagram,

$A_R = 2A \sin 60° = \sqrt{3}\ A$

$I_R = \frac{1}{2}\rho A_R^2\omega^2 C = \frac{1}{2}\rho 3A^2\omega^2 C$

$= 3\left(\frac{1}{2}\rho A^2\omega^2 C\right) = 3I_0 = nI_0 \Rightarrow n = 3$

3. $R = \frac{dA}{dt} = \frac{d}{dt}(\lambda N) = \frac{\lambda dN}{dt} = \lambda^2 N$

Given: $\lambda_P N_0 = \lambda_Q N_0$

$\Rightarrow \frac{N_{0P}}{\tau_P} = \frac{N_{0Q}}{\tau_Q} \Rightarrow \frac{N_{OP}}{N_{OQ}} = \frac{\tau_P}{\tau_Q} = \frac{\tau}{2\tau} = \frac{1}{2}$

$\therefore\ \dfrac{R_P}{R_Q} = \dfrac{\lambda_P^2 N_P}{\lambda_Q^2 N_Q} = \dfrac{\tau_Q^2}{\tau_P^2}\dfrac{N_P}{N_Q}$

$= \dfrac{\tau_Q^2}{\tau_P^2}\cdot\dfrac{N_{OP}e^{-t/\tau_P}}{N_{OQ}e^{-t/\tau_Q}}$

$= \dfrac{\tau_Q^2}{\tau_P^2}\cdot\dfrac{\tau_P}{\tau_Q}.e^{-t\left(\frac{1}{\tau_P}-\frac{1}{\tau_Q}\right)}$

$= \dfrac{\tau_Q}{\tau_P}.e^{-2\tau\left(\frac{1}{\tau}-\frac{1}{2\tau}\right)}$

$= \dfrac{\tau_Q}{\tau_P}.e^{-1} = \dfrac{\tau_Q}{\tau_P.e} = \dfrac{2}{e}$

$\Rightarrow n = 2$

4. $\sin 60 = n \sin r_1 \Rightarrow \sin r_1 = \dfrac{\sqrt{B}}{2n}$

$\Rightarrow \cos r_1 = \dfrac{\sqrt{4n^2-3}}{2n}$

$\Rightarrow \dfrac{\sqrt{3}}{2} = n \sin r_1$

$\sin\theta = n \sin(60 - r_1)$

$= n\left(\dfrac{\sqrt{3}}{2}\cos T_1 - \dfrac{1}{2}\sin r_1\right)$

$= n\left(\dfrac{\sqrt{3}}{2}\dfrac{\sqrt{4n^2-3}}{2n} - \dfrac{1}{2}.\dfrac{\sqrt{3}}{2n}\right)$

$\Rightarrow \sin\theta = \left(\dfrac{\sqrt{3}}{4}\sqrt{4n^2-3} = \dfrac{\sqrt{3}}{4}\right)$

$\Rightarrow$ differentiating,

$\cos\theta.\dfrac{d\theta}{dn} = \dfrac{\sqrt{3}}{4}.\dfrac{1}{2}.\dfrac{8n}{\sqrt{4n^2-3}}$

At $\theta = 60°$

$\dfrac{m}{2} = \dfrac{\sqrt{3}n}{\sqrt{4n^2-3}}$

$\Rightarrow m = \dfrac{2\sqrt{3}n}{\sqrt{4n^2-3}},\quad n = \sqrt{3}$

$\Rightarrow m = \dfrac{6}{3} = 2$

5. $R_Z = 2 + \dfrac{6\times 18}{6+18} = 6.5\ \Omega$

$I = \dfrac{V}{R_Z} = \dfrac{6.5}{6.5} = 1$ A

6. $Z = 3$

$L = \dfrac{nh}{2\pi} = \dfrac{3h}{2\pi} \Rightarrow n = 3$

$r_n = \dfrac{n^2 a_0}{Z} = \dfrac{3^2.a_0}{3} = 3a_0$

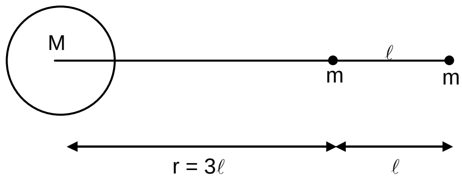
$\delta = 2\pi r_A = 2\pi \times 3a_0 = 6\pi a_0$

$S = n\lambda_D = 3\lambda_D$

$6\pi a_0 = 3\lambda_D \Rightarrow \lambda_D = 2\pi a_0$

$= P\pi a_0 \Rightarrow P = 2$

7.



$a_1 = \dfrac{\left[\dfrac{Gm^2}{\ell^2} + \dfrac{GMm}{16\ell^2}\right]}{m} = a_2 = \dfrac{\left[\dfrac{GMm}{\ell^2} - \dfrac{Gm^2}{\ell^2}\right]}{m}$

$\dfrac{2Gm^2}{\ell^2} = \dfrac{GMm}{9\ell^2} - \dfrac{GMm}{16\ell^2}$

$2Gm = GM\left(\dfrac{1}{9} - \dfrac{1}{16}\right) = \dfrac{GM\times 7}{16\times 9}$

$m = \dfrac{7M}{288} = \dfrac{kM}{288} \Rightarrow k = 7$

8. $E = A^2 e^{-\alpha t}$

$dE = A^2.e^{-\alpha t} \times (-\alpha)dt + e^{-\alpha t} \times 2A \times dA$

$\dfrac{dE}{E} = \alpha dt + 2\dfrac{dA}{A}$

$= (0.2 \times 1.5 \times 5) + 2 \times 1.25$

$= 1.5 + 2.5 = 4$

9. $Q = \Delta U + W$

$W = \dfrac{1}{2}kx^2 = \dfrac{1}{2}kx.x = \dfrac{F.x}{2}$

$= \dfrac{F}{A}\times\dfrac{Ax}{2}$

$= \dfrac{1}{2}p\times\Delta V$

[p = Final pressure]

(A) $\dfrac{p_1V_1}{T_1} = \dfrac{p_2V_2}{T_2}$

$p_2 = \dfrac{p_1V_1}{T_1}\times\dfrac{T_2}{V_2} = \dfrac{p_1\times V_1\times 3T_1}{T_1\times 2V_1}$

$= \dfrac{3}{2}p_1$

$W = \dfrac{1}{2}p_2\times[V_2 - V_1]$

$= \dfrac{1}{2}\times\dfrac{3}{2}p_1\times[2V_1 - V_1] = \dfrac{3}{4}p_1V_1$

A $\rightarrow$ wrong

B : $p_2 = \dfrac{3}{2}p_1$ $\qquad \Delta U = nC_V\Delta T$

$= n\times\dfrac{3R}{2}(3T_1 - T_1)$

$= 3nRT_1 = 3p_1V_1$

B $\rightarrow$ correct

C: $W = \frac{1}{2} p_{final} \times \Delta V$

$$\frac{p_2V_2}{T_2} = \frac{p_1V_1}{T_1} \Rightarrow p_2 \times \frac{3V_1}{4T_1} = p_1\frac{V_1}{T_1}$$

$$p_2 = \frac{4}{3}p_1$$

$$W = \frac{1}{2} \times \frac{4}{3} p_1 \times [3V_1 - V_1] = \frac{4}{3}p_1V_1$$

C $\rightarrow$ wrong

D: Q = ΔU + W

$$\Delta U = nC_V\Delta T = n \times \frac{3R}{2} \times [4T_1 - T_1]$$

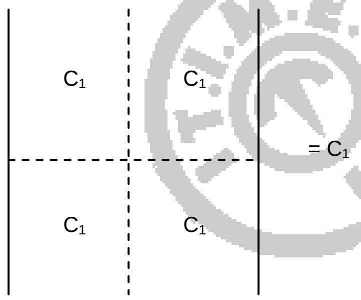
$$= \frac{9}{2}nRT_1 = \frac{9}{2}p_1V_1$$

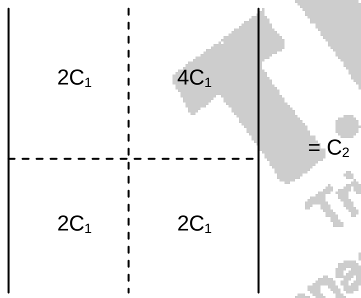
$$Q = \frac{9}{2}p_1V_1 + \frac{4}{3}p_1V_1 \quad \text{[From (C) above]}$$

$$= \frac{35}{6}p_1V_1$$

D $\rightarrow$ wrong

10.





$$C_2 = \frac{2C_1 \times 4C_1}{6C_1} + C_1 = \frac{4}{3}C_1 + C_1 = \frac{7}{3}C_1$$

$$\frac{C_2}{C_1} = \frac{7}{3}$$

Section II

11. $\frac{dp}{dr} = -\rho g_{(r)}$

$$= -\rho\frac{4}{3}\pi G\rho r$$

$$\Rightarrow dp = -\frac{4}{3}\pi G\rho^2 r dr$$

$$\int_{p(r)}^{p(R)} dp = \int_r^R -\frac{4}{3}\pi G\rho^2 r dr$$

$$\Rightarrow p(R) - p(r) = -\frac{4}{3}\pi G\rho^2 \left.\frac{r^2}{2}\right]_r^R$$

Take p(R) = 0 (atmospheric pressure)

$$\therefore -p(r) = -\frac{2\pi G\rho^2}{3}\left[R^2 - r^2\right]$$

$$\Rightarrow p(r) = \frac{2\pi G\rho^2}{3}\left[R^2 - r^2\right]$$

A: $r = 0 \rightarrow p(0) = \frac{2\pi G\rho^2}{3}R^2 \neq 0$

$\Rightarrow$ A is not correct

$$\frac{p(r_1)}{p(r_2)} = \frac{\left(R^2 - r_1^2\right)}{\left(R^2 - r_2^2\right)}$$

B: $$\frac{p\left(\frac{3R}{4}\right)}{p\left(\frac{2R}{3}\right)} = \frac{\left(R^2 - \frac{9R^2}{16}\right)}{\left(R^2 - \frac{4R^2}{9}\right)}$$

$$= \frac{(16-9)\times 9}{16 \times (9-4)} = \frac{7 \times 9}{16 \times 5} = \frac{63}{80}$$

B is correct

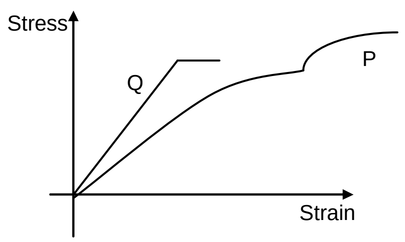
C: $$\frac{p\left(\frac{3R}{5}\right)}{p\left(\frac{2R}{5}\right)} = \frac{\left(R^2 - \frac{9R^2}{25}\right)}{\left(R^2 - \frac{4R^2}{25}\right)} = \frac{(25-9)\times 25}{25 \times (25-4)} = \frac{16}{21}$$

C is correct

D: $$\frac{p\left(\frac{R}{2}\right)}{p\left(\frac{R}{3}\right)} = \frac{\left(R^2 - \frac{R^2}{4}\right)}{\left(R^2 - \frac{R^2}{9}\right)} = \frac{(4-1)\times 9}{4 \times (9-1)} = \frac{27}{32}$$

D is not correct

12.



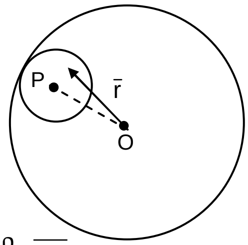
A: Breaking stress of P is slightly more than Q. Hence P has more tensile strength.

B: P is more ductile as it can undergo more strain before rupture.

C: Q is more brittle than P as it breaks at a lesser strain and stress.

D: $Y_Q > Y_P$ (slope of Q is more)

13.



$\overline{E} = \frac{\rho}{3\varepsilon_0}\overline{OP}$

Uniform field

$|\,|\overline{OP}| = a = R_2 - R_1$

A: $E(\bar{r})$ independent of R_2

Also direction independent of $\bar{r}$

A $\rightarrow$ incorrect

B: $E(\bar{r})$ independent of R_2 and also direction independent of $\bar{r}$

B $\rightarrow$ incorrect

C: $\overline{E}$ depends on a & $\bar{a}$

C $\rightarrow$ incorrect

D: $\rightarrow$ correct

14. $\mu_0 = \frac{\text{henry}}{\text{m}}$ $\quad \varepsilon_0 = \frac{\text{farad}}{\text{m}}$

A: $\mu_0 I^2 = \epsilon_0 V^2$ (Both energy)

$\left(\frac{1}{2}LI^2 = \frac{1}{2}CV^2\right)$

A $\rightarrow$ correct

B: $\varepsilon_0 I = \mu_0 V \rightarrow$ incorrect

since A is correct

C: $I = \varepsilon_0 CV$

$= \frac{\text{farad}}{\text{m}} \times \frac{\text{m}}{\text{s}} \times \text{volt}$

C $\rightarrow$ correct

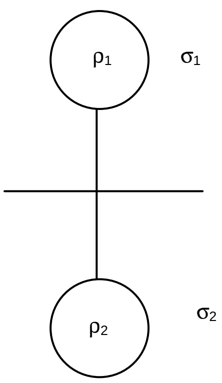
D: $\mu_0 CI = \varepsilon_0 V$

$\frac{\text{henry}}{\text{m}} \times \frac{\text{m}}{\text{S}} \times \frac{\text{coulomb}}{\text{S}} = \frac{\text{farad}}{\text{m}} \times \text{volt}$

$= \frac{\text{columb}}{\text{m}}$

D $\rightarrow$ incorrect

15.



For system

$U_1 + U_2 = W_1 + W_2$

$V\sigma_1 g + V\sigma_2 g$

$= V\rho_1 g + V\rho_2 g$

$\sigma_1 - \rho_1 = \rho_2 - \sigma_2$

$V_T = \frac{2R^2 g}{9\eta} \times$ (Diff in densities)

$\frac{|\overline{V}_P|}{|\overline{V}_Q|} = \frac{(\sigma_1 - \rho_1)}{\eta_1(\rho_2 - \sigma_2)/\eta_2} = \frac{\eta_2}{\eta_1}$

A $\rightarrow$ incorrect B $\rightarrow$ correct

The spheres attain terminal velocities in opposite direction. $\therefore$ $\overline{V}_P.\overline{V}_Q < 0$

D $\rightarrow$ correct

16. U $\rightarrow$ Xe + Sr + x + y

U $\rightarrow$ Xe +	Sr	+ x	+ y
86	129	2	2
MeV	MeV	MeV	MeV

A: Mass number conservation

236 = 140 + 94 + 1 + 1

= 236

Charge conservation

92 = 54 + 38 = 92

Energy conservation 86 + 129 + 2 + 2

= 219

A $\rightarrow$ correct

B: Energy conserved

Charge conserved

Mass number not conserved

B $\rightarrow$ wrong

C: Energy conserved

Charge not conserved

Mass no. conserved

C $\rightarrow$ wrong

D: $K.E = \frac{p^2}{2m}$ $\quad K.E \propto \frac{1}{\text{mass}}$

$\therefore$ $K_{Sr} > K_{Xe}$

D $\rightarrow$ wrong

Section III

17. $\sin i = n_1 \cos\theta$

$n_1 \sin\theta > n_2 \Rightarrow \Rightarrow \sin\theta > \frac{n_2}{n_1}$

$\Rightarrow \cos\theta < \sqrt{1 - \frac{n_2^2}{n_1^2}}$

$\Rightarrow \sin i < n_1\sqrt{1 - \frac{n_2^2}{n_1^2}} = \sqrt{n_1^2 - n_2^2}$

$\Rightarrow NA = \sin i_m = \sqrt{n_1^2 - n_2^2}$

A: S_1: $NA = \frac{\sqrt{\frac{45}{16} - \frac{9}{4}}}{\frac{4}{3}} = \frac{\frac{3}{4}}{\frac{4}{3}} = \frac{9}{16}$

$$S_2 = \frac{\sqrt{\frac{64}{25}-\frac{49}{25}}}{\frac{16}{\frac{3}{\sqrt{15}}}} = \frac{\frac{\sqrt{15}}{5}}{\frac{16}{3\sqrt{15}}} = \frac{45}{80} = \frac{9}{16}$$

$\therefore$ A $\rightarrow$ correct

B: $S_1 : \frac{\frac{3}{4}}{\frac{6}{\sqrt{15}}} = \frac{\sqrt{15}}{8}$;

$S_2 = \frac{\frac{\sqrt{15}}{5}}{\frac{4}{3}} = \frac{3\sqrt{15}}{20}$ $\quad\therefore$ B $\rightarrow$ incorrect

C: $S_1 : \frac{\frac{3}{4}}{1}$; $S_2 = \frac{\frac{\sqrt{15}}{5}}{\frac{4}{\sqrt{15}}} = \frac{3}{4}$

$\therefore$ C $\rightarrow$ correct

D: $S_1 = \frac{3}{4}$: $S_2 = \frac{\frac{\sqrt{15}}{5}}{\frac{4}{3}} = \frac{3\sqrt{15}}{20}$

$\therefore$ D $\rightarrow$ incorrect

18. Least sin i_m, values.

19. Drift velocity: $v_d = \frac{I}{Ane} = \frac{I}{dwne}$

$\Rightarrow v_d \propto \frac{1}{dw}$

$qv_dB = qE \Rightarrow E \propto v_d$

$\Rightarrow E \propto \frac{1}{dw}$; $\Delta V = E\omega$

$\therefore \Delta V \propto \frac{1}{d}$ $\therefore \frac{V_1}{V_2} = \frac{d_2}{d_1}$

20. $v_d = \frac{I}{Ane} \Rightarrow v_d \propto \frac{1}{n}$

$qv_dB = qE \Rightarrow E \propto v_dB$

$\Rightarrow E \propto \frac{B}{n} \Rightarrow \Delta V \propto \frac{B}{n}$

$\therefore \frac{V_1}{V_2} = \frac{B_1}{B_2}.\frac{n_2}{n_1}$

A: $\frac{V_1}{V_2} = 1.\frac{1}{2} \Rightarrow V_2 = 2V_1$ $\quad\therefore$ correct

$\therefore$ B $\rightarrow$ incorrect

C: $\frac{V_1}{V_2} = 2.1 \Rightarrow V_2 = \frac{V_1}{2}$: correct

D: $\therefore$ incorrect

PART II

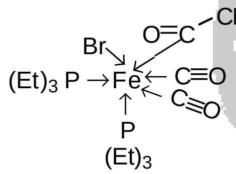
21	22	23	24	25	26	27	28
3	**6**	**6**	**3**	**9**	**8**	**4**	**4**

29	30	31	32
B, C, D	**B**	**C, D**	**B, C**

33	34	35	36
A	**B**	**A**	**C**

37	38	39	40
A	**B**	**C**	**D**

Section I

21. 

22. All the 6 given complexes show geometrical isomerism

23. B_2H_6 on alcoholysis gives trimethyl borate and H_2
3 moles of $B_2H_6 \equiv$ 6 moles $B(OEt)_3$

24. Given, $\Lambda^\circ_{HX} = \Lambda^\circ_{HY}$

$$\frac{\alpha_{HY}}{\alpha_{HX}} = 10$$

$$\frac{K_{a(HY)}}{K_{a(HX)}} = \frac{C\alpha^2_{(HY)}}{C\alpha^2_{(HX)}} = \frac{0.1}{0.01} \times 100 = 10^3$$

$$pK_{a(HX)} - pK_{a(HY)} = \log\frac{K_{a(HY)}}{K_{a(HX)}} = 3$$

25. $\frac{P_2}{P_1} = \frac{n_2}{n_1} = \frac{1+8}{1} = 9$

1 mole of uranium produces 8 moles of helium

26. $MnO_4^- + 8H^+ + 5e^- \rightarrow Mn^{2+} + 4H_2O$

$Fe^{2+} \rightarrow Fe^{3+} + e^-$

$2C_2O_4^{2-} \rightarrow 4CO_2 + 4e^-$

$\therefore 1\ MnO_4^- \equiv 1$ complex

For 1 mole MnO_4^- 8 moles of H^+ is consumed

27. When heated with H^+, the given alcohol undergoes dehydration to form an additional double bond. Cis-hydroxylation with aqueous dilute $KMnO_4$ will give a compound containing 4 OH group

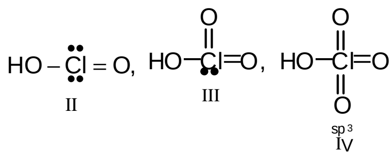
28. All four reactions give benzaldehyde

Section II

29. Adsorption of O_2 on metal surface is an example for chemisorption, where in O_2 changes to O_2^- and O_2^{2-}

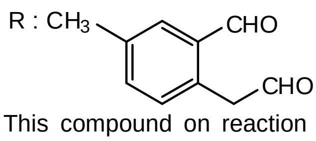
30. CH_3SiCl_2 on hydrolysis gives bifunctional molecules. So polymerisation is linear
$(CH_3)_2SiCl$ gives only one functional group so termination takes place

31. Cu^{2+}, Pb^{2+}
Hg^{2+} and Bi^{3+}
belongs to II group in qualitative analysis

32. 

33. β-Naphthol undergoes coupling at position-1

34. T is cumene and U is cumene hydroperoxide

35. 

This compound on reaction with NH_3 gives the isoquinoline derivative (A)

36. The equation P(V–b) = RT is true for the monoatomic gas helium. There is no bond formation between the atoms of helium

Section III

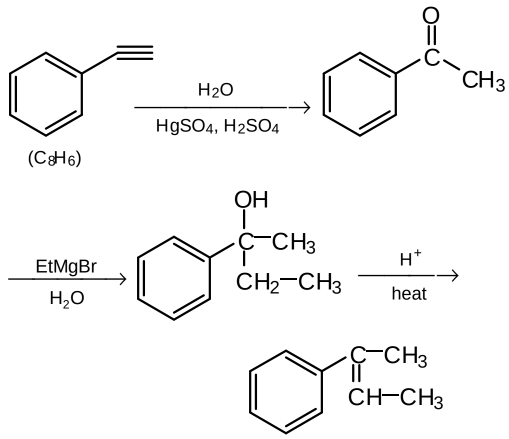
37. Heat liberated in experiment 1 = 5700 J
Calorimeter constant = 1000 J
Heat of ionisation of acidic acid

$$= (5700 - 5600) \times 10$$
$$= 1 \text{ kJ mol}^{-1}$$

38. $$\text{pH} = \text{pK}_a + \log \frac{[\text{Salt}]}{[\text{Acid}]}$$
$$= 4.7 + \log \frac{100}{100} = 4.7$$

39. Hydroboration-oxidation results in the formation of alcohol with anti-Markovnikov addition of water to alkene

40.



PART III

41	42	43	44	45	46	47	48
8	**4**	**2**	**9**	**7**	**9**	**4**	**9**

49	50	51	52
A, B	**B, C**	**A, C**	**A, B, D**

53	54	55	56
A, B	**B, C, D**	**A, D**	**D**

57	58	59	60
A, B, C	**C, D**	**A, B**	**C, D**

Section I

41. The required coefficient = coefficient of x^9m

$=(1 + x)(1+ x^2) \ldots\ldots(1 + x^9)$

$= 1 +$ coeft of x^9 in $(1 +x)(1+ x^2) \ldots\ldots(1+ x^8)$

$= 1 +1 +$ coefficient of x^9 in $(1 + x) + (1+ x^2) \ldots\ldots\ldots(1 +x^7)$

$= 2 + 1 +$ coeft of x^9 in $(1+ x)(1 + x^2)\ldots\ldots(1+ x^6)$

$3 +$ coeft of x^3 in $(1+ x)(1+ x^2)(1 + x^3)$

$+$ coeft of x^9 in $(1 + x)(1 +x^2) \ldots (4x^5)$

$= 3 + 2 +$ coeft of x^4 in $(1 +x)(1+ x^2)(1+ x^3)$

$(1+ x^4) +$ coeft of x^9 in $(1+ x)(1 + x^2)(1 +x^3)(1+ x^4)$

$= 5 + 2 + 1 = 8$

42. $\dfrac{x^2}{9}+\dfrac{y^2}{5}=1$

$a^2 = 9$

$b^2 = 5$

$5 = 9(1 – e^2)$

$\Rightarrow e = \dfrac{2}{3}$

Foci: (f, 0) $(f_2, 0)$

: (+2, 0) (–2, 0)

Parabola with focus at (2, 0)
$y^2 = 8(x– 2)$ } P_1

Parabola with focus at (–4, 0)
$y^2 = –16(x + 4)$ } P_2

T_1 : Tangent to P_1 passing through (–4, 0)

T_2 : Tangent to P_2 passing through (2, 0)

$y^2 = 8(x – 2)$

$y^2 – 8x + 16 = 0$ —— (1)

T, be $y – 0 = m_1(x + 4)$

$y = m_1(x + 4)$

$\therefore C = \dfrac{a}{m}$

$4m_1 = \dfrac{2}{m_1}$

$\Rightarrow m_1^2 = \dfrac{1}{2}$

Let T_2 be

$y – 0 = m_2(x – 2)$

Substituting in $y^2 = –16(x + 4)$

Eq: T_2

$y = m_2 (x – 2)$

$–2m_2 = \dfrac{-4}{m_2}$

$m_2^2 = 2$

$\dfrac{1}{m_1^2}+m_2^2 = 2+2=4$

43. $\lim\limits_{\alpha\to 0}\left(\dfrac{e^{\cos(\alpha^n)}-e}{\alpha^m}\right)=-\left(\dfrac{e}{2}\right)$

then value of $\dfrac{m}{n}$ is

$\lim\limits_{\alpha\to 0}\dfrac{e^{\cos(\alpha^n)}\left(-\sin(\alpha^n)\right)n\alpha^{n-1}}{m\alpha^{m-1}}=-\left(\dfrac{e}{2}\right)$

$\lim\limits_{\alpha\to 0}\dfrac{-e^{\cos(\alpha^n)}\sin(\alpha^n)}{\dfrac{m}{n}\alpha^{m-n}}=-\dfrac{e}{2}$

$\lim\limits_{\alpha\to 0} e^{\cos(\alpha^n)}\dfrac{\sin(\alpha^n)}{(\alpha^{-n})}\cdot\dfrac{1}{\alpha^m}=\dfrac{-e}{2}\times\dfrac{m}{n}$

$\therefore \dfrac{m}{n}=2$

44. $9x + 3\tan^{-1}x = t$

$\dfrac{12+9x^2}{1+x^2}=dt$

$x = 0 \Rightarrow t = 0$

$x = 1 \Rightarrow t = 9 + \frac{3\pi}{4}$

$\therefore \alpha = \int_0^{9+\frac{3\pi}{4}} e^t dt = e^{9+\frac{3\pi}{4}} - 1$

$\therefore \log |1 + \alpha| - \left.\frac{3\pi}{4}\right|$

$= \log \left|1 + e^{9+\frac{3\pi}{4}} - 1\right| - \frac{3\pi}{4}$

$= \log\left(e^{9+\frac{3\pi}{4}} - \frac{3\pi}{4}\right)$

$= 9 + \frac{3\pi}{4} - \frac{3\pi}{4} = 9$

45. $f(-x) = -f(x)$

$f(1) = \frac{1}{2}$

$\Rightarrow f(-1) = \frac{-1}{2}$

$F(x) = \int_{-1}^{x} f(t)dt$ for all $x \in [-1, 2]$

$G(x) = \int_{-1}^{x} t|f(f(t))|dt$ for $x \in [-1, 2]$

Given $\lim_{x\to 1} \frac{F(x)}{G(x)} = \frac{1}{14}$

We have $F(1) = \int_{-1}^{1} f(t)dt = 0$, since f(x) is an odd function

$G(1) = \int_{-1}^{1} t|f(f(t))|dt = 0$

Therefore,

$\frac{1}{14} = x \to \lim_{x\to 1} \frac{F(x)}{G(x)} = \lim_{x\to 1} \frac{F'(x)}{G'(x)}$

$= \lim_{x\to 1} \frac{f(x)}{x|f(f(x))|}$

$= \frac{f(1)}{1\left|f\left(\frac{1}{2}\right)\right|}$

$= \frac{f(1)}{1\left|f\left(\frac{1}{2}\right)\right|} = \frac{1}{14}$

$\left|f\left(\frac{1}{2}\right)\right| = 14f(1)$

$= 7$

46. $\bar{s} = 4\bar{p} + 3\bar{q} + 5\bar{r}$

$x(-\bar{p} + \bar{q} + \bar{r}) + y(\bar{p} - \bar{q} + \bar{r}) + z(-\bar{p} - \bar{q} + \bar{r})$

$= = 4\bar{p} + 3q + 5\bar{r}$

$-x + y - z = 4$

$x - y - z = 3$

$x + y + z = 5$

$\begin{vmatrix} -1 & 1 & -1 \\ 1 & -1 & -1 \\ 1 & 1 & 1 \end{vmatrix}$ $\begin{vmatrix} -1 & 4 & -1 \\ 1 & -3 & -1 \\ 1 & 5 & 1 \end{vmatrix}$

$-1(2) - 1(2) = -4$ $-1(8) - 1(9) + 1(-4 + 3)$

$-8 - 9 - 1 = -18$

$\begin{vmatrix} 4 & 1 & -1 \\ 3 & -1 & -1 \\ 5 & 1 & 1 \end{vmatrix}$

$4(0) - 3(2) + 5(-2)$

$= -6 - 10 = -16$

$x = 4$

$y = \frac{9}{2}$ $2x + y + z$

$z = -\frac{7}{2}$ $= 8 + \frac{9}{2} - \frac{7}{2}$

$= 9$

$= -1(-8) - 1(1) + 1(7)$

$8 - 1 + 7 = 14$

47. $\alpha_k = e^{\frac{i\pi}{7}k}$

$\therefore \alpha_{k+1} = e^{\frac{i\pi}{7}(k+1)} = e^{\frac{i\pi}{7}k} \; e^{\frac{i\pi}{7}}$

$= \alpha_k \alpha_1$

$\therefore |\alpha_{k+1} - \alpha_k| = |(\alpha_1 - 1)\alpha_k|$

$= |\alpha_1 - 1|$

Similarly,

$|\alpha_{4k-1} - \alpha_{4k-2}| = |\alpha_1 - 1|$

$\therefore \frac{\sum_{k=1}^{12} |\alpha_{k+1} - \alpha_k|}{\sum_{k=1}^{3} |\alpha_{4k-1} - \alpha_{4k-2}|} = \frac{12|\alpha_1 - 1|}{3|\alpha_1 - 1|} = 4$

48. $\frac{\frac{7}{2}(2a + 6d)}{\frac{11}{2}(2a + 10d)} = \frac{6}{11}$

$7(2a + 6d) = 6(2a + 10d)$

$14a + 42d = 12d + 60d$

$2a = 18d$

$a = 9d$

$130 < a + 6d < 140$

$130 < 9d + 6d < 140$

$130 < 15d < 140$

$\therefore d = 9$

Section II

49. $\int_0^{\frac{\pi}{4}} f(x)dx =$

$$\int_0^{\frac{\pi}{2}} \left(7\tan^8 x + 7\tan^6 x - 3\tan^4 x - 3\tan^2 x\right)dx$$

$$= \int_0^{\frac{\pi}{4}} \left[7\tan^6 x\left(1+\tan^2 x\right) - 3\tan^2 x\left(1+\tan^2 x\right)\right]dx$$

$$= \int_0^{\frac{\pi}{4}} \left(7\tan^6 x - 3\tan^2 x\right)\sec^2 dx \quad \text{put } \tan x = u$$

$\sec^2 x dx = du$

$\left(0, \frac{\pi}{4}\right) \to (0, 1)$

$$= \int_0^1 \left(7u^6 - 3u^2\right)du$$

$$= \left(u^7 - u^3\right)_0^1$$

$= 0$

(b) true

$$\int_0^{\frac{\pi}{4}} x f(x) dx = \int_0^{\frac{\pi}{4}} x\left(7\tan^6 x - 3\tan^2 x\right)\sec^2 x \, dx$$

$$= \int_0^1 \tan^{-1} u\left(7u^6 - 3u^2\right)du$$

$$= \left[\tan^{-1} u \left(u^7 - u^3\right)\right]_0^1 - \int_0^1 \frac{1}{1+u^2} \times \left(u^7 - u^3\right)du$$

$$= 0 - \int_0^1 \frac{u^3\left(u^4 - 1\right)}{1+u^2}du$$

$$= \int_0^1 u^3\left(1 - u^2\right)du$$

$$= \int_0^1 \left(u^3 - u^5\right)du = \left[\frac{u^4}{4} - \frac{u^6}{6}\right]_0^1$$

$$= \frac{1}{4} - \frac{1}{6} - 0$$

$$= \frac{2}{24} = \frac{1}{12}$$

50. From the table at x = –1, 0, 2 (f –3g) = 3
∴ Using Rolle's theorem, (f' –3g') = 0
at least one point in (–1, 0)
and at atleast one point in (0, 2)
Now let (f' – 3g') = 0 at more than one point, (i.e.) at 3 points in (–1, 0) ⇒ (f – 3g)" = 0 at atleast 2 point in (–1, 0) ⇒ contradicts given fact (f – 3g)" ≠ 0 in (–1, 0)
same applies to (0, 2)

51. We have,

$$\sin^6 at = \frac{-1}{32}\left[\cos 6at - 6\cos 4at + 15\cos 2at - 20\right]$$

$$\cos^4 at = \frac{1}{8}\left[\cos 4at - 4\cos 2at + 6\right]$$

∴ e^t

$$\left[\sin^6 at + \cos^4 at\right] = \frac{1}{32}\left[1 - e^t \cos 6at + 10e^t \cos 4at - 31e^t \cos 2at + 44e^t\right]$$

Now, $\int_0^{4\pi} e^t \cos at \, dt = \frac{e^{4\pi} - 1}{a^2 + 1}$

Also, $\int_0^{\pi} e^t \cos at \, dt = \frac{e^{4\pi} - 1}{a^2 + 1}$, for a = 2 and 4

∴ Required integral

$$= \frac{e^{4\pi} - 1}{e^{\pi} - 1} \text{ for all even a}$$

52. Equation of PM is
$xx_1 - yy_1 = 1$
∴ coordinates of M
are $\left(\frac{1}{x}, 0\right)$

Centroid = (ℓ, m)

$\Rightarrow x_1 + x_2 + \frac{1}{x_1} = 3\ell$ and $y_1 = 3m$

$\Rightarrow \ell = \frac{1}{3}\left(x_1 + \frac{1}{x_1} + x_2\right)$ —— (1)

Slope of PM = $\dfrac{y_1}{x_1 - \frac{1}{x_1}}$

Slope of PN = $\dfrac{y_1}{x_1 - x_2}$

PM ⊥PN $\Rightarrow \dfrac{y_1}{x_1 - \frac{1}{x_1}} + \dfrac{y_1}{x_1 - x_2} = -1$

$\Rightarrow y_1^2 + \left(x_1 - \frac{1}{x_1}\right)\left(x_1 - x_2\right) = 0$

$\Rightarrow x_1^2 - 1 + \left(x_1 - \frac{1}{x_1}\right)\left(x_1 - x_2\right) = 0$

$\Rightarrow x_2 = 2x_1$

$\therefore \ell = \frac{1}{3}\left(x_1 + \frac{1}{x_1} + 2x_1\right)$

$= \frac{1}{3}\left(3x_1 + \frac{1}{x_1}\right)$

$\therefore \frac{d\ell}{dx_1} = \frac{1}{3}\left(3 - \frac{1}{x_1^2}\right) = 1 - \frac{1}{3x_1^2}, \; x_1 > 1$

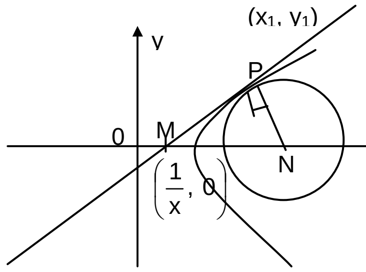
Also $y_1 = 3m$

Also, $y_1 = 3m$

$\Rightarrow m = \frac{1}{3}, \; y_1$

$= \frac{1}{3}, \sqrt{x_1^2 - 1}$

$\Rightarrow \dfrac{dm}{dx_1} = \dfrac{x_1}{3\left(\sqrt{x_1^2 - 1}\right)}, \; x_1 > 1$



$\therefore\ 1 = 3\dfrac{dm}{dy_1} \Rightarrow \dfrac{dm}{dy_1} = \dfrac{1}{3}, \; y_1 > 0$

53. Point of intersection of $x^2 + (y-1)^2 = 2$ and

$x + 3y = 3$ is $P(2, 1) \Rightarrow PQ = PR = \dfrac{2\sqrt{2}}{3}$

$\Rightarrow R\left(\dfrac{8}{3}, \dfrac{1}{3}\right)$ and $Q\left(\dfrac{4}{3}, \dfrac{5}{3}\right)$

Tangent at Q is $\dfrac{4x}{3a_1^2} + \dfrac{5y_1}{3b_1^2} = 1$

$\Rightarrow \dfrac{4x}{a_1^2} + \dfrac{5y}{b_1^2} = 3$ is tangent

Same as $a_1^2 = 4$, $b_1^2 = 5$

$\therefore\ e_1^2 = \dfrac{b_1^2 - a_1^2}{b_1^2} = \dfrac{1}{5}$

Tangent at $R\left(\dfrac{8}{3}, \dfrac{1}{3}\right) \Rightarrow \dfrac{8x}{a_2^2} + \dfrac{y}{b_2^2} = 3$

$\Rightarrow a_2^2 = 8$ and $b_2^2 = 1$

$\therefore e_2^2 = \dfrac{7}{8}$

Now, $e_1^2 + e_2^2 = \dfrac{43}{40}$ is true

$e_1^2 e_2^2 = \dfrac{7}{40} \Rightarrow e_1 e_2 = \dfrac{\sqrt{7}}{2\sqrt{10}}$ is true

$e_1^2 - e_2^2 = \dfrac{7}{8} - \dfrac{1}{5} = \dfrac{35-8}{40} = \dfrac{27}{40}$

$\therefore$ (C) is not true

54. $\alpha = 3\sin^{-1}\dfrac{6}{11}$

$\sin\alpha = 3\times\dfrac{6}{11} - 4\times\dfrac{6^3}{11^3} = \dfrac{18}{11}\left(1 - \dfrac{48}{121}\right)$

$= \dfrac{18\times 73}{11\times 121} > 0$

$\cos\alpha = \cos 3\theta,\ \theta = \sin^{-1}\dfrac{6}{11}$

$= 4\cos^3\theta - 3\cos\theta$

$= \cos\theta\left[4\left(1 - \dfrac{36}{121}\right) - 3\right]$

$= \sqrt{1 - \dfrac{36}{121}}\left[1 - \dfrac{144}{121}\right] = \dfrac{\sqrt{85}}{11}\left(\dfrac{-23}{121}\right)$

$= \dfrac{-23\sqrt{85}}{1331} < 0$

$\sin\beta = \sin 3\phi,\ \phi = \cos^{-1}\dfrac{4}{9}$

$= 3\sin\phi - 4\sin^3\phi$

$= \sin\phi\,(4\cos^2\phi - 1)$

$= \sqrt{\dfrac{65}{81}}\left(\dfrac{64}{81} - 1\right) = -\dfrac{\sqrt{65}}{9}\cdot\dfrac{17}{81} = -\dfrac{17\sqrt{65}}{729} < 0$

$\cos\beta = \cos 3\phi,\ \phi = \cos^{-1}\dfrac{4}{9}$

$= 4\cos^3\phi - 3\cos\phi$

$= \dfrac{4}{9}\left(\dfrac{64}{81} - 3\right) = -\dfrac{4\times 179}{729} < 0$

$\cos(\alpha + \beta) =$

$$\frac{-23\sqrt{85}}{1331}\,\frac{-4\times 179}{729} + \frac{18\times 73}{11\times 121}\,\frac{17\sqrt{65}}{729} > 0$$

55. $x_1 + x_2 = \dfrac{1}{\alpha}$ $\quad x_1, x_2 = \dfrac{a}{a} = 1$

given $|x_1 - x_2| < 1 \Rightarrow$

$\dfrac{1}{\alpha^2} - 4 < 1 \Rightarrow \dfrac{1}{\alpha^2} < 5$

$\therefore\ \alpha^2 > \dfrac{1}{5}$

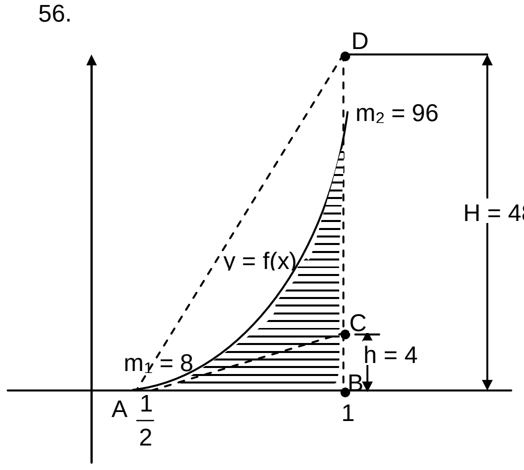
$\left(\alpha^2 - \dfrac{1}{5}\right) > 0$

$\left(\alpha + \dfrac{1}{\sqrt{5}}\right)\left(\alpha - \dfrac{1}{\sqrt{5}}\right) > 0$

$+ \qquad -\dfrac{1}{\sqrt{5}} \qquad - \qquad \dfrac{1}{\sqrt{5}} \qquad +$

$\alpha < -\dfrac{1}{\sqrt{5}};\ \ \alpha > \dfrac{1}{\sqrt{5}}$

56.



$$m \le \int_{\frac{1}{2}}^{1} f(x)dx \sum M$$

$$m = \frac{1}{2}bh = \text{Area of } \Delta ABC = \frac{1}{2}\times\frac{1}{2}\times 4 = 1$$

$$M = \frac{1}{2}bh = \text{Area of } \Delta ABD = \frac{1}{2}\times\frac{1}{2}\times 48 = 12$$

$\therefore 1 \le A \le 12$

Section III

57. $f(x) = x\,F(x),\ x \in R$

$\therefore f(1) = 0;\ f(3) = -12 \Rightarrow f(2) < 0$

Option (B) is correct

$f'(x) = x\,F'(x) + F(x)$

$f'(x) = F'(1) + F(1) = F'(1) < 0$

Option (A) is correct

$f'(x) = X\,F'(x) + F(x)$

in (1, 3) $F(x) < 0$ and $F'(x) < 0$

$\therefore f'(x) < 0$ in (1, 3)

$\therefore$ Option (C) is correct

58. $\int_1^3 x^3F'(x)dx = -12$

$\int_1^3 x^3F''(x)dx = 40$

$f(x) = xF(x)$

$F'(x) = \frac{f(x)}{x}$

$F'(x) = \frac{xf'(x) - f(x)}{x^2}$

$\int_1^3 x^2F'(x)dx = -12$

$\Rightarrow \int_1^3 [xf'(x) - f(x)]dx = -12$

$\Rightarrow \int_1^3 xf'(x)dx - \int_1^3 f(x)dx = -12$

$\Rightarrow \int_1^3 x\ d(f(x)) - \int_1^3 f(x)dx = -12$

$[xf(x)]_1^3 - 2\int_1^3 f(x)dx = -12$

$3f(3) - 1f(1) - 2\int_1^3 f(x)dx$

$= -12$

$3x - 12 - 0 - 2\int_1^3 f(x)dx$

$= -12$

$\int_1^3 f(x)dx = -12$

Option (D) is correct

We have $f(x) = xF(x)$

$f'(x) = xF'(x) + F(x)$

$f'(1) = F'(1) + F(1) = F'(1)$

$f'(3) = 3F'(3) + 3F(3) - 4$

$\int_1^3 x^3F''(x)dx = 40$

$\int_1^3 x^3d(F'(x)) = 40$

$[x^3F'(x)]_1^3 - \int_1^3 3x^2F'(x)dx = 40$

$27F'(3) - F'(1) + 3\times 12 = 40$

$\frac{27(f'(3)+4)}{3} - f'(1) = 4$

$9f'(3) + 36 - f'(1) = 4$

$9f'(3) - f'(1) + 32 = 0 \Rightarrow$ Option (C) is correct

59. I $\Rightarrow$ Red – n_1 Black – n_2

II $\Rightarrow$ Red – n_3 Black – n_4

Let E be the even of drawing a red ball

$$P(E) = \frac{1}{2}\left(\frac{n_1}{n_1+n_2} + \frac{n_3}{n_3+n_4}\right)$$

$$P\left(\frac{II}{E}\right) = \frac{\frac{1}{2}\left(\frac{n_3}{n_3+n_4}\right)}{\frac{1}{2}\left(\frac{n_1}{n_1+n_2} + \frac{n_3}{n_3+n_4}\right)}$$

$$= \frac{1}{3}(\text{given})$$

Checking the options:

Option (A) : $\frac{\frac{5}{20}}{\frac{3}{6}+\frac{5}{20}} = \frac{\frac{1}{4}}{\frac{1}{2}+\frac{1}{4}} = \frac{1}{3}$

Option (B) $\frac{\frac{10}{60}}{\frac{3}{9}+\frac{10}{60}} = \frac{\frac{1}{6}}{\frac{1}{3}+\frac{1}{6}} = \frac{1}{3}$

Option (C) $\frac{\frac{5}{25}}{\frac{8}{14}+\frac{5}{25}} = \frac{\frac{1}{5}}{\frac{4}{7}+\frac{1}{5}} \ne \frac{1}{3}$

Option (D) $\frac{\frac{5}{25}}{\frac{6}{18}+\frac{5}{25}} = \frac{\frac{1}{5}}{\frac{1}{3}+\frac{1}{5}} \ne \frac{1}{3}$

60. Ball transferred from Box I to Box II

$\therefore$ No of balls in I is $n_1 + n_2 - 1$

After transfer P(Red) =

$$\frac{n_1}{n_1+n_2}\times\frac{(n-1)}{(n_1+n_2-1)} + \frac{n_2}{n_1+n_2}\times\frac{n_1}{n_1+n_2-1}$$

$$\frac{n_1(n_1+n_2-1)}{(n_1+n_2)(n_1+n_2-1)}$$

$\Rightarrow \frac{n_1}{n_1+n_2} = \frac{1}{3}$

$\therefore 3n_1 = n_1 + n_2 \Rightarrow 2n_1 = n_2$

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