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Triumphant Institute of
Management Education Pvt. Ltd.

MODEL SOLUTIONS TO IIT JEE ADVANCED 2014

Paper I – Code 0

PART I

1	2	3	4	5
C	**B, D**	**A, C**	**D**	**C, D**
6	**7**	**8**	**9**	**10**
A, B, D	**A, B, C**	**D, C**	**A, C, D**	**A, D**

11	12	13	14	15	16	17	18	19	20
5	**3**	**4**	**4**	**2**	**5**	**2**	**3**	**5**	**8**

Section I

1. $E_1 = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2}$; $E_2 = \frac{\lambda}{2\pi\varepsilon_0 r}$

$E_3 = \frac{\sigma}{2\varepsilon_0}$

$E_1 = E_3 \Rightarrow \frac{Q}{4\pi\varepsilon_0 r^2} = \frac{\sigma}{2\varepsilon_0} \Rightarrow Q = 2\pi\sigma r^2$

(A) incorrect

$E_2 = E_3 \Rightarrow \frac{\lambda}{2\pi\varepsilon_0 r} = \frac{\sigma}{2\varepsilon_0} \Rightarrow r = \frac{\lambda}{\pi\sigma}$

(B) incorrect

$E_1 = E_2 \Rightarrow \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2} = \frac{\lambda}{2\pi\varepsilon_0 r}$

$Q = \lambda 2r$

(c) $\frac{\lambda 2r}{4\pi\varepsilon_0 \times \frac{r^2}{4}} = \frac{2\times\lambda}{2\pi\varepsilon_0 \times \frac{r}{2}} \Rightarrow$ (c) correct

(d) $\frac{\lambda}{2\pi\varepsilon_0 \frac{r}{2}} \neq \frac{4\times\sigma}{2\varepsilon_0} = \frac{4^2}{2\varepsilon_0}\times\frac{\lambda}{\pi r} \Rightarrow$ (d) incorrect

2. $R = \frac{\rho L}{\pi\frac{d^2}{4}}$

$R' = \frac{\rho L}{\pi\frac{(2d)^2}{4}} = \frac{R}{4}$

H is constant

$t \propto R$

$R_S = 2R' = 2.\frac{R}{4} = \frac{R}{2}$

$t' = \frac{4}{2} = 2$ min

$R_p = \frac{R'}{2} = \frac{R}{8}$

$t'' = \frac{4}{8} = 0.5$ min

3. $\frac{1}{f_1} = (\mu - 1)\frac{1}{R} = \frac{1}{2R}$

$f_1 = 2R$

$f_2 = f_1 + \frac{t}{\mu} = f_1 = 2R$

$f_1 = f_2 = 2R$

4. $\frac{\lambda}{4} = 0.350$ m $\Rightarrow \lambda = 1.4$ m

% error $= \frac{0.005}{0.350}\times 100 = \frac{100}{70} = 1.4\%$

$f = \frac{v}{4\ell} \Rightarrow v = f \times 4\ell$

$= f \times \lambda$

$= 244 \times 1.4$ m s^{-1}

$= 341.6$ m s^{-1}

(A) $v = \sqrt{\frac{\gamma RT}{M}} = \sqrt{\frac{\frac{5}{3}\times RT}{10^{-3}\times 20}\left(\frac{\times 10}{10}\right)}$

$= \sqrt{167RT}\sqrt{\frac{10}{20}} = 640\times\frac{7}{10}$

$= 448\ m\ s^{-1}$

(B) $v = \sqrt{\frac{\frac{2}{5}\times RT}{10^{-3}\times 28}\left(\frac{10}{10}\right)}$

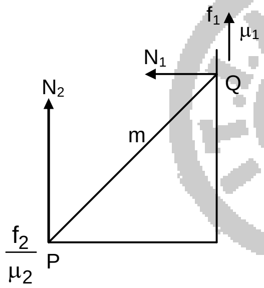
$= \sqrt{140RT}\times\sqrt{\frac{10}{20}} = 590\times\frac{3}{5} = 354\ m\ s^{-1}$

(C) $v = \sqrt{\frac{\frac{2}{5}\times RT}{10^{-3}\times 32}\left(\frac{10}{10}\right)} = 590\times\frac{9}{16} = \frac{331}{m/s}$

(D) $v = \sqrt{\frac{5}{3}\frac{RT}{10^{-3}\times 36}\left(\frac{10}{10}\right)} = 640\times\frac{17}{32}$

$= 340\ m\ s^{-1}$ (only correct)

5.



Taking torque about P

$mg\frac{\ell}{2}\cos\theta = N_1\ell\sin\theta$

$N_1\tan\theta = \frac{mg}{2}$

$\tan\theta = \frac{mg}{2N_1} = \frac{mg}{2\mu}\frac{f_2}{\mu_2}$

$\tan\theta = \frac{mg}{2}$

$N_2.\ell\cos\theta = f_2.\ell\sin\theta = \mu N_2\ell\sin\theta$

Vertical equilibrium

$N_2 = mg + f_1 = mg + 0 = mg$

$\tan\theta = \frac{mg}{2}$

$N_2 + \mu_1 N_1 = mg$

$N_2 + \mu_1 f_2 = mg$

$N_2 = \mu_1\mu_2 N_2 = mg$

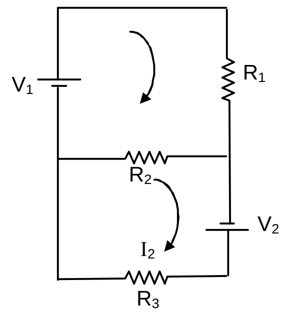
$N_2 = \frac{mg}{1+\mu_1\mu_2}$

6. Current through

$R_2 = 0$ $[I_1 = I_2]$

$\therefore$ $V_1 = I_1R_1$ and

$V_2 = I_2R_3$



A, B and D possible.

7. $\beta = \frac{D\lambda}{d}$

$\beta_1 = 400\times\frac{D}{d}$

$\beta_2 = 600\times\frac{D}{d} = 1.5\beta_1$

$\beta_2 > \beta_1$

$m_1 > m_2$

$3\beta_2 = 4.5\beta_1$

8. $I = I_0\cos\omega t$

$\frac{dQ}{dt} = I$ $\qquad Q = \frac{I_0}{500}\sin\omega t$

$= \frac{1}{500}\times\sin 210° = 10^{-3}$

B^X R negative

$C^XV = \frac{Q}{C} = \frac{10^{-3}}{20\times 10^{-6}} = 50\ V$

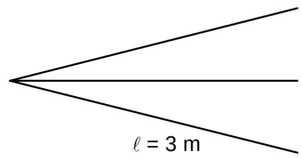
$V_1 + V_2 = 100 \Rightarrow I = \frac{100}{10} = 10\ A$

$\Delta V = 100$

$Q = CV = 100\times 20\times 10^{-6}$

$= 2\times 10^{-3}$ C

9.



$v = 100\ m\ s^{-1}$

$\ell = 3m = (2n-1)\times\frac{\pi}{4}$

$\Rightarrow \lambda = \frac{4\ell}{2n-1} = \frac{4\times 3}{(2n-1)}$

$\lambda = \frac{12}{(2n-1)}$ —(1)

(A) $K = \frac{\pi}{6} = \frac{2\pi}{\lambda} \Rightarrow \lambda = 12$

(1) is satisfied for n = 1

$\Rightarrow$ A is correct.

(B) K = $\frac{\pi}{3} = \frac{2\pi}{\lambda} \Rightarrow \lambda = 6$ m $\Rightarrow n = \frac{3}{2}$

$\Rightarrow$ B incorrect

(C) K = $\frac{5\pi}{6} = \frac{2\pi}{\lambda} \Rightarrow \lambda = \frac{12}{5} = \frac{12}{2n-1}$

$2n - 1 = 5 \Rightarrow n = 3$

Hence (C) is correct.

(D) K = $\frac{5\pi}{2} = \frac{2\pi}{\lambda} \Rightarrow \lambda = \frac{4}{5} = \frac{12}{2n-1}$

$2n - 1 = 15 \Rightarrow n = 8$

$\Rightarrow$ D correct

10. $C_1' = \frac{\varepsilon_0}{d}\frac{A}{3} = \frac{Cl0}{3}$

$C' = \frac{\varepsilon_0 A}{d}.\frac{2}{3} = \frac{2}{3}C_0$

$C_1 = \frac{KC_0}{3}$ $\qquad$ $i = \frac{2}{3}C_0$

$E_1 = E_2 = \frac{V}{d}$ $\qquad$ $\frac{E_1}{E_2} = 1$

$Q_1 = C_1V = \frac{KC_0}{3}V$

$Q_2 = C'V = \frac{2}{3}C_0V$

$\frac{Q_1}{Q_2} = \frac{KCV}{3} \times \frac{3}{2}.CV = \frac{K}{2}$

$\frac{C}{C_1} = \frac{2+K}{K}$

Section II

11. $i_g = 0.006$ A

V = 30 V

R = 4990 Ω

$R = \frac{V}{i_g} - G$

$4990 = \frac{30}{6} \times 10 - G$

4990 = 5000 − G

G = 10 Ω

$S = \frac{G}{\frac{i}{i_g} - 1}$

$= \frac{10}{\frac{1.5}{6} \times 10^3 - 1}$

$= \frac{10}{\frac{1500}{6} - 1}$

$= \frac{10}{24g} = \frac{2M}{24g}$

n = 5

12. $d \propto \rho^2 s^b f^c$

$L' \,(ML^{-3})^a\,(MT^{-3})^b\,(T^{-1})^c$

$-3a = 0 + 0 = 1$

$a = -\frac{1}{3}$ $\qquad$ for L

$a + b = \theta$

$b = -a$

$= +\frac{1}{3}$

$\frac{1}{n} = \frac{1}{3}$

n = 3

13. MSD = 3.2 cm = 32 mm

vernier = $20 \times 1 \times 10^{-5}$ m

$= 20 \times 10^{-3}$ cm

= 0.02 cm = 0.2 mm

Reading = 32.2 mm

Next MSD = 32 mm

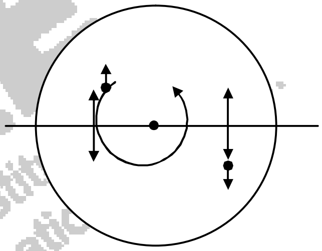
Vernier = 45^{th} = 45×10^{-3} cm = 0.45 mm

Reading = 32.45 mm

L.C = 1×10^{-3} cm = 0.01 mm

% error = $\frac{0.01 \times 100}{0.25} = 4\%$

14.



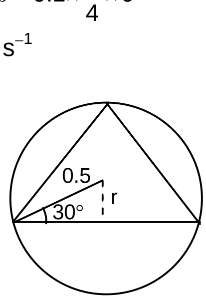
$I_1\omega = 2 \times mr^2 \times \frac{v}{r}$

$\frac{0.45 \times (0.5)^2}{2}\omega = 2 \times 0.05 \times \left(\frac{1}{4}\right)^2 \times \frac{9}{\left(\frac{1}{4}\right)}$

$0.45 \times \frac{1}{2}\omega = 0.1 \times \frac{1}{4} \times 9$

$\omega = 4$ rad s^{-1}

15.



$r = 0.5 \sin 30° = \frac{r}{4}$

Torque = r.F = $\frac{1}{4} \times 0.5 \times 3 = \frac{3}{8}$ N m

$L = I\omega$

$I = \frac{mr^2}{2} = 1.5\frac{(0.5)^2}{2} = \frac{3}{2}\times\frac{1}{2}\times\frac{1}{2}\times\frac{1}{2} = \frac{3}{16}$

$\Delta L = \tau \times t = \tau \times 1 = I\omega$

$\frac{3}{8} = \frac{3}{16}\times\omega \Rightarrow \omega = 2$

16. $W_{net} = K_f - K_i$
$F.5 - mg4 = K$
$18 \times 5 - 1 \times 10 \times 4 = K$
$90 - 40 = 10 \times n$
$n = 5$

17. $W_{ib} = +50$ J
$W_{af} = +200$ J
$W_{bf} = +100$ J
$u_i = 100$ J
$u_b = 200$ J
$Q_{iaf} = 500$ J
$Q_{iaf} = (u_f - u_i) + W_{ia} + W_{af}$
$500 = u_f - 100 + 0 + 200$
$u_f = 400$ J
$Q_bf = u_f - u_b + W_{bf}$

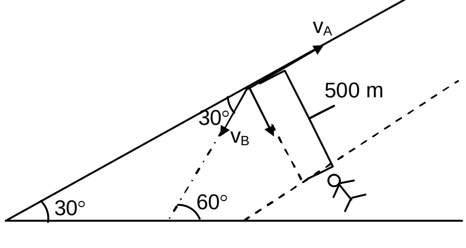
$= 200 + 100 = 300$ J
$Q_{ib} = u_b - u_i + W_{ib}$
$= 100 + 50 = 150$ J
$\frac{Q_{bf}}{a_{ib}} = \frac{300}{1.10} = 2$

18. $R_1 = \frac{mu}{B_1 - B_2 q}$

$R_2 = \frac{mu}{B_1 + B_2 q}$

$\frac{R_1}{R_2} = \frac{B_1 + B_2}{B_1 - B_2} = \frac{\frac{1}{\frac{1}{3}} + \frac{1}{\frac{2}{3}}}{\frac{1}{\frac{1}{3}} - \frac{1}{\frac{2}{3}}} = \frac{9}{3} = 3$

19.



$v_A = v_B \cos 30°$

$100\sqrt{3} = v_B \frac{\sqrt{3}}{2}$

$v_B = 200$ m s^{-1}
$S_{rel} = 500$ m
$500 = v_B \sin 30° \times t_0$

$500 = 200 \times \frac{1}{2} \times t_0$

$t_0 = 5$ sec^2

20. $u_{rel} = 0.5$,, $a_{vel} = 0$
$S_{rel} = u_{rel}.t$
$u = 0.5 \times t$
$t = 8$

PART II

21	22	23	24	25
B, C, D	**A, B, C**	**A, B, D**	**A, C, D**	**A, B, C**

26	27	28	29	30
A, B, C	**A, B, D**	**A**	**A, B, C**	**B, C, D**

31	32	33	34	35	36	37	38	39	40
1	**2**	**4**	**8**	**6**	**7**	**4**	**5**	**3**	**7**

Section I

21. $CuO \xrightarrow[temp]{higher} Cu_2O + O_2$

    $CuSO_4 \xrightarrow{\text{strong heating}} CuO + SO_3$

    $Cu_2S + 2Cu_2O \longrightarrow 6Cu + SO_2$

22. In a galvanic cell, the salt bridge does not participate chemically in the cell reaction, it stops the diffusion of ions from one electrode to another, it is required to complete the cell circuit

23. Ice is lighter than water due to extensive hydrogen bonding. Intramolecular hydrogen bonding in acetic acid makes it a dimer in benzene solvent
    Primary amines are more basic than 3° amines due to greater solvation of their conjugate acids by hydrogen bonding

24. The IUPAC name of tert-butyl alcohol given in (B) is not correct

25. The given pattern of electrophilic substitution can be explained by the steric effect of the halogen, the steric effect of the tert-butyl group and the electronic effect of the phenolic group.

26. $Na + (x + y)NH_3 \longrightarrow [Na(NH_3)_x]^+ \underset{\text{ammoniated } e^-\text{s (paramagnetic)}}{[e^-(NH_3)_y]^-}$

    $K + O_2 \longrightarrow \underset{\text{superoxide (paramagnetic)}}{KO_2}$

    $3Cu + 8HNO_3 \longrightarrow 3Cu(NO_3)_2 + \underset{\text{(paramagnetic)}}{2NO} + 4H_2O$

    $\text{2-Ethylanthraquinol} \underset{H_2/Pd}{\overset{O_2}{\rightleftharpoons}} \text{2-ethylanthraquinone} + \underset{\text{(diamagnetic)}}{H_2O_2}$

27. The balanced equation is

    $6I^- + ClO_3^- + 6H_2SO_4 \rightarrow$

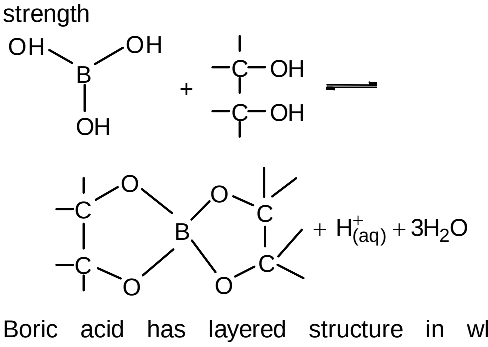
    $Cl^- + 3I_2 + 6HSO_4^- + 3H_2O$

    I^- is oxidised to I_2
    There is no change in the oxidation number of sulphur

28. Amines undergo acetylation when treated with acetic anhydride

29. For an adiabatic process, q = 0
    Since the expansion is against vaccum, w = 0
    $\therefore \Delta U = 0$
    When $\Delta U = 0$, then the temperature remains as constant.
    $\therefore T_1 = T_2$ and $P_1V_1 = P_2V_2$

30. $B(OH)_3 + H_2O \longrightarrow [B(OH)_4]^- + H^+$
    It behaves as a weak Lewis acid by accepting electrons from OH^- ions and in turn releasing H^+ ions. Since it produces ions in H_2O, it behaves as a weak electrolyte
    Complex formation with polyhydric alcohol containing cis-diol group increases acidic strength



Boric acid has layered structure in which molecules are linked by H-bonding

Section II

31. Glycine is the only naturally occurring amino acid obtained by the hydrolysis of the given peptide

32. $MX_2 \rightarrow M^{2+} + 2X^-$

$i = 1 - \alpha + n\alpha$
$= 1 + 2\alpha$
$= 1 + 2 \times 0.5$
$= 2$

33. $R = K_B \times N_A$
$= 1.380 \times 10^{-23} \times 6.023 \times 10^{23}$
$= 8.312$

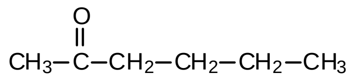
34. 3.2 molar $\Rightarrow$ 3.2 moles in 1000 mL solution given that, there is no change in volume up on dissolution
$\therefore$ Volume of solvent = 1000 mL
Wt. of solvent = 400 g
$\therefore$ Molality $= \frac{3.2}{0.4} = 8$

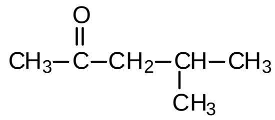
35. For n = 4
$\ell = 0$
$\ell = 1$ $m_\ell = -1$ and $+1$
$\ell = 2$ $m_\ell = -1$ and $+1$
$\ell = 3$ $m_\ell = -1$ and $+1$
} Total 6 orbitals and 12 electrons of which 6 electrons are having $m_s = -1/2$

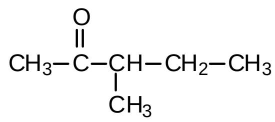
36. $K_2Cr_2O_7$ – nascent oxygen
$CuSO_4$ – instability of CuI_2
H_2O_2 – nascent oxygen
Cl_2 – displacement
O_3 – nascent oxygen
$FeCl_3$ – $Fe^{+3} \rightarrow Fe^{+2}$
HNO_3 – decomposition by temperature

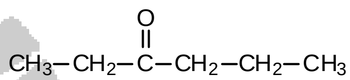
37. XeF_4 – sp^3d^2 with 2 lone pairs – square planar
SF_4 – sp^3d with 1 lone pair – see saw
SiF_4 – sp^3 – tetrahedral
BF_4^- – sp^3 – tetrahedral
BrF_4^- – sp^3d^2 with 2 lone pairs – square planar
$[Cu(NH_3)_4]^{2+}$ – dsp^2 – square planar
$[FeCl_4]^{2-}$ – sp^3 – tetrahedral
$[CoCl_4]^{2-}$ – sp^3 – tetrahedral
$[PtCl_4]^{2-}$ – dsp^2 – square planar

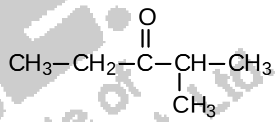
38. The ketones are

(i) 

(ii) 

(iii) 

(iv) 

(v) 

39. (3)

40. MnS – Flesh coloured
SnS_2 – Yellow coloured
All other sulphides are black coloured

PART III

41	42	43	44	45
A, D	**B, C**	**B , C**	**A, B, C**	**A, B, C**
46	47	48	49	50
A, C	**A, C**	**C, D**	**B, D**	**A, B**

51	52	53	54	55	56	57	58	59	60
6	**4**	**2**	**3**	**3**	**7**	**5**	**8**	**4**	**2**

Section I

41. Let a, b, $\in$ (0, 1) such that f'(a) =g'(b) = 0 and
  f(a) = g(b)

  Now f'(a) = $\lim\limits_{h_1 \to 0} \frac{f(a+h_1)-f(a)}{h_1}$

  g'(b) = $\lim\limits_{h_2 \to 0} \frac{g(b+h_2)-g(b)}{h_2}$

  $\therefore \lim\limits_{h_1 \to 0}(f(a+h_1)-f(a)) = \lim\limits_{h_2 \to 0} g(b+h_2)-g(b))$

  $\Rightarrow \lim\limits_{h \to 0} f(a+h_1) = \lim\limits_{h \to 0} g(b+h_2)$

  $\therefore$ There is a 'c' = a + h_1 = b + h_2
  So the curve is intersecting at c$\in$ (0, 1)

42. Equation of the circle is
  $x^2 + y^2 + 2gx + 2fy + c = 0$ ——(1)
  given circles $x^2 + y^2 - 2x - 15 = 0$ ——(2) and
  $x^2 + y^2 - 1 = 0$ ———(3)
  (1) is orthogonal to (2) $\Rightarrow$ –2g = c – 15
  (1) is orthogonal to (3) $\Rightarrow$ c – 1 = 0 $\Rightarrow$ c = 1
  $\therefore$ g = 7
  Passing through (0, 1) $\Rightarrow$ f = –1
  $\therefore$ Equation of circle is
  $x^2 + y^2 + 14x - 2y + 1 = 0$
  Centre (–7, 1), Radius = 7

43. Foot of the perpendicular of P(λ, λ, λ)
  on $\frac{x-0}{1} = \frac{y-0}{1} = \frac{z-1}{0}$ is given by

  $\frac{x-0}{1} = \frac{y-0}{1} = \frac{z-1}{0} = \mu$

  Where $\mu = \frac{1(\lambda-0)+1(\lambda-0)+0(\lambda-1)}{1+1} = \lambda$

  $\therefore$ Q. is (λ, λ, 1)
  Again, foot of the $\perp$ar of P(λ, λ, λ)
  on $\frac{x-0}{-1} = \frac{y-0}{1} = \frac{z+1}{0}$ is given by

  $\frac{x-0}{-1} = \frac{y-0}{1} = \frac{z+1}{0} = \mu$, where

  $\mu = \frac{(\lambda-0)+(\lambda-0)+0}{2} = 0$

  Thus, R(0, 0, –1) is the foot of perpendicular on the second line

  $\therefore \overrightarrow{PQ} = 0i + 0j + (\lambda-1)k$

  $\overrightarrow{PR} = \lambda i + \lambda j + (\lambda+1)k$

  given $\overrightarrow{PQ} \cdot \overrightarrow{PR} = 0 \Rightarrow \lambda^2 - 1 = 0 \Rightarrow \lambda = \pm 1$

44. $\bar{a} = k_1\bar{y} + k_2\bar{z}$
  $\bar{a}.\bar{x} = 0 \Rightarrow (k_1\bar{y} + k_2\bar{z}).\bar{b}\bar{c} = 0$
  $\Rightarrow k_1 + k_2 = 0$
  Hence $\bar{a} = k_1\bar{y} - k_2\bar{z}$
  $= k_1 (\bar{y} - \bar{z})$ where k_1 is a scalar
  $\bar{a}.\ \bar{y} = k_1(\bar{y} - \bar{z}).\bar{y}$
  $= 2k_1 - k_1 = k_1$
  Therefore, $\bar{a} = (\bar{a}.\bar{y})\ (\bar{y} - \bar{z})$
  $\bar{b}, \bar{z}, \bar{x}$ are coplanar
  $\bar{b} = c_1\bar{z} + c_2\bar{x}$
  $\bar{b}.\bar{y} = (c_1\bar{z} - +c_2\bar{x}).\bar{y}$
  $= c_1 + c_2$
  Hence, $\bar{b} = c_1(\bar{z} - \bar{x})$
  $\bar{b}.\bar{z} = c(\bar{z} - \bar{x}).\ \bar{z}$
  $2c_1 - c_1 = c_1$
  Therefore, $\bar{b} = (\bar{b}.\ \bar{z})(\bar{z} - \bar{x})$
  $\bar{a}.\ \bar{b} =$
  $(\bar{a}.\ \bar{y})\ (\bar{b}.\bar{z})\ \{\bar{y}.\bar{z} - \bar{y}.\bar{x} - |\bar{z}|^2 + \bar{z}.\bar{x}\}$
  $= (\bar{a}.\ \bar{y})\ (\bar{b}.\bar{z})\ (1-1+2+1)$
  $= -(\bar{a}.\ \bar{y})\ (\bar{b}.\bar{z})$

45. $f: \left(-\frac{\pi}{2}, \frac{\pi}{2}\right) \to R$

$f(x) = (\log(\sec x + \tan x))^3$

$f'(x) = 3(\log(\sec x+ \tan x))^2$

$\frac{1}{(\sec x + \tan x)}\left(\sec x \tan x + \sec^2 x\right)$

$= 3(\log(\sec x + \tan x))^2 . \sec x$

$f'(x) > 0$

$\therefore$ f(x) is increasing function

$\therefore$ if is one- one and onto

$f(0) = 0$ and $f(-x) = (\log(\sec x - \tan x))^3$

$= \left(\log\left(\frac{1}{\sec x + \tan x}\right)\right)^3$

$= -(\log(\sec x+ \tan x))^3$

$= -f(x)$

$\therefore$ f(x) is odd

46. $g(x) = \begin{cases} 0, & x < a \\ \int_a^x f(t)dt, & a \le x \le b \\ \int_a^b f(t)dt, & x > b \end{cases}$

$g(a) = 0 = g(a^{\pm})$

g(x) is continuous at x = a

$g(\bar{b}) = \int_a^{\bar{b}} f(t)dt = g(b^+)$

g(x) is continuous at x = b

$g'(x) = \begin{cases} 0, & x < a \\ f(x) & a \le x \le b \\ 0 & x > b \end{cases}$

$g(\bar{a}) = 0$, $g'(a^+) = 1$

g(x) is not differentiable at x = a

$g'(\bar{b}) = f(\bar{b}),\ g'(b^+) = 0$

g(x) is not differentiable at x = b

47. $f(x) = \int_{\frac{1}{x}}^{x} \frac{e^{-\left(t+\frac{1}{t}\right)}}{t}dt$

$f'(x) = \frac{2}{x}e^{-\left(u+\frac{1}{x}\right)} > 0\ \forall\ x\ (1, \infty)$

Option (A)

$f(x) = \int_{\frac{1}{x}}^{x} e^{-\left(t+\frac{1}{t}\right)}\frac{dt}{t}$

$f\left(\frac{1}{x}\right) = \int_x^{\frac{1}{x}} e^{-\left(t+\frac{1}{t}\right)}\frac{dt}{t}$

Now $f(x) = -\int_x^{\frac{1}{x}} e^{-\left(u+\frac{1}{u}\right)}\frac{du}{u},\ u = \frac{1}{t}$

$= -f\left(\frac{1}{x}\right)$

$\Rightarrow f(x) + f\left(\frac{1}{x}\right) = 0\ \forall\ x \in (0, \infty)$

Option (c)

$f'(x) = \frac{2}{x}e^{-\left(x+\frac{1}{x}\right)}$

$f'(2^x) = \frac{2}{2^x}\left(e^{-\left(2^x+2^{-x}\right)}\right) = g(x)$

$g(x) = 2^{1-x}e^{-\left(2^x+2^{-x}\right)}$

$g(-x) = 2^{1+x}e^{-\left(2^{-x}+2^{-x}\right)}$

$g(-x) \ne g(x)$ or

$g(x) \ne -g(x)$

$\therefore$ f'(x) is neither even nor odd. Thus f(x) is neither even nor odd.

48. Let M be $\begin{pmatrix} a & b \\ b & c \end{pmatrix}$ where a, b, c $\in Z$

For M to be invertible $ac \ne b^2$

$\Rightarrow$ Option D is correct

Let 1st column of M be the transpose of 2nd row of M $\Rightarrow \begin{pmatrix} a \\ b \end{pmatrix} = \begin{pmatrix} b \\ c \end{pmatrix}$

$\Rightarrow$ a = b & b – c $\Rightarrow M = \begin{pmatrix} b & b \\ b & b \end{pmatrix} \Rightarrow$ Not invertible

Let 2nd row of M be the transpose of 1st column of M $\Rightarrow (b\ \ c) = (a\ \ b)$

$\Rightarrow$ Not invertible

Let M be a diagonal matrix $\Rightarrow M = \begin{pmatrix} a & 0 \\ 0 & c \end{pmatrix}$

M is symmetric & M is invertible

$\Rightarrow$ Option (c) and (D)

49. Let us consider $F(x) = x^5 - 5x$

$F(x) = x^5 - 5x = x(x^4 - 5)$

$= x\left(x^2 + \sqrt{5}\right)\left(x^2 - \sqrt{5}\right)$

$= x\left(x^2 + 5^{\frac{1}{2}}\right)\left(x - 5^{\frac{1}{4}}\right)\left(+x + 5^{\frac{1}{4}}\right)$

There will be true real roots if a = 0

$F'(x) = 5x^4 - 5 = 5(x^2 + 1)(x - 1)(x + 1)$

$\Rightarrow$ There exists a maximum at x = –1 and minimum at x = 1

$F(-1) = -1 + 5 = 4$

$F(1) = 1 - 5 = -4$

So for $f(x) = x^5 - 5x + a$,

$a < -4 \Rightarrow$ there exists only one root

$-4 < a < 4 \Rightarrow$ there exists three real roots.

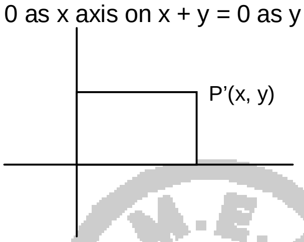
$a > 4 \Rightarrow$ there exists only one root.

50. $M^2 + MN^2 = N^4 + MN(N)$

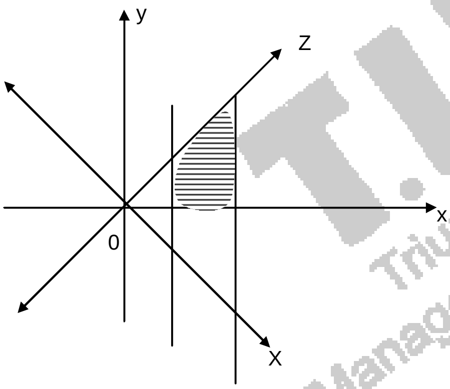
$= N^4 + NM(N)$
$= N^4 + NNM$
$= N^4 + N^2M$
$= N^2(N^2 + M)$
$|M|^2 = |N|^4$
$M^2 - N^4 = 0$
$(M + N^2)(M - N^2) = 0$
Either $|M + N^2| = 0$ or $|M - N^2| = 0$
Let $|M + N^2| = 0$
Then, $|M^2 + MN^2| = |N|^2\,|N^2 + M|$
$= 0$

Section II

51. Choose x – y = 0 as x axis on x + y = 0 as y – axis.



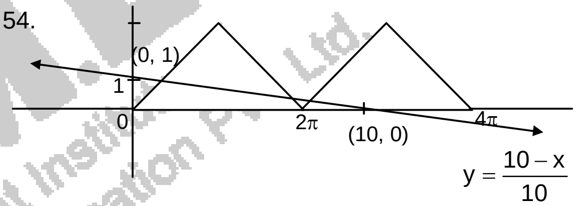
Now locus of $d_1(P) + d_2(P) = k$ is the locus of P'(X, Y) in new axis and it is a straight line.
$X + Y = k$
∴ Area that satisfies $2 \le X + Y \le 4$ is
is $2\left[\frac{1}{2}\times 4\times 4-\frac{1}{2}\times 2\times 2\right]$ = 12 units



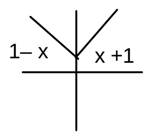
But in our case area in the first quadrant is required.
$\therefore \frac{1}{2}(12)$ = 6 sq. units

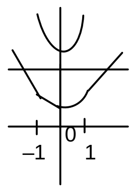
52. $a \times b + b \times c = p\bar{a} + q\bar{b} + r\bar{c}$
taking dot on either side with $\bar{a}, \bar{b}$, and $\bar{c}$
$\therefore [a\ b\ c] = p+\frac{q}{2}+\frac{r}{2}$
$\frac{\sqrt{3}}{4} = p+\frac{q}{2}+\frac{r}{2}$ ——— (1)
$\frac{\sqrt{3}}{4} = \frac{p}{2}+\frac{q}{2}+r$
and $0 = \frac{p}{2}+q+\frac{r}{2}$
$\therefore p + 2q + r = 0$
$p + q + 2r = \frac{\sqrt{3}}{2}$
$2p + q + r = \frac{\sqrt{3}}{2}$
Solving p = r = –q
$\therefore \frac{p^2+2q^2+r^2}{q^2} = \frac{4p^2}{p^2} = 4$

53. $\frac{1-x}{1-\sqrt{x}} = 1+\sqrt{x}$
$\lim_{x\to 1}\frac{1-x}{1-\sqrt{x}} = 2$
$\therefore \lim_{x\to 1}\left(\frac{-ax+\sin x-1+a}{x+\sin(x-1)-1}\right)^2 = \frac{1}{4}$
$\Rightarrow \lim_{x\to 1}\frac{-a+\cos(x-1)}{1+\cos(x-1)} = \frac{\pm 1}{2}$
$\frac{-a+1}{2} = \pm\frac{1}{2}$
$-a + 1 = \pm 1$
$\Rightarrow a = 0, 2 \Rightarrow a = 2$
a = 2, is the greatest non zero integer.

54.


Number of points of intersection is (3)

55. f(x) = |x| + 1 ⇒


g(x) = x^2 + 1 ⇒
h(x) = is


$\therefore h(x) = \begin{cases} x^2+1 & x<-1 \\ 1-x & -1\le x<0 \\ x^2+1 & 0\le x<1 \\ x+1 & x>1 \end{cases}$

$$h'(x) = \begin{cases} 2x & x < -1 \\ -1 & -1 \le x < 0 \\ 2x & 0 \le x < 1 \\ 1 & x > 1 \end{cases}$$

$\therefore$ h(x) is not differentiate at x = –1, x = 0 and x = 1

$\therefore$ Ans (3)

56. Since $n_i \neq 0$ and $n_1 < n_2 < n_3 < n_4 < n_5$
Let us allocate 1, 2, 3, 4 and 5 to the 5 numbers respectively.

1 + 2 + 3 + 4+ 5 = 15 and

We have 5 more left.

Possible distributors are

1,	1,	1,	1,	1
0,	1,	1,	1,	2
0,	0,	0,	2,	3
0,	0,	1,	1,	3
0,	0,	1,	2,	2
0,	0,	0,	1,	4
0,	0,	0,	0,	5

7 possible solutions

57. When n points lie on a circle, there are ${}^nC_2$ lines possible.
Of which n lines will form sides of the polygon as they are from adjacent points.

$\therefore {}^nC_2 - n = n$

$n(n-1) - 2n = 2n$

$n^2 - 5n = 0 \Rightarrow n = 5$

58. $(y - x^5)^2 = x(1+x^2)^2$

$2(y - x^5)(y' - 5x^4) = x\,2(1+x^2).\,2x + (1+x^2)^2$

$2(3-1)(y'-5) = 1.2\,(1+1)2 + (1+1)^2$

$2(2)\ (y' - 5) = 8 + 4$

$y' - 5 = \frac{12}{4} = 3$

$y' = 8$

59. Let $\frac{b}{a} = k$ (an integer)

$b = ak$

Given $b^2 = ac$

$a^2k^2 = ac$

$c = ak^2$

and a + b +c c = 3b +6

$a - 2b + c = 6$

$a - 2ak + ak^2 = 6$

$a(k-1)^2 = 6$

$a = \frac{6}{(k-1)^2}$

Since a is a positive

Integer, $(k-1)^2 = 1$ is the only solution

Therefore, a = 6

Again, we have $\rightarrow k - 1 = \pm 1$

$k = 2$ or 0

$k \neq 0$

$k = 2$

Therefore, $b = 6 \times 2 = 12$

and $c = 6 \times 4 = 24$

$\frac{a^2 + a - 14}{a+1} = a - \frac{14}{a+1}$

$= 6 - \frac{14}{7} = 4$

60. $\int_0^1 4x^3 \frac{d^2}{dx^2}\left(1-x^2\right)^5 dx$

$= \left[4x^3 \frac{d}{dx}\left(1-x^2\right)^5\right]_0^1 - \int_0^1 12x^2 \frac{d}{dx}\left(1-x^2\right)^5$

$0 - \left[12x^2\left(1-x^2\right)^5\right]_0^1 - \int_0^1 24x\left(1-x^2\right)^5 dx$

$= 12.\ \frac{u^6}{6}\Big]_0^1 = 2$