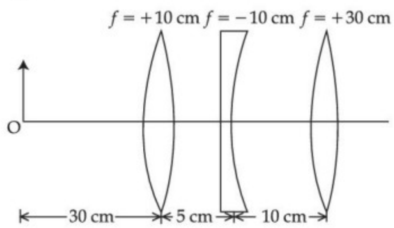
SOLUTIONS & ANSWERS FOR JEE MAINS-2021
27th August Shift 1

[PHYSICS, CHEMISTRY & MATHEMATICS]

PART – A – PHYSICS

Section A

Q.1 Find the distance of the image from object O, formed by the combination of lenses in the figure :



Options
1. infinity
2. 20 cm
3. 75 cm
4. 10 cm

Ans: 75 cm

Sol: $\frac{1}{V_1}+\frac{1}{30}=\frac{1}{10}$

$\frac{1}{V_1}+\frac{2}{30} \Rightarrow V_1 = 15\text{cm}$ $\qquad \frac{1}{V_1}=\frac{2}{30}$

$\frac{1}{V_2}-\frac{1}{10}=\frac{1}{10}$

$\frac{1}{V_2}=0$ $\qquad V_2 = \infty$

$V_3 = 30$ cm

$OV_3 = 75$cm

Q.2 Electric field in a plane electromagnetic wave is given by

$E = 50 \sin(500x - 10 \times 10^{10}t)$ V/m

The velocity of electromagnetic wave in this medium is :
(Given C = speed of light in vacuum)

Options
1. C
2. $\frac{3}{2}C$
3. $\frac{2}{3}C$
4. $\frac{C}{2}$

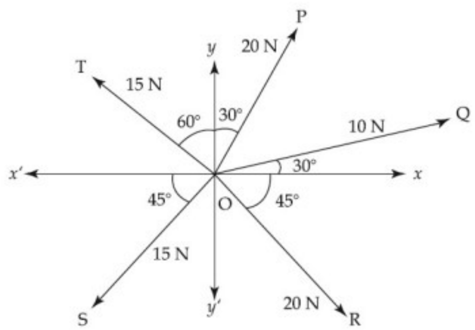
Ans: $\frac{2C}{3}$

Sol: $V = \frac{\omega}{K} = \frac{10 \times 10^{10}}{500} = 2 \times 10^{8}$

$V = \frac{2C}{3}$

Q.3 The resultant of these forces $\vec{OP}$, $\vec{OQ}$, $\vec{OR}$, $\vec{OS}$ and $\vec{OT}$ is approximately ________ N.

[Take $\sqrt{3} = 1.7$, $\sqrt{2} = 1.4$ Given $\hat{i}$ and $\hat{j}$ unit vectors along x, y axis]

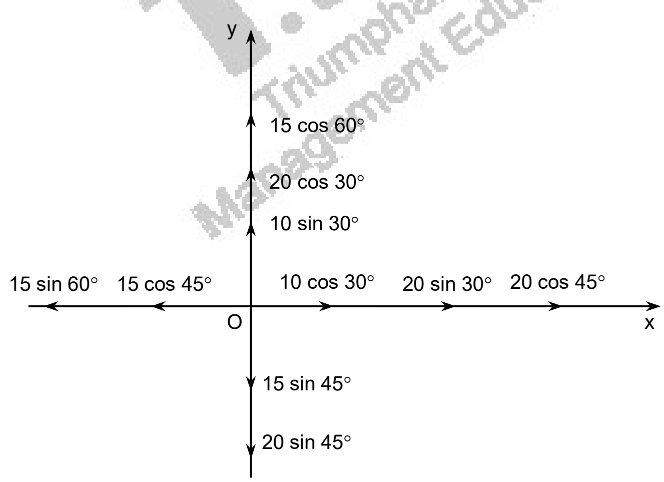


Options

1. $9.25\hat{i} + 5\hat{j}$
2. $2.5\hat{i} - 14.5\hat{j}$
3. $3\hat{i} + 15\hat{j}$
4. $-1.5\hat{i} - 15.5\hat{j}$

Ans: $9.25\hat{i} + 5\hat{j}$

Sol:



$$F_x = \left(10 \times \frac{\sqrt{3}}{2} + 20\left(\frac{1}{2}\right) + 20\left(\frac{1}{\sqrt{2}}\right) - 15\left(\frac{1}{\sqrt{2}}\right) - 15\left(\frac{\sqrt{3}}{2}\right)\right)\hat{i} = 9.25\ \hat{i}$$

$$F_y = \left(15\left(\frac{1}{2}\right)+20\left(\frac{\sqrt{3}}{2}\right)+10\left(\frac{1}{2}\right)-15\left(\frac{1}{\sqrt{2}}\right)-20\left(\frac{1}{\sqrt{2}}\right)\right)\hat{j} = 5\ \hat{j}$$

Q.4 A balloon carries a total load of 185 kg at normal pressure and temperature of 27°C. What load will the balloon carry on rising to a height at which the barometric pressure is 45 cm of Hg and the temperature is −7°C. Assuming the volume constant ?

Options
1. 123.54 kg
2. 219.07 kg
3. 181.46 kg
4. 214.15 kg

Ans: 123. 54 kg

Sol: $P_m = \rho RT$

$$\therefore \frac{P_1}{P_2} = \frac{\rho_1 T_1}{\rho_2 T_2}$$

$$\frac{\rho_1}{\rho_2} \Rightarrow \frac{P_1 T_2}{P_2 T_1} = \left(\frac{76}{45}\right)\times\frac{266}{300}$$

$$\frac{\rho_1}{\rho_2} \Rightarrow \frac{M_1}{M_2} = \frac{76\times 266}{45\times 300}$$

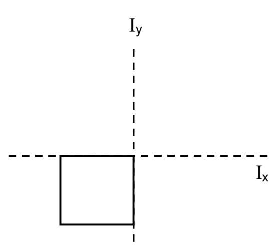
$$\therefore M_2 \Rightarrow \frac{45\times 300\times 185}{76\times 266} = 123.54\ \text{kg}$$

Q.5 Moment of inertia of a square plate of side l about the axis passing through one of the corner and perpendicular to the plane of square plate is given by :

Options
1. Ml^2
2. $\frac{2}{3}Ml^2$
3. $\frac{Ml^2}{6}$
4. $\frac{Ml^2}{12}$

Ans: $\frac{2}{3}M\ell^2$

Sol: According to perpendicular axis theorem



$I_x + I_y = I_z$

$$I_z \Rightarrow \frac{M\ell^2}{3} + \frac{M\ell^2}{3} = \frac{2M\ell^2}{3}$$

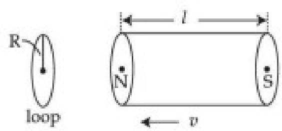
Q.6 In Millikan's oil drop experiment, what is viscous force acting on an uncharged drop of radius 2.0×10^{-5} m and density 1.2×10^3 kgm^{-3} ? Take viscosity of liquid $= 1.8 \times 10^{-5}$ Nsm^{-2}. (Neglect buoyancy due to air).

Options
1. 3.8×10^{-11} N
2. 1.8×10^{-10} N
3. 5.8×10^{-10} N
4. 3.9×10^{-10} N

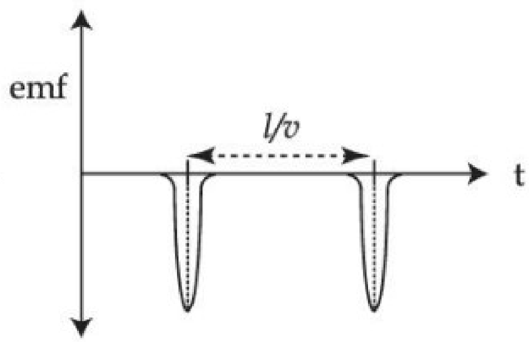
Ans: 3.9×10^{-10} N

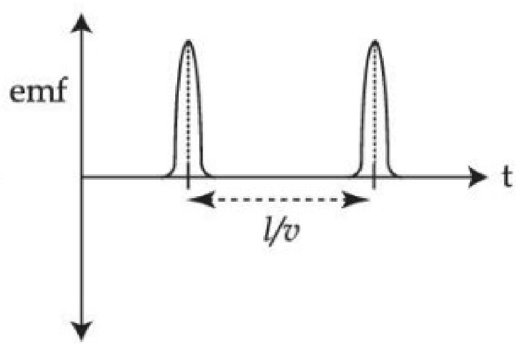
Sol: Viscous force = weight $= \rho \times \left(\frac{4}{3}\pi r^3\right) g = 3.9 \times 10^{-10}$ N

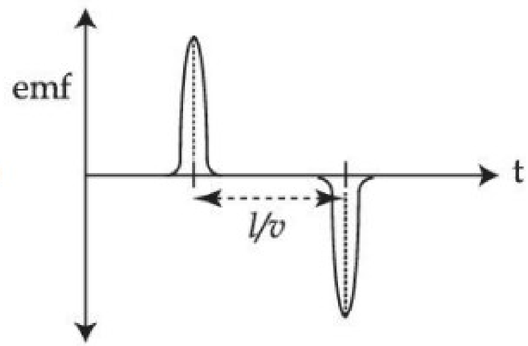
Q.7 A bar magnet is passing through a conducting loop of radius R with velocity v. The radius of the bar magnet is such that it just passes through the loop. The induced e.m.f. in the loop can be represented by the approximate curve :

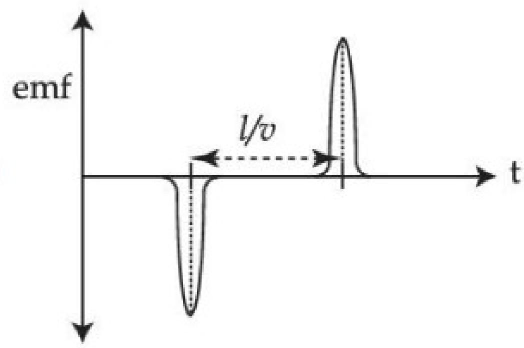


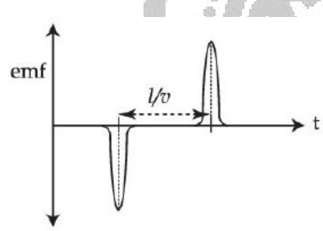
Options

1.


2.


3. 

4. 

Ans: 

Sol: 

- When magnet passes through centre region of solenoid, no current/ emf is induced in loop
- While entering flux increases so negative induced emf
- While leaving flux decreases so positive induced emf

Q.8 If E and H represents the intensity of electric field and magnetising field respectively, then the unit of E/H will be :

Options
1. mho
2. newton
3. ohm
4. joule

Ans: ohm

Sol: Unit of $\frac{E}{H}$ is $\frac{\text{Volt}/\text{metre}}{\text{Ampere}/\text{metre}} = \frac{\text{Volt}}{\text{Ampere}} = \text{ohm}$

Q.9 There are 10^{10} radioactive nuclei in a given radioactive element. Its half-life time is 1 minute. How many nuclei will remain after 30 seconds ?

$(\sqrt{2} = 1.414)$

Options
1. 7×10^9
2. 4×10^{10}
3. 10^5
4. 2×10^{10}

Ans: 7×10^9

Sol: $\frac{N}{N_0} = \left(\frac{1}{2}\right)^{\frac{1}{t_{\frac{1}{2}}}}$

$$\frac{N}{10^{10}} = \left(\frac{1}{2}\right)^{\frac{30}{60}}$$

$$\Rightarrow N = 10^{10} \times \left(\frac{1}{2}\right)^{\frac{1}{2}} = \frac{10^{10}}{\sqrt{2}} \approx 7 \times 10^9$$

Q.10 In a photoelectric experiment, increasing the intensity of incident light :

Options
1. increases the number of photons incident and also increases the K.E. of the ejected electrons.
2. increases the frequency of photons incident and increases the K.E. of the ejected electrons.
3. increases the number of photons incident and the K.E. of the ejected electrons remains unchanged.
4. increases the frequency of photons incident and the K.E. of the ejected electrons remains unchanged.

Ans: increase the number of photons incident and the K.E. of the ejected electrons remains unchanged

Sol: As the intensity increases number of photoelectrons increases but intensity does not depend on kinetic energy of photoelectrons.

Q.11 Two ions of masses 4 amu and 16 amu have charges $+2e$ and $+3e$ respectively. These ions pass through the region of constant perpendicular magnetic field. The kinetic energy of both ions is same. Then :

Options
1. both ions will be deflected equally
2. lighter ion will be deflected more than heavier ion
3. lighter ion will be deflected less than heavier ion
4. no ion will be deflected

Ans: lighter ion will be deflected less than heavier ion

Sol: $r = \frac{P}{qB} = \frac{\sqrt{2mk}}{qB}$

Given they have same kinetic energy

$r \propto \frac{\sqrt{m}}{q}$

$\frac{r_1}{r_2} = \frac{\sqrt{4}}{2} \times \frac{3}{\sqrt{16}} = \frac{3}{4}$

$\boxed{r_2 = \frac{4r_1}{3}}$

(r_2 is for hearier ion and r_1 is for lighter ion)

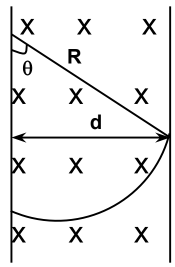
$\sin\theta = \frac{d}{R}$

$\theta \to$ Deflection

$\theta \propto \frac{1}{R}$

($R \to$ Radius of path)

$\because R_2 > R_1 \Rightarrow \theta_2 < \theta_1$



Q.12 A uniformly charged disc of radius R having surface charge density σ is placed in the xy plane with its center at the origin. Find the electric field intensity along the z-axis at a distance Z from origin :

Options

1. $E = \frac{\sigma}{2\varepsilon_0}\left(1 + \frac{Z}{(Z^2 + R^2)^{1/2}}\right)$

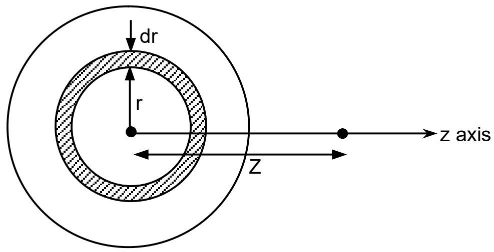
2. $E = \frac{\sigma}{2\varepsilon_0}\left(\frac{1}{(Z^2 + R^2)} + \frac{1}{Z^2}\right)$

3. $E = \frac{2\varepsilon_0}{\sigma}\left(\frac{1}{(Z^2 + R^2)^{1/2}} + Z\right)$

4. $E = \frac{\sigma}{2\varepsilon_0}\left(1 - \frac{Z}{(Z^2 + R^2)^{1/2}}\right)$

Ans: $E = \frac{\sigma}{2\in_0}\left[1 - \frac{z}{(z^2 + R^2)^{1/2}}\right]$

Sol: Consider a small ring of radius r and thickness dr on disc



Area of elemental ring on disc

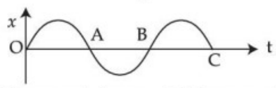
$dA = 2\pi r dr$

change on this ring $dq = \sigma dA$

$$dEz = \frac{kdqz}{\left(z^2 + r^2\right)^{3/2}}$$

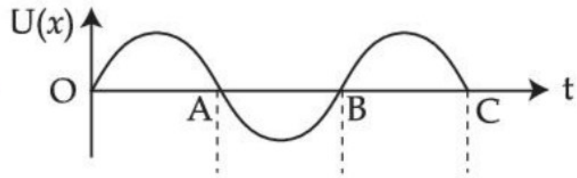
$$E = \int_0^R dE_z = \frac{\sigma}{2 \epsilon_0}\left[1 - \frac{z}{\sqrt{R^2 + z^2}}\right]$$

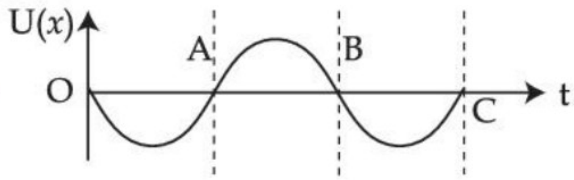
Q.13 The variation of displacement with time of a particle executing free simple harmonic motion is shown in the figure.

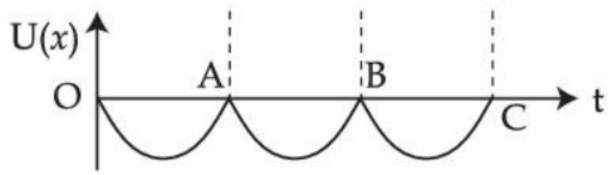


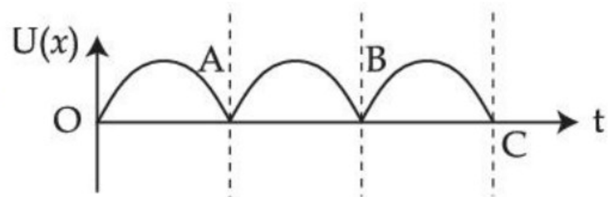
The potential energy U(x) versus time (t) plot of the particle is correctly shown in figure :

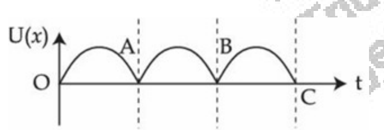
Options

1. 

2. 

3. 

4. 

Ans: 

Sol: Potential energy is maximum at maximum distance from mean position

Q.14 Which of the following is not a dimensionless quantity ?

Options

1. Relative magnetic permeability (μ_r)
2. Permeability of free space (μ_0)
3. Power factor
4. Quality factor

Ans: permeability of free space (μ_0)

Sol: $[\mu_r] = 1$ as $\mu_r = \dfrac{\mu}{\mu_m}$

[power factor ($\cos\phi$)] = 1

$\mu_0 = \dfrac{B_0}{H}\left(\text{unit} = NA^{-2}\right)$: Not dimensionless

$[\mu_0] = [MLT^{-2}A^{-2}]$

Quality factor (Q) = $\dfrac{\text{Energy stored}}{\text{Energy dissipated per cycle}}$

So Q is unitless and dimensionless

Q.15 An object is placed beyond the centre of curvature C of the given concave mirror. If the distance of the object is d_1 from C and the distance of the image formed is d_2 from C, the radius of curvature of this mirror is :

Options

1. $\dfrac{d_1 d_2}{d_1 - d_2}$
2. $\dfrac{2d_1 d_2}{d_1 + d_2}$
3. $\dfrac{2d_1 d_2}{d_1 - d_2}$
4. $\dfrac{d_1 d_2}{d_1 + d_2}$

Ans: $\dfrac{2d_1 d_2}{d_1 - d_2}$

Sol: Using Newton's formula

$(f + d_1)(f - d_2) = f^2$

$f^2 + fd_1 - fd_2 - d_1d_2 = f_2$

$f = \dfrac{d_1 d_2}{d_1 - d_2}$

$\therefore R = \dfrac{2d_1 d_2}{d_1 - d_2}$

Q.16 For a transistor in CE mode to be used as an amplifier, it must be operated in :

Options

1. Cut-off region only
2. Saturation region only
3. Both cut-off and Saturation
4. The active region only

Ans: The active region only

Sol: CE transistor in active region can be best used as an amplifier

Q.17 Five identical cells each of internal resistance 1 Ω and emf 5 V are connected in series and in parallel with an external resistance 'R'. For what value of 'R', current in series and parallel combination will remain the same ?

Options
1. $1\ \Omega$
2. $10\ \Omega$
3. $5\ \Omega$
4. $25\ \Omega$

Ans: 1 Ω

Sol: $i_1 = \dfrac{25}{5+R}$

$i_2 = \dfrac{5}{R+\frac{1}{5}}$

$i_1 = i_2 \Rightarrow 5\left(R+\dfrac{1}{5}\right) = 5+R$

4R = 4

R = 1 Ω

Q.18 A huge circular arc of length 4.4 ly subtends an angle '4s' at the centre of the circle. How long it would take for a body to complete 4 revolution if its speed is 8 AU per second ?

Given : $1\ \text{ly} = 9.46 \times 10^{15}$ m

$1\ \text{AU} = 1.5 \times 10^{11}$ m

Options
1. 3.5×10^{6} s
2. 4.1×10^{8} s
3. 7.2×10^{8} s
4. 4.5×10^{10} s

Ans: 4.5×10^{10} s

Sol: $R = \dfrac{\ell}{\theta}$

$\text{Time} = \dfrac{4 \times 2\pi R}{v} = \dfrac{4 \times 2\pi}{v}\left(\dfrac{\ell}{\theta}\right)$

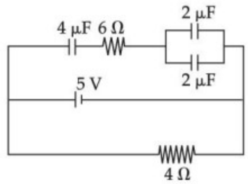
Put $\ell = 4.4 \times 9.46 \times 10^{15}$

$v = 8 \times 1.5 \times 10^{11}$

$\theta = \dfrac{4}{3600} \times \dfrac{\pi}{180}$ rad.

We get time = 4.5×10^{10} sec

Q.19 Calculate the amount of charge on capacitor of 4 μF. The internal resistance of battery is 1 Ω :



Options
1. zero
2. 8 μC
3. 16 μC
4. 4 μC

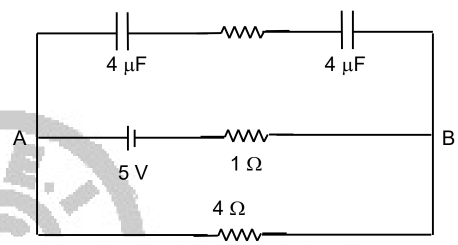
Ans: 8 μC

Sol: On simplifying circuit we get
No current in upper wire

$\therefore V_{AB} = \frac{5}{4+1} \times 4 = 4v$

$\therefore \theta = (C_{eq})v$

$\Rightarrow 2 \times 4 = 8\mu C$



Q.20 An ideal gas is expanding such that PT^3 = constant. The coefficient of volume expansion of the gas is :

Options
1. $\frac{1}{T}$
2. $\frac{3}{T}$
3. $\frac{2}{T}$
4. $\frac{4}{T}$

Ans: $\frac{4}{T}$

Sol: PT^3 = constant

$\left(\frac{nRT}{v}\right)T^3 = \text{constant}$

T^4V^{-1} = constant

$T^4 = kV$

$\Rightarrow 4\frac{\Delta T}{T} = \frac{\Delta V}{V} - - - - - - - - (1)$

$\Delta V = V\gamma\Delta T$-----------(2)

Comparing (1) and (2)

We get

$\gamma = \frac{4}{T}$

Section B

Q.1 If the velocity of a body related to displacement x is given by $v = \sqrt{5000 + 24x}$ m/s, then the acceleration of the body is ________ m/s^2.

Given **12**
Answer :

Ans: 12

Sol: $V = \sqrt{5000 + 24x}$

$$\frac{dV}{dx} = \frac{1}{2\sqrt{5000+24x}} \times \frac{12}{\sqrt{5000+24x}}$$

$a = 12\ m/s^2$

Q.2 First, a set of n equal resistors of 10 Ω each are connected in series to a battery of emf 20 V and internal resistance 10 Ω. A current I is observed to flow. Then, the n resistors are connected in parallel to the same battery. It is observed that the current is increased 20 times, then the value of n is _________.

Given **--**
Answer :

Ans: 20

Sol: In series

$R_{eq} = nR = 10\ n$

$$i_s = \frac{20}{10+10n} = \frac{2}{1+n}$$

In parallel

$$R_{eq} = \frac{10}{n}$$

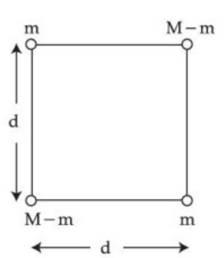
$$i_p = \frac{20}{\frac{10}{n}+10} = \frac{2n}{1+n}$$

$$\frac{i_p}{i_s} = 20$$

$$\frac{\left(\frac{2n}{1+n}\right)}{\left(\frac{2}{1+n}\right)} = 20$$

n = 20

Q.3 A body of mass (2M) splits into four masses {m, M − m, m, M − m}, which are rearranged to form a square as shown in the figure. The ratio of $\frac{M}{m}$ for which, the gravitational potential energy of the system becomes maximum is $x : 1$. The value of x is _________.



Given **2**
Answer :

Ans: 2

Sol: Energy is maximum when mass is split equally so $\frac{M}{m} = 2$

Q.4 A uniform conducting wire of length is 24a, and resistance R is wound up as a current carrying coil in the shape of an equilateral triangle of side 'a' and then in the form of a square of side 'a'. The coil is connected to a voltage source V_0. The ratio of magnetic moment of the coils in case of equilateral triangle to that for square is $1 : \sqrt{y}$ where y is __________.

Given Answer : **3**

Ans: 3

Sol: In triangle shape $N_t = \frac{24a}{3a} = 8$

In square $N_s = \frac{24a}{4a} = 6$

$$\frac{M_t}{M_s} = \frac{N_t I A_t}{N_s I A_s} \text{ [I will be same in both]} = \frac{8 \times \frac{\sqrt{3}}{4} \times a^2}{6 \times a^2}$$

$$\frac{M_t}{M_s} = \frac{1}{\sqrt{3}}$$

y = 3

Q.5 Two persons A and B perform same amount of work in moving a body through a certain distance d with application of forces acting at angles 45° and 60° with the direction of displacement respectively. The ratio of force applied by person A to the force applied by person B is $\frac{1}{\sqrt{x}}$. The value of x is __________.

Given Answer : **3**

Ans: 2

Sol: Given $W_A = W_B$

$$F_A d \cos 45° = F_B d \cos 60°$$

$$F_A \times \frac{1}{\sqrt{2}} = F_B \times \frac{1}{2}$$

$$\frac{F_A}{F_B} = \frac{\sqrt{2}}{2} = \frac{1}{\sqrt{2}}$$

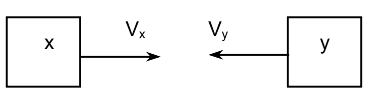
x = 2

Q.6 Two cars X and Y are approaching each other with velocities 36 km/h and 72 km/h respectively. The frequency of a whistle sound as emitted by a passenger in car X, heard by the passenger in car Y is 1320 Hz. If the velocity of sound in air is 340 m/s, the actual frequency of the whistle sound produced is __________ Hz.

Given Answer : **--**

Ans: 1210

Sol:

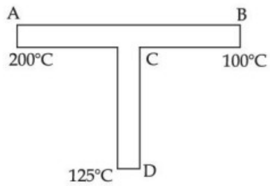


V_x = 36 km/hr = 10 m/s
V_y = 72 km/hr = 20 m/s
By Doppler's effect

$$F' = F_0\left(\frac{V \pm V_0}{V \pm V_s}\right)$$

$$1320 = F_0\left(\frac{340+20}{340-10}\right) \Rightarrow F_0 = 1210 \text{ Hz}$$

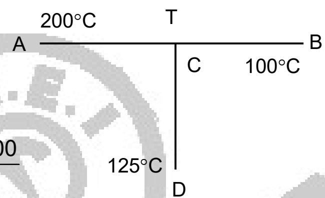
Q.7 A rod CD of thermal resistance 10.0 KW^{-1} is joined at the middle of an identical rod AB as shown in figure. The ends A, B and D are maintained at 200°C, 100°C and 125°C respectively. The heat current in CD is P watt. The value of P is __________.



Given --
Answer :

Ans: 2

Sol: Rods are identical so



$R_{AB} = R_{CD} = 10\ Kw^{-1}$

C is mid-point of AB, so

$R_{AC} = R_{CB} = 5\ Kw^{-1}$

At point C

$$\frac{200-T}{5} = \frac{T-125}{10} + \frac{T-100}{5}$$

$2(200 - T) = T - 125 + 2(T - 100)$

$400 - 2T = T - 125 + 2T - 200$

$$T = \frac{725}{5} = 145°C$$

$$I_h = \frac{145-125}{10} w = \frac{20}{10} w$$

$I_h = 2w$

Q.8 A transmitting antenna has a height of 320 m and that of receiving antenna is 2000 m. The maximum distance between them for satisfactory communication in line of sight mode is 'd'. The value of 'd' is __________ km.

Given --
Answer :

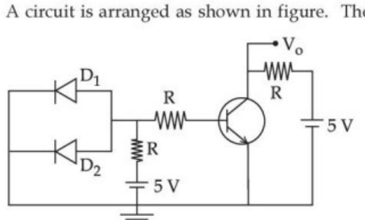
Ans: 224

Sol: $d_m = \sqrt{2Rh_T} + \sqrt{2Rh_R}$

$$d_m\left(\sqrt{2 \times 6400 \times 10^3 \times 320} + \sqrt{2 \times 6400 \times 10^3 \times 2000}\right) m$$

$d_m = 224$ km

Q.9 A circuit is arranged as shown in figure. The output voltage V_o is equal to __________V.



Given **4**
Answer :

Ans: 5

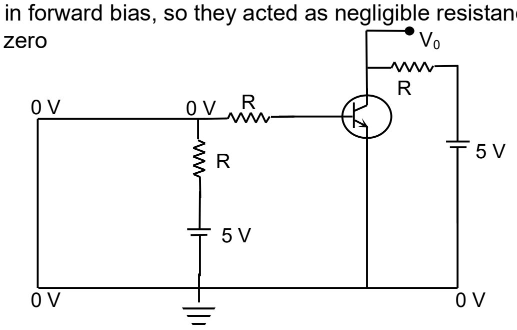
Sol: As diodes D_1 and D_2 are in forward bias, so they acted as negligible resistances

$\Rightarrow$ input voltage become zero

$\Rightarrow$ input current is zero

$\Rightarrow$ output current is zero

$\Rightarrow$ V_0 = 5 volt



Q.10 The alternating current is given by

$$i = \left\{\sqrt{42}\,\sin\left(\frac{2\pi}{T}\,t\right)+10\right\}A$$

The r.m.s. value of this current is __________ A.

Given --
Answer :

Ans: 11

Sol: $f_{rms}^2 = f_{1rms}^2 + f_{2rms}^2$

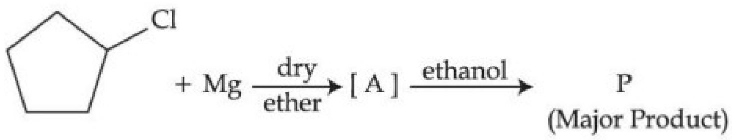
$$= \left(\frac{\sqrt{42}}{\sqrt{2}}\right)^2 + 10^2$$

$= 121 \Rightarrow f_{rms} = 11$ A

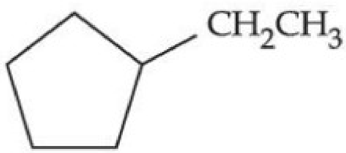
PART – B – CHEMISTRY

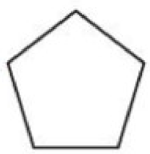
Section A

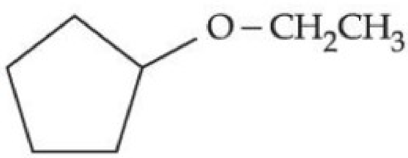
Q.1 In the following sequence of reactions the P is :

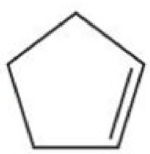


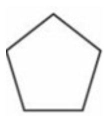
Options

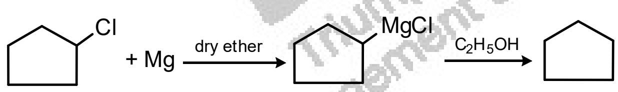
1. 

2. 

3. 

4. 

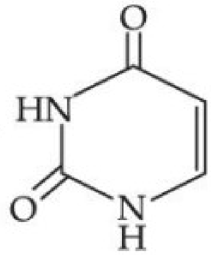
Ans: 

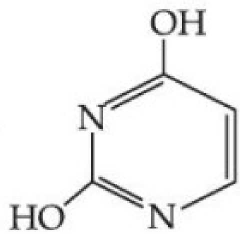
Sol: 

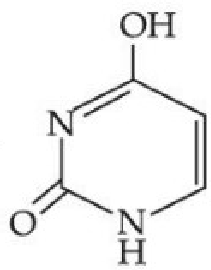
The reaction between ethanol and Grignard reagent is an acid-base reaction. Due to the presence of an acidic hydrogen in alcohol, Grignard reagent is converted to hydrocarbon.

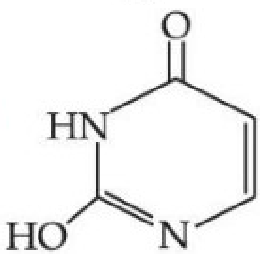
Q.2 Out of following isomeric forms of uracil, which one is present in RNA ?

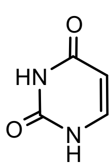
Options

1. 

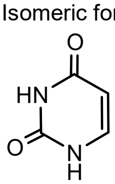
2. 

3. 

4. 

Ans: 

Sol: Isomeric form of uracil present in RNA is

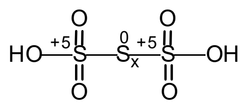


Q.3 In polythionic acid, $H_2S_xO_6$ ($x = 3$ to 5) the oxidation state(s) of sulphur is/are :

Options
1. $+5$ only
2. 0 and $+5$ only
3. $+6$ only
4. $+3$ and $+5$ only

Ans: 0 and +5 only

Sol:



Q.4 Given below are two statements : one is labelled as **Assertion (A)** and the other is labelled as **Reason (R)**.

Assertion (A) : Synthesis of ethyl phenyl ether may be achieved by Williamson synthesis.

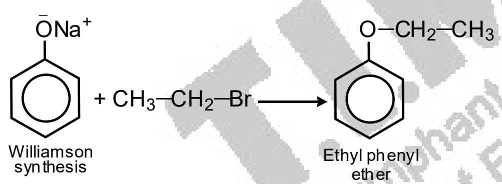
Reason (R) : Reaction of bromobenzene with sodium ethoxide yields ethyl phenyl ether.

In the light of the above statements, choose the **most appropriate** answer from the options given below :

Options
1. Both **(A)** and **(R)** are correct but **(R)** is NOT the correct explanation of **(A)**
2. **(A)** is correct but **(R)** is not correct
3. Both **(A)** and **(R)** are correct and **(R)** is the correct explanation of **(A)**
4. **(A)** is not correct but **(R)** is correct

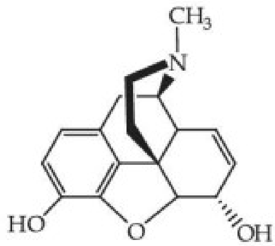
Ans: (A) is correct but (R) is not correct

Sol:

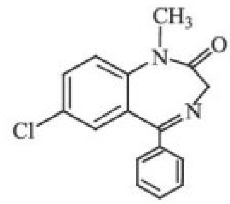


Because of partial double bond character of C–Br bond in bromobenzene, it does not undergo Wiliamson synthesis to yield ethyl phenyl ether.

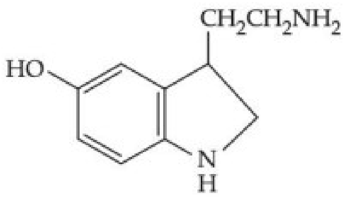
Q.5



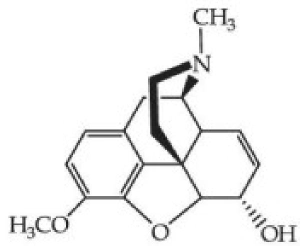
(A)



(B)



(C)



(D)

The correct statement about (A), (B), (C) and (D) is :

Options
1. (B), (C) and (D) are tranquillizers
2. (B) and (C) are tranquillizers
3. (A), (B) and (C) are narcotic analgesics
4. (A) and (D) are tranquillizers

Ans: (B) and (C) tranquilizers

Sol:
(A) – Morphine
(B) – Codeine } both are narcotic analgesics

(B) – Valium
(C) – Serotonin } both are tranquilizers

Q.6 Match items of **List - I** with those of **List - II** :

	List - I (Property)		List - II (Example)
(a)	Diamagnetism	(i)	MnO
(b)	Ferrimagnetism	(ii)	O_2
(c)	Paramagnetism	(iii)	NaCl
(d)	Antiferromagnetism	(iv)	Fe_3O_4

Choose the **most appropriate** answer from the options given below :

Options
1. (a)-(iv), (b)-(ii), (c)-(i), (d)-(iii)
2. (a)-(i), (b)-(iii), (c)-(iv), (d)-(ii)
3. (a)-(iii), (b)-(iv), (c)-(ii), (d)-(i)
4. (a)-(ii), (b)-(i), (c)-(iii), (d)-(iv)

Ans: (a)-(iii), (b)-(iv), (c)-(ii), (d)-(i)

Sol: Diamagnetism – NaCl
Ferrimagnetism – Fe_3O_4
Paramagnetism – O_2
Antiferromagnetism – MnO

Q.7 The gas 'A' is having very low reactivity reaches to stratosphere. It is non-toxic and non-flammable but dissociated by UV-radiations in stratosphere. The intermediates formed initially from the gas 'A' are :

Options

1. $Cl\dot{O} + \dot{C}F_2Cl$
2. $\dot{C}l + \dot{C}F_2Cl$
3. $\dot{C}H_3 + \dot{C}F_2Cl$
4. $Cl\dot{O} + \dot{C}H_3$

Ans: $\dot{C}l + \dot{C}F_2Cl$

Sol: $CF_2Cl_{2(g)} \xrightarrow{uv} \dot{C}l_{(g)} + \dot{C}F_2Cl_{(g)}$

In stratosphere, CFC get broken down by uv radiations releasing $\dot{C}l + \dot{C}F_2Cl$

Q.8 The major product of the following reaction is :

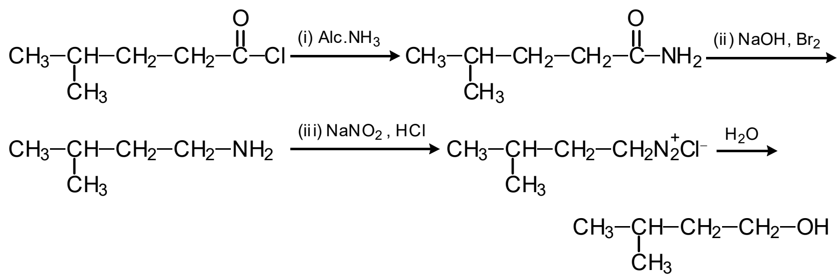
$$CH_3-\underset{}{\overset{CH_3}{\overset{|}{C}H}}-CH_2-CH_2-\overset{O}{\overset{\|}{C}}-Cl \xrightarrow[\text{(iii) } NaNO_2,\ HCl \quad \text{(iv) } H_2O]{\text{(i) alcoholic } NH_3 \quad \text{(ii) } NaOH,\ Br_2} \text{Major Product}$$

Options

1. $CH_3-\underset{CH_3}{\underset{|}{CH}}-\overset{Br}{\overset{|}{CH}}-CH_2OH$
2. $CH_3-\underset{CH_3}{\underset{|}{CH}}-CH_2-CH_2-CH_2OH$
3. $CH_3-\underset{CH_3}{\underset{|}{CH}}-CH_2-CH_2-Cl$
4. $CH_3-\underset{CH_3}{\underset{|}{CH}}-CH_2-CH_2OH$

Ans: $CH_3-\underset{CH_3}{\underset{|}{CH}}-CH_2-CH_2OH$

Sol:



Q.9 Acidic ferric chloride solution on treatment with excess of potassium ferrocyanide gives a Prussian blue coloured colloidal species. It is :

Options

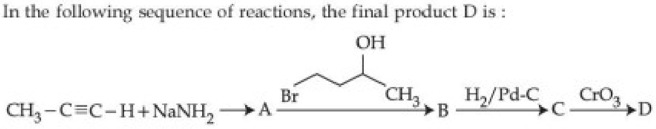
1. $KFe[Fe(CN)_6]$
2. $HFe[Fe(CN)_6]$
3. $K_5Fe[Fe(CN)_6]_2$
4. $Fe_4[Fe(CN)_6]_3$

Ans: $KFe[Fe(CN)_6]$

Sol: $FeCl_3 + K_4[Fe(CN)_6] \rightarrow KFe[Fe(CN)_6] + 3KCl$

If we add ferric chloride over potassium ferrocyanide, it forms a white precipitate that will become blue. Since amount of ferrichloride is low compared to potassium ferrocyanide, not all potassium is displaced.

Q.10 In the following sequence of reactions, the final product D is :

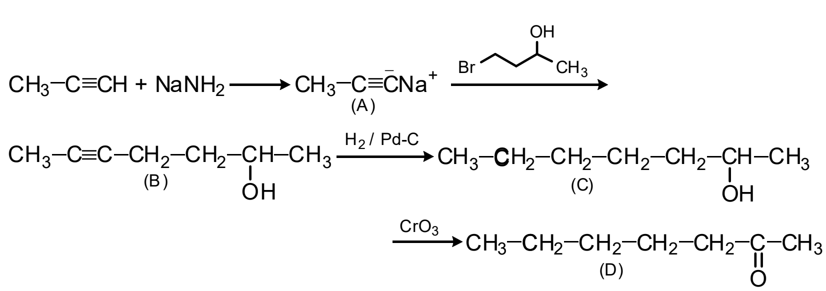


Options

1. $CH_3-CH=CH-CH_2-CH_2-CH_2-COOH$
2. $H_3C-CH=CH-CH(OH)-CH_2-CH_2-CH_3$
3. $H_3C-CH_2-CH_2-CH_2-CH_2-\overset{O}{\overset{\|}{C}}-H$
4. $CH_3-CH_2-CH_2-CH_2-CH_2-\overset{O}{\overset{\|}{C}}-CH_3$

Ans: $CH_3-CH_2-CH_2-CH_2-CH_2-\overset{O}{\overset{\|}{C}}-CH_3$

Sol:



Q.11 The nature of oxides V_2O_3 and CrO is indexed as 'X' and 'Y' type respectively. The correct set of X and Y is :

Options
1. $X = \text{basic}$ $Y = \text{basic}$
2. $X = \text{amphoteric}$ $Y = \text{basic}$
3. $X = \text{basic}$ $Y = \text{amphoteric}$
4. $X = \text{acidic}$ $Y = \text{acidic}$

Ans: X = basic Y = basic

Sol: $\overset{+3}{V_2O_3}$ is basic
X

$\overset{+2}{CrO}$ is basic
Y

Oxide in lower oxidation state of element is more basic.

Q.12 The number of water molecules in gypsum, dead burnt plaster and plaster of Paris, respectively are :

Options
1. 2, 0 and 1
2. 0.5, 0 and 2
3. 2, 0 and 0.5
4. 5, 0 and 0.5

Ans: 2, 0 and 0.5

Sol: Gypsum – $CaSO_4.2H_2O$
Dead burnt – $CaSO_4$
Plaster of Paris – $CaSO_4.\frac{1}{2}H_2O$

Q.13 In which one of the following molecules strongest back donation of an electron pair from halide to boron is expected ?

Options
1. BF_3
2. BBr_3
3. BI_3
4. BCl_3

Ans: BF_3

Sol: Order of back bonding in boron trihalides is
$BF_3 > BCl_3 > BBr_3 > BI_3$

Q.14 The unit of the van der Waals gas equation parameter 'a' in

$$\left(P + \frac{an^2}{V^2}\right)(V - nb) = nRT \text{ is :}$$

Options
1. $atm\ dm^6\ mol^{-2}$
2. $dm^3\ mol^{-1}$
3. $kg\ m\ s^{-1}$
4. $kg\ m\ s^{-2}$

Ans: atm dm^6 mol^{-2}

Sol: $\frac{an^2}{v^2} = atm$

$$a = \frac{atm \times (dm^3)^2}{(mol)^2} = atm.dm^6.mol^{-2}$$

Q.15 Which of the following is **not** a correct statement for primary aliphatic amines ?

Options
1. Primary amines are less basic than the secondary amines.
2. The intermolecular association in primary amines is less than the intermolecular association in secondary amines.
3. Primary amines on treating with nitrous acid solution form corresponding alcohols except methyl amine.
4. Primary amines can be prepared by the Gabriel phthalimide synthesis.

Ans: The intermolecular association in primary amines is less than the intermolecular association in secondary amines

Sol: The intermolecular association in primary amines is greater than the secondary due to the availability of two hydrogen atoms.

Q.16 Which refining process is generally used in the purification of low melting metals ?

Options
1. Electrolysis
2. Chromatographic method
3. Liquation
4. Zone refining

Ans: Liquation

Sol: To purity those impure metals which has lower melting point than the melting point of impurities liquation method is used

Q.17 Match **List - I** with **List - II** :

	List - I (Species)		**List - II** (No. of lone pairs of electrons on the central atom)
(a)	XeF_2	(i)	0
(b)	XeO_2F_2	(ii)	1
(c)	XeO_3F_2	(iii)	2
(d)	XeF_4	(iv)	3

Choose the **most appropriate** answer from the options given below :

Options
1. (a)-(iii), (b)-(ii), (c)-(iv), (d)-(i)
2. (a)-(iv), (b)-(ii), (c)-(i), (d)-(iii)
3. (a)-(iv), (b)-(i), (c)-(ii), (d)-(iii)
4. (a)-(iii), (b)-(iv), (c)-(ii), (d)-(i)

Ans: (a)-(iv), (b)-(ii), (c)-(i), (d)-(iii)

Sol:

Species	**No. of lone pairs of electrons on the central atom**
XeF_2	3
XeO_2F_2	1
XeO_3F_2	0
XeF_4	2

Q.18 Deuterium resembles hydrogen in properties but :

Options
1. reacts slower than hydrogen
2. reacts just as hydrogen
3. reacts vigorously than hydrogen
4. emits β^+ particles

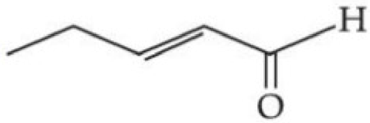
Ans: reacts slower than hydrogen

Sol: Bond dissociation energy of Deuterium is greater than hydrogen. Therefore D_2 reacts slower than H_2

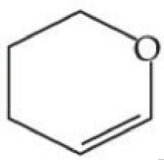
Q.19 The structure of the starting compound **P** used in the reaction given below is :

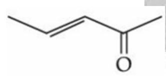
Options

1. 

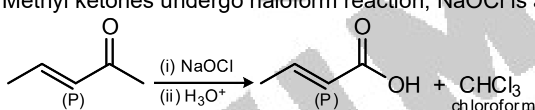
2. 

3. 

4. 

Ans: 

Sol: Methyl ketones undergo haloform reaction, NaOCl is a reagent used in haloform reaction



Q.20 Tyndall effect is more effectively shown by :

Options
1. lyophobic colloid
2. true solution
3. suspension
4. lyophilic colloid

Ans: lyophobic colloid

Sol: Lyophobic colloids show Tyndall effect

Section B

Q.1 In Carius method for estimation of halogens, 0.2 g of an organic compound gave 0.188 g of AgBr. The percentage of bromine in the compound is ________. (Nearest integer)
[Atomic mass : Ag = 108, Br = 80]

Given **69**
Answer :

Ans: 40

Sol: % of bromine $= \frac{80}{188} \times \frac{\text{Wt. of AgBr formed}}{\text{Wt. of the compound}} \times 100 = \frac{80}{188} \times \frac{0.188}{0.2} \times 100 = 40\%$

Q.2 The number of moles of CuO, that will be utilized in Dumas method for estimating nitrogen in a sample of 57.5 g of N,N-dimethylaminopentane is _________ $\times 10^{-2}$. (Nearest integer)

Given --
Answer :

Ans: 1125

Sol: Moles in N,N-dimethyl amino pentane $= \frac{57.5}{115} = 0.5$ mol

$$C_7H_{17}N + \frac{45}{2}CuO \longrightarrow 7CO_2 + \frac{17}{2}H_2O + \frac{1}{2}N_2 + \frac{45}{2}Cu$$

1 mol $C_7H_{17}N$ reacts with $\frac{45}{2}$ mol of CuO

$\therefore$ 0.5 mol $C_7H_{17}N$ reacts with $\frac{45}{2} \times 0.5$ mol of CuO = 11.25 = 1125 $\times 10^{-2}$

Q.3 The number of f electrons in the ground state electronic configuration of Np $(Z = 93)$ is _________. (Integer answer)

Given 7
Answer :

Ans: 18

Sol: Np $(Z = 93) = 1s^2\ 2s^2\ 2p^6\ 3s^2\ 3p^6\ 4s^2\ 3d^{10}\ 4p^6\ 5s^2\ 4d^{10}\ 5p^6\ 6s^2\ \underline{4f^{14}}\ 5d^{10}\ 6p^6\ 7s^2\ \underline{5f^4}\ 6d^1$

No. of electrons = 14 + 4 = 18

Q.4 The kinetic energy of an electron in the second Bohr orbit of a hydrogen atom is equal to $\frac{h^2}{x\,m\,a_0^2}$. The value of 10 x is _________. (a_0 is radius of Bohr's orbit)

(Nearest integer)

[Given : $\pi = 3.14$]

Given 4
Answer :

Ans: 3155

Sol: n = 2

$$KE = \frac{n^2h^2}{8\pi^2mr^2} = \frac{4h^2}{8\pi^2m(4a_0)^2} = \left(\frac{4}{8\pi^2 \times 16}\right)\frac{h^2}{ma_0^2}$$

Given, $KE = \frac{h^2}{xma_0^2}$

$$\frac{1}{x} = \frac{4}{8\pi^2 \times 16}$$

$$x = \frac{8\pi^2 \times 16}{4} = \frac{8 \times (3.14)^2 \times 16}{4} = 8 \times (3.14)^2 \times 4$$

x = 315.507

10x = 315.507 $\times$ 10 = 3155.07

Q.5 The number of moles of NH_3, that must be added to 2 L of 0.80 M $AgNO_3$ in order to reduce the concentration of Ag^+ ions to 5.0×10^{-8} M ($K_{formation}$ for $[Ag(NH_3)_2]^+ = 1.0 \times 10^8$) is _________. (Nearest integer)

[Assume no volume change on adding NH_3]

Given --
Answer :

Ans: 4

Sol: Let the number of moles = x

$$Ag^{+}_{(aq)} + 2NH_{3(aq)} \rightleftharpoons [AgNH_3)]^{+}_{(aq)}$$

t = o 0.8 $x/2$

t = ∝ (infinity) 5×10^{-8} $\left(\frac{x}{2} - 1.61\right)$ 0.8

$$\frac{0.8}{(5\times10^{-8})\left(\frac{x}{2}-1.6\right)^2} = 1 \times 10^8$$

$$\frac{x}{2} - 1.6 = 0.4$$

x = 4

Q.6 The reaction that occurs in a breath analyser, a device used to determine the alcohol level in a person's blood stream is

$$2K_2Cr_2O_7 + 8H_2SO_4 + 3C_2H_6O \rightarrow 2Cr_2(SO_4)_3 + 3C_2H_4O_2 + 2K_2SO_4 + 11H_2O$$

If the rate of appearance of $Cr_2(SO_4)_3$ is 2.67 mol min^{-1} at a particular time, the rate of disappearance of C_2H_6O at the same time is __________ mol min^{-1}. (Nearest integer)

Given --
Answer :

Ans: 4

Sol: $\frac{1}{3}\frac{d}{dt}[C_2H_6O] = \frac{1}{2}\frac{d}{dt}[Cr_2(SO_4)_3]$

$$\frac{d}{dt}[C_2H_6O] = \frac{3}{2}\frac{d}{dt}[Cr_2(SO_4)_3]$$

$$= \frac{3}{2} \times 2.76 \text{ mol/min} = 4.005$$

Q.7 200 mL of 0.2 M HCl is mixed with 300 mL of 0.1 M NaOH. The molar heat of neutralization of this reaction is −57.1 kJ. The increase in temperature in °C of the system on mixing is $x \times 10^{-2}$. The value of x is __________. (Nearest integer)

[Given : Specific heat of water = 4.18 J g^{-1} K^{-1}

Density of water = 1.00 g cm^{-3}]

(Assume no volume change on mixing)

Given **1**
Answer :

Ans: 82

Sol: Milli moles of HCl = 200 × 0.2 = 40

Milli moles of NaOH = 300 × 0.1 = 30

Heat released $= \frac{300}{1000} \times 5.7 \times 1000 = 1713$ J

Mass of solution = V × d = 500 × 1 = 500 gm

$$\Delta T = \frac{q}{mc} = \frac{1713}{500 \times 4.18} = 0.8196 K = 81.96 \times 10^{-2} K$$

Q.8 When 10 mL of an aqueous solution of $KMnO_4$ was titrated in acidic medium, equal volume of 0.1 M of an aqueous solution of ferrous sulphate was required for complete discharge of colour. The strength of $KMnO_4$ in grams per litre is __________ $\times 10^{-2}$. (Nearest integer)

[Atomic mass of K = 39, Mn = 55, O = 16]

Given **7**
Answer :

Ans: 316

Sol: $2KMnO_4 + 8H_2SO_4 + 10FeSO_4 \rightarrow K_2SO_4 + 2MnSO_4 + 8H_2O + 5Fe_2(SO_4)_3$

(Or)

$2KMnO_4 + 16H^+ + 10Fe^{2+} \rightarrow 2Mn^{2+} + 10Fe^{3+} + 8H_2O$

1 mol $KmnO_4$ require $\frac{10}{2}$ mol $FeSO_4$

n = 1

$N_1V_1 = N_2V_2$

$5 \times M \times 10 = 1 \times 10 \times 0.1$

$M = \frac{0.1}{5} = 0.02$

Molar mass of $KMnO_4$ = 158 g / mol

Strength = $0.02 \times 158 = 3.16$ g / L = 316×10^{-2}

Q.9 1 mol of an octahedral metal complex with formula $MCl_3 \cdot 2L$ on reaction with excess of $AgNO_3$ gives 1 mol of AgCl. The denticity of Ligand L is ________. (Integer answer)

Given --
Answer :

Ans: 2

Sol: $MCl_3.2L \xrightarrow{\text{Excess } AgNO_3} 1 \text{ mole } AgCl$

This means Cl^- ion present in ionization sphere

$\therefore$ Formula = $[MCl_2.L_2]$ Cl

For octahedral complex coordination number is 6

$\therefore$ L is a didentate ligand

Q.10 1 kg of 0.75 molal aqueous solution of sucrose can be cooled up to $-4°C$ before freezing. The amount of ice (in g) that will be separated out is ________. (Nearest integer)

[Given : $K_f(H_2O) = 1.86$ K kg mol^{-1}]

Given --
Answer :

Ans: 518

Sol: Let the mass of water initially present is 'a' gm

$\therefore$ Mass of sucrose in 1 kg soln = (1000 − a) g

$\therefore n_{sucrose} = \frac{1000 - a}{342}$

$m = \frac{n_{solute}}{w_{solvent\ in\ kg}} = \frac{\frac{1000-a}{342}}{\frac{a}{1000}}$

$0.75 = \frac{(1000 - a)1000}{342\,a}$

Solving, a = 795.86 g

$\therefore n_{sucrose} = \frac{1000 - 795.86}{342} = 0.5969$

Let the new mass of water be 'x' g

i.e., $4 = \frac{0.5969}{a} \times 1.86$

a = 0.2775 kg = 277.5 g

$\therefore$ Ice separated = 795.86 − 277.5 g = 518.4 g

PART – C – MATHEMATICS

Section A

Q.1

If α, β are the distinct roots of $x^2+bx+c=0$, then $\lim\limits_{x\to\beta}\dfrac{e^{2(x^2+bx+c)}-1-2(x^2+bx+c)}{(x-\beta)^2}$ is equal to :

Options

1. b^2-4c
2. b^2+4c
3. $2(b^2+4c)$
4. $2(b^2-4c)$

Ans: $2(b^2-4c)$

Sol: $\lim\limits_{x\to\beta}\dfrac{e^{2(x^2+bx+c)}-1-2(x^2+bx+c)}{(x-\beta)^2}$

$$\Rightarrow \lim_{x\to\beta}\frac{1\left(1+\frac{2(x^2+bx+c)}{1!}+\frac{2^2(x^2+bx+c)^2}{2!}\right)-1-2(x^2+bx+c)}{(x-\beta)^2}$$

$$\Rightarrow \lim_{x\to\beta}\frac{2(x^2+bx+1)^2}{(x-\beta)^2}$$

$$\Rightarrow 2(\beta-\alpha)^2=2\left[(\alpha+\beta)^2-4\alpha\beta\right]=2(b^2-4c)$$

Q.2

When a certain biased die is rolled, a particular face occurs with probability $\frac{1}{6}-x$ and its opposite face occurs with probability $\frac{1}{6}+x$. All other faces occur with probability $\frac{1}{6}$. Note that opposite faces sum to 7 in any die. If $0<x<\frac{1}{6}$, and the probability of obtaining total sum = 7, when such a die is rolled twice, is $\frac{13}{96}$, then the value of x is :

Options

1. $\frac{1}{8}$
2. $\frac{1}{16}$
3. $\frac{1}{9}$
4. $\frac{1}{12}$

Ans: $\frac{1}{8}$

Sol: Given, probability of obtaining total sum 7 = probability of getting opposite faces.

Probability of getting opposite faces = $2\left[\left(\frac{1}{6}-x\right)\left(\frac{1}{6}+x\right)+\frac{1}{6}\times\frac{1}{6}+\frac{1}{6}\times\frac{1}{6}\right]=\frac{13}{96}$

$\Rightarrow 2x^2=\frac{1}{6}-\frac{13}{96}=\frac{1}{32}\Rightarrow x^2=\frac{1}{64}$

$x=\frac{1}{8}$

Q.3 If for $x, y \in \mathbf{R}$, $x > 0$, $y=\log_{10}x+\log_{10}x^{1/3}+\log_{10}x^{1/9}+\ldots$ upto ∞ terms and $\frac{2+4+6+\ldots+2y}{3+6+9+\ldots+3y}=\frac{4}{\log_{10}x}$, then the ordered pair (x, y) is equal to :

Options
1. $(10^2, 3)$
2. $(10^6, 9)$
3. $(10^4, 6)$
4. $(10^6, 6)$

Ans: $(10^6, 9)$

Sol: $\frac{2(1+2+3+\ldots.+y)}{3(1+2+3+\ldots.+y)}=\frac{4}{\log_{10}x}$

$\Rightarrow \log_{10}x=6\Rightarrow x=10^6$

Also, $y=(\log_{10}x)+\left(\log_{10}x^{\frac{1}{3}}\right)+\left(\log_{10}x^{\frac{1}{9}}\right)+\ldots\infty = \log_{10}x^{1+\frac{1}{3}+\frac{1}{9}+\ldots\ldots\infty}$

$=\left(1+\frac{1}{3}+\frac{1}{9}+\ldots.\infty\right)\log_{10}x$

$=\left(\frac{1}{1-\frac{1}{3}}\right)\log_{10}x=\frac{3}{2}\times 6=9$

Q.4 Equation of a plane at a distance $\sqrt{\frac{2}{21}}$ from the origin, which contains the line of intersection of the planes $x-y-z-1=0$ and $2x+y-3z+4=0$, is :

Options
1. $-x+2y+2z-3=0$
2. $3x-y-5z+2=0$
3. $4x-y-5z+2=0$
4. $3x-4z+3=0$

Ans: $4x-y-5z+2=0$

Sol: Required equation of plane

$(x-y-z-1)+\lambda(2x+y-3z+4)=0$

Since distance from origin is $\frac{2}{\sqrt{21}}$,

$$\frac{|4\lambda - 1|}{\sqrt{(2\lambda+1)^2 + (\lambda-1)^2 + (-3\lambda-1)^2}} \frac{\sqrt{2}}{\sqrt{21}}$$

Solving we get

$\Rightarrow \lambda = \frac{1}{2}$ or $\frac{15}{154}$

When $\lambda = \frac{1}{2}$, The plane is $4x - y - 5z + 2 = 0$

Q.5 If $(\sin^{-1}x)^2 - (\cos^{-1}x)^2 = a$; $0 < x < 1$, $a \neq 0$, then the value of $2x^2 - 1$ is :

Options

1. $\cos\left(\frac{2a}{\pi}\right)$
2. $\sin\left(\frac{4a}{\pi}\right)$
3. $\sin\left(\frac{2a}{\pi}\right)$
4. $\cos\left(\frac{4a}{\pi}\right)$

Ans: $\sin\left(\frac{2a}{\pi}\right)$

Sol: Given a = $(\sin^{-1}x)^2 - (\cos^{-1}x)^2$

$= (\sin^{-1}x + \cos^{-1}x)(\sin^{-1}x - \cos^{-1}x)$

$= \frac{\pi}{2}\left(\frac{\pi}{2} - 2\cos^{-1}x\right) \Rightarrow \frac{2\pi}{a} = \frac{\pi}{2} - 2\cos^{-1}x$

$\Rightarrow 2\cos^{-1}x = \frac{\pi}{2} - \frac{2a}{\pi}$

Now $2\cos^{-1}x = \cos^{-1}(2x^2 - 1)$

$\Rightarrow \cos^{-1}(2x^2 - 1) = \frac{\pi}{2} - \frac{2a}{\pi}$

$\Rightarrow 2x^2 - 1 = \cos\left(\frac{\pi}{2} - \frac{2a}{\pi}\right)$

$= \sin\left(\frac{2a}{\pi}\right)$

Q.6

Let $y = y(x)$ be the solution of the differential equation $\frac{dy}{dx} = 2(y + 2\sin x - 5)\,x - 2\cos x$ such that $y(0) = 7$. Then $y(\pi)$ is equal to :

Options

1. $7e^{\pi^2} + 5$
2. $3e^{\pi^2} + 5$
3. $e^{\pi^2} + 5$
4. $2e^{\pi^2} + 5$

Ans: $2e^{\pi^2} + 5$

Sol: $\frac{dy}{dx} = 2xy = 2(2\sin x - 5)x - 2\cos x$

$IF = e^{-x^2}$

So, $y.e^{-x^2} = \int e^{-x^2}(2x(2\sin x - 5) - 2\cos x)dx$

$\Rightarrow y.e^{-x^2} = e^{-x^2}(5 - 2\sin x) + C$

$\Rightarrow y = 5 - 2\sin x + C.e^{x^2}$

$y(0) = 7$

$\Rightarrow 7 = 5 + C \Rightarrow C = 2$

So, $\Rightarrow y = 5 - 2\sin x + 2.e^{x^2}$

At $x = \pi$

$y = 5 + 2e^{\pi^2}$

Q.7 The distance of the point $(1, -2, 3)$ from the plane $x - y + z = 5$ measured parallel to a line, whose direction ratios are $2, 3, -6$ is :

Options

1. 3
2. 2
3. 1
4. 5

Ans: 1

Sol:

Equation of the line passing through $(1,-2,3)$ and dr's 2, 3, –6 is $\frac{x-1}{2} = \frac{y+2}{3} = \frac{z-3}{-6} = \lambda$

$\therefore$ General point is $(2\lambda + 1, 3\lambda + 2, -6\lambda + 3)$. Since its satisfies $x - y + z = 5$,

$\lambda = \frac{1}{7}$

So, point = $\left(\frac{9}{7}, -\frac{11}{7}, \frac{15}{7}\right)$

Distance from $(1,-2,3)$ = $\sqrt{\left(1 - \frac{9}{7}\right)^2 + \left(-2 + \frac{11}{7}\right)^2 + \left(3 - \frac{15}{7}\right)^2}$

$= \sqrt{\left(\frac{4}{49}\right) + \frac{9}{49} + \frac{36}{49}} = 1$

Q.8 $\sum_{k=0}^{20} \left({}^{20}C_k\right)^2$ is equal to :

Options
1. ${}^{41}C_{20}$
2. ${}^{40}C_{20}$
3. ${}^{40}C_{21}$
4. ${}^{40}C_{19}$

Ans: ${}^{40}C_{20}$

Sol: $\sum_{k=0}^{20} {}^{40}C_k \, {}^{40}C_{20-k}$

$= {}^{40}C_{20}$

Q.9 A tangent and a normal are drawn at the point P(2, −4) on the parabola $y^2 = 8x$, which meet the directrix of the parabola at the points A and B respectively. If Q(a, b) is a point such that AQBP is a square, then $2a + b$ is equal to :

Options
1. −20
2. −18
3. −16
4. −12

Ans: –16

Sol:

Equation of tangent at (2, –4) is x + y + 2 = 0 .... (1)

equation of normal $x - y + k = 0$

$k = -6$

Directrix of the parabola is x = –2

$\Rightarrow A = (-2, 0), B = (-2, -8)$

$\Rightarrow m_{AQ} . m_{AP} = -1$

$\Rightarrow \left(\frac{b}{a+2}\right)\left(\frac{4}{-4}\right) = -1 \Rightarrow a + 2 = b$ ....(1)

Also, PQ is parallel to x-axis

$\Rightarrow$ b = –4 and a = –6

$\Rightarrow$ 2a + b = –16

Q.10 A wire of length 20 m is to be cut into two pieces. One of the pieces is to be made into a square and the other into a regular hexagon. Then the length of the side (in meters) of the hexagon, so that the combined area of the square and the hexagon is minimum, is :

Options

1. $\dfrac{10}{2+3\sqrt{3}}$
2. $\dfrac{5}{3+\sqrt{3}}$
3. $\dfrac{5}{2+\sqrt{3}}$
4. $\dfrac{10}{3+2\sqrt{3}}$

Ans: $\dfrac{10}{3+2\sqrt{3}}$

Sol: Let x be the length of the wire cut into square and $20-x$ be the length of the wire cut into regular hexagon

Area of square = $\left(\dfrac{x}{4}\right)^2$; Area of regular hexagon $=6\times\dfrac{\sqrt{3}}{4}\left(\dfrac{20-x}{6}\right)^2$

Total area = A(x) = $\dfrac{x^2}{16}+\dfrac{3\sqrt{3}}{2}\dfrac{(20-x)^2}{36}$

A'(x) = $\dfrac{2x}{16}+\dfrac{3\sqrt{3}\times 2}{2\times 36}(20-x)(-1)$

A'(x) = 0 at x = $\dfrac{40\sqrt{3}}{3+2\sqrt{3}}$

Length of side of regular Hexagon = $\dfrac{1}{6}(20-x)$

$=\dfrac{1}{6}\left(20-\dfrac{4\sqrt{3}}{3+2\sqrt{3}}\right)=\dfrac{10}{3+2\sqrt{3}}$

Q.11 If $S=\left\{z\in\mathbb{C}:\ \dfrac{z-i}{z+2i}\in\mathbb{R}\right\}$, then :

Options

1. S is a circle in the complex plane
2. S contains exactly two elements
3. S contains only one element
4. S is a straight line in the complex plane

Ans: S is straight line in complex

Sol: Given $\dfrac{z-i}{z+2i}\in R$

$\Rightarrow \arg\left(\frac{z-i}{z+2i}\right)$ is 0 or π

$\Rightarrow$z is the locus of the joining i and –2i

$\Rightarrow$ S is straight line in complex

Q.12 If the matrix $A = \begin{pmatrix} 0 & 2 \\ K & -1 \end{pmatrix}$ satisfies $A(A^3+3I)=2I$, then the value of K is :

Options
1. 1
2. $-\frac{1}{2}$
3. -1
4. $\frac{1}{2}$

Ans: $\frac{1}{2}$

Sol: Given matrix A = $\begin{bmatrix} 0 & 2 \\ k & -1 \end{bmatrix}$

$A^4 + 3IA = 2I$

$\Rightarrow A^4 = 2I - 3A$ ......(1)

Also characteristic equation of A is

$\Rightarrow A + A^2 = 2k.I$

$\Rightarrow A^2 = 2kI - A$

$\Rightarrow A^4 = 4k^2I + A^2 - 4Ak$

= $(2k + 4k^2) - (4k+1)A$ ......(2)

Comparing (1) and (2) $4k+1=3$

$\Rightarrow$k = $\frac{1}{2}$

Q.13 If $x^2+9y^2-4x+3=0$, $x, y \in \mathbb{R}$, then x and y respectively lie in the intervals :

Options
1. $[1, 3]$ and $[1, 3]$
2. $\left[-\frac{1}{3}, \frac{1}{3}\right]$ and $[1, 3]$
3. $[1, 3]$ and $\left[-\frac{1}{3}, \frac{1}{3}\right]$
4. $\left[-\frac{1}{3}, \frac{1}{3}\right]$ and $\left[-\frac{1}{3}, \frac{1}{3}\right]$

Ans: $[1,3]$ and $\left[-\frac{1}{3}, \frac{1}{3}\right]$

Sol: $x^2 + 9y^2 - 4x + 3 = 0$

$(x^2 - 4x) + (9y^2) + 3 = 0$

$(x^2 - 4x + 4) + (9y^2) + 3 - 4 = 0$

$(x-2)^2 + (3y)^2 = 1$

$\frac{(x-2)^2}{(1)^2} + \frac{y^2}{\left(\frac{1}{3}\right)^2} = 1$ (Equation of an ellipse)

Since it is an ellipse,

$x - 2 \in [-1, 1]$ and $y \in \left[-\frac{1}{3}, \frac{1}{3}\right]$

$x \in [1, 3]$ $y \in \left[-\frac{1}{3}, \frac{1}{3}\right]$

Q.14 The statement $(p \wedge (p \to q) \wedge (q \to r)) \to r$ is :

Options
1. a fallacy
2. equivalent to $q \to \sim r$
3. equivalent to $p \to \sim r$
4. a tautology

Ans: a tautology

Sol: $(p \wedge (p \to q) \wedge (q \to r)) \to r$

$\equiv (p \wedge (\sim p \vee q) \wedge (\sim q \vee r)) \to r$

$\equiv (p \wedge q) \wedge (\sim p \vee r)) \to r$

$\equiv (p \wedge q \wedge r) \to r$

$\equiv \sim (p \wedge q \wedge r) \vee r$

$\equiv (\sim p) \vee (\sim q) \vee (\sim r) \vee r$

$\Rightarrow$ tautology

Q.15 $\int_6^{16} \frac{\log_e x^2}{\log_e x^2 + \log_e (x^2 - 44x + 484)} dx$ is equal to :

Options
1. 6
2. 10
3. 5
4. 8

Ans: 5

Sol: Let $I = \int_6^{16} \frac{\log_e x^2}{\log_e x^2 + \log_e (x^2 - 44x + 484)} dx$

$$I = \int_6^{16} \frac{\log_e x^2}{\log_e x^2 + \log_e (x-22)^2} dx$$

We know that

$$\int_a^b f(x)dx = \int_a^b f(a+b-x)dx$$

$$I = \int_6^{16} \frac{\log_e (22-x)^2}{\log_e (22-x)^2 + \log_e (22-(22-x))^2} dx$$

$$I = \int_6^{16} \frac{\log_e (22-x)^2}{\log_e x^2 + \log_e (22-x)^2} dx \quad ....(2)$$

(1) + (2)

I = 5

Q.16 Let $\frac{\sin A}{\sin B} = \frac{\sin (A-C)}{\sin (C-B)}$, where A, B, C are angles of a triangle ABC. If the lengths of the sides opposite these angles are a, b, c respectively, then :

Options

1. a^2, b^2, c^2 are in A.P.
2. $b^2 - a^2 = a^2 + c^2$
3. b^2, c^2, a^2 are in A.P.
4. c^2, a^2, b^2 are in A.P.

Ans: b^2, c^2, a^2 are in A.P

Sol: $\frac{\sin A}{\sin B} = \frac{\sin(A-C)}{\sin(C-B)}$

A + B + C = π

A = π – (B + C)

$\Rightarrow$ sinA = sin (B + C) .... (1)

Similarly sinB = sin (A + C) ... (2)

From (1) and (2)

$$\frac{\sin(B+C)}{\sin(A+C)} = \frac{\sin(A-C)}{\sin(C-B)}$$

sin (C + B). sin (C – B) = sin(A – C) sin(A + C)

$\sin^2 C - \sin^2 B = \sin^2 A - \sin^2 C$

$\{\because \sin(x+y)\sin(x-y) = \sin^2 x - \sin^2 y\}$

$2\sin^2 C = \sin^2 A + \sin^2 B$

By sine rule

$2c^2 = a^2 + b^2$

$\Rightarrow b^2, c^2$ and a^2 are in A.P

Q.17 Let us consider a curve, $y=f(x)$ passing through the point $(-2, 2)$ and the slope of the tangent to the curve at any point $(x, f(x))$ is given by $f(x)+xf'(x)=x^2$. Then :

Options

1. $x^3+xf(x)+12=0$
2. $x^3-3xf(x)-4=0$
3. $x^2+2xf(x)-12=0$
4. $x^2+2xf(x)+4=0$

Ans: $x^3-3xf(x)-4=0$

Sol: Given $y+\frac{xdy}{dx}=x^2$

$\Rightarrow \frac{dy}{dx}+\frac{y}{x}=x$

I.F = $e^{\int\frac{1}{x}dx}=x$

Solution is

$\Rightarrow xy=\frac{x^3}{3}+C$

Since it passes through $(-2,2)$,

$-4=\frac{-8}{3}+C \Rightarrow C=-\frac{4}{3}$

$\therefore 3xy=x^3-4$

i.e $3x.f(x)=x^3-4$

Q.18

If $U_n=\left(1+\frac{1}{n^2}\right)\left(1+\frac{2^2}{n^2}\right)^2 \ldots \left(1+\frac{n^2}{n^2}\right)^n$, then $\lim_{n\to\infty}(U_n)^{\frac{-4}{n^2}}$ is equal to :

Options

1. $\frac{4}{e^2}$
2. $\frac{4}{e}$
3. $\frac{16}{e^2}$
4. $\frac{e^2}{16}$

Ans: $\frac{e^2}{16}$

Sol: $U_n=\prod_{r=1}^{n}\left(1+\frac{r^2}{n^2}\right)^r$

$$L = \lim_{n\to\infty} (U_n)^{-4/n^2}$$

$$\log L = \lim_{n\to\infty} \frac{-4}{n^2}\sum_{r=1}^{n}\log\left(1+\frac{r^2}{n^2}\right)^r$$

$$\Rightarrow \log L = \lim_{n\to\infty} \frac{-4r}{n}.\frac{1}{n}\log\left(1+\frac{r^2}{n^2}\right)$$

$$\Rightarrow \log L = -4\int_0^1 x\log(1+x^2)dx = \log\left(\frac{4}{e}\right)^{-2}$$

$$L = \frac{e^2}{16}$$

Q.19 Let A be a fixed point (0, 6) and B be a moving point (2t, 0). Let M be the mid-point of AB and the perpendicular bisector of AB meets the *y*-axis at C. The locus of the mid-point P of MC is :

Options

1. $2x^2 - 3y + 9 = 0$
2. $3x^2 + 2y - 6 = 0$
3. $2x^2 + 3y - 9 = 0$
4. $3x^2 - 2y - 6 = 0$

Ans: $2x^2 + 3y - 9 = 0$

Sol: A(0,6) and B(2t,0)

Perpendicular bisector of AB is

$$(y-3) = \frac{t}{3}(x-t)$$

So, C = $\left(0, 3 - \frac{t^2}{3}\right)$

Let P be (a, b)

$$a = \frac{t}{2}; b = \left(3 - \frac{t^2}{6}\right)$$

$$\Rightarrow b = 3 - \frac{4a^2}{6} \Rightarrow 2x^2 + 3y - 9 = 0$$

Q.20

If $0 < x < 1$, then $\frac{3}{2}x^2 + \frac{5}{3}x^3 + \frac{7}{4}x^4 + \ldots$, is equal to :

Options

1. $x\left(\frac{1+x}{1-x}\right) + \log_e(1-x)$
2. $x\left(\frac{1-x}{1+x}\right) + \log_e(1-x)$
3. $\frac{1-x}{1+x} + \log_e(1-x)$
4. $\frac{1+x}{1-x} + \log_e(1-x)$

Ans: $x\left(\frac{1+x}{1-x}\right) + \log_e(1-x)$

Sol: $\frac{3}{2}x^2 + \frac{5}{3}x^3 + \frac{7}{4}x^4 + \ldots\ldots\infty$

$= \left(2-\frac{1}{2}\right)x^2 + \left(2-\frac{1}{3}\right)x^3 + \left(2-\frac{1}{4}\right)x^4 + \ldots\ldots\infty$

$= 2\left(x^2 + x^3 + x^4 + \ldots\infty\right) - \left(\frac{x^2}{2} + \frac{x^3}{3} + \frac{x^4}{4} + \ldots\infty\right)$

$= \frac{2x^2}{1-x} - (\ln(1-x) - x)$

$= \frac{2x^2}{1-x} + x - \ln(1-x)$

$= \frac{x(1+x)}{(1-x)} - \ln(1-x)$

Section B

Q.1 A number is called a palindrome if it reads the same backward as well as forward. For example 285582 is a six digit palindrome. The number of six digit palindromes, which are divisible by 55, is _________.

Given Answer : **17**

Ans: 100

Sol: The palindrome divisible by 55 will be of the form 5pqqp5

It will be divisible by 5 and 11.

Now, p and q can be filled in 10×10 ways

Required number = 10 × 10 = 100

Q.2 If $y^{1/4} + y^{-1/4} = 2x$, and $(x^2-1)\frac{d^2y}{dx^2} + \alpha x\frac{dy}{dx} + \beta y = 0$, then $|\alpha - \beta|$ is equal to _________.

Given Answer : **--**

Ans: 17

Sol: $y^{\frac{1}{4}} + \frac{1}{y^{\frac{1}{4}}} = 2x$

$\Rightarrow \left(y^{\frac{1}{4}}\right)^2 - 2xy^{\left(\frac{1}{4}\right)} + 1 = 0$

$\Rightarrow y^{\frac{1}{4}} = x + \sqrt{x^2 - 1}$ or $x - \sqrt{x^2 - 1}$

$\Rightarrow \frac{dy}{dx} = \frac{4y}{\sqrt{x^2 - 1}}$ ......(1)

Differentiating again, $\frac{d^2y}{dx^2} = 4\frac{\left(\sqrt{x^2 - 1}\right)y' - \frac{yx}{\sqrt{x^2 - 1}}}{x^2 - 1}$

Rearranging we get

$\Rightarrow (x^2 - 1)y'' + xy' - 16y = 0$

$\alpha = 1, \beta = 16$

So, $|\alpha - \beta| = 17$

Q.3 The number of distinct real roots of the equation $3x^4 + 4x^3 - 12x^2 + 4 = 0$ is _________.

Given Answer : **2**

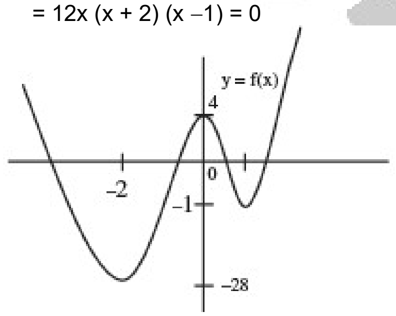
Ans: 4

Sol:

Let $f(x) = 3x^4 + 4x^3 - 12x^2 + 4$

$\therefore f'(x) = 12x(x^2 + x - 2) = 0$

$= 12x (x + 2) (x - 1) = 0$



$\therefore$ The no. of real roots = 4

Q.4 If $\int \frac{dx}{(x^2 + x + 1)^2} = a \tan^{-1}\left(\frac{2x + 1}{\sqrt{3}}\right) + b\left(\frac{2x + 1}{x^2 + x + 1}\right) + C$, $x > 0$ where C is the constant of integration, then the value of $9(\sqrt{3}\,a + b)$ is equal to __________.

Given Answer : **--**

Ans: 15

Sol: Given integral can be written as ,

$$I = \int \frac{dx}{\left[\left(x + \frac{1}{2}\right)^2 + \frac{3}{4}\right]^2}$$

$\left(\text{Put } x+\frac{1}{2}=t\right) \int \frac{dt}{\left(t^2+\frac{3}{4}\right)^2}$

$= \frac{4\sqrt{3}}{9}\left[\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)+\frac{\sqrt{3}(2x+1)}{3+(2x+1)^2}\right]+C$

$= \frac{4\sqrt{3}}{9}\ \tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)+\frac{1}{3}\left(\frac{2x+1}{x^2+x+1}\right)+C$

a.= $\frac{4\sqrt{3}}{9}$ b= $\frac{1}{3}$

Hence , $9\left(\sqrt{3}a+b\right)=15$

Q.5 Let n be an odd natural number such that the variance of 1, 2, 3, 4, ..., n is 14. Then n is equal to ___________.

Given **13**
Answer :

Ans: 13

Sol: $\frac{n^2-1}{12}=14 \Rightarrow n^2=169 \Rightarrow n=13$

Q.6 If the system of linear equations

$2x+y-z=3$

$x-y-z=\alpha$

$3x+3y+\beta z=3$

has infinitely many solution, then $\alpha+\beta-\alpha\beta$ is equal to ___________.

Given **--**
Answer :

Ans: 5

Sol: Since, the system has infinitely many solutions

$\Delta=0$

$\begin{vmatrix} 2 & 1 & -1 \\ 1 & -1 & -1 \\ 3 & 3 & \beta \end{vmatrix}=0 \Rightarrow \beta=-1$

$\Delta_3=0$

$\begin{vmatrix} 2 & 1 & 3 \\ 1 & -1 & \alpha \\ 3 & 3 & 3 \end{vmatrix}=0 \Rightarrow \alpha=3$

Hence, $\alpha+\beta-\alpha\beta=5$

Q.7 Let the equation $x^2+y^2+px+(1-p)y+5=0$ represent circles of varying radius $r \in (0, 5]$. Then the number of elements in the set $S=\{q : q=p^2$ and q is an integer$\}$ is __________.

Given **--**
Answer :

Ans: 61

Sol: $r=\sqrt{\frac{p^2}{4}+\frac{(1-p)^2}{4}-5}=\frac{\sqrt{2p^2-2p-19}}{2}$

Now, $r \in (o, 5]$

$\Rightarrow 0 < 2p^2 - 2p - 19 \leq 100$

$$\Rightarrow p \in \left[\frac{1-\sqrt{239}}{2}, \frac{1-\sqrt{39}}{2}\right) \cup \left(\frac{1+\sqrt{39}}{2}, \frac{1+\sqrt{239}}{2}\right]$$

So, number of integral values of p^2 is 61

Q.8 If the minimum area of the triangle formed by a tangent to the ellipse $\frac{x^2}{b^2} + \frac{y^2}{4a^2} = 1$ and the co-ordinate axis is kab, then k is equal to ___________.

Given **4**
Answer :

Ans: 2

Sol: Equation of the tangent to the ellipse is $\frac{x\cos\theta}{b} + \frac{y\sin\theta}{2a} = 1$

Required area = $\frac{1}{2} \times \frac{b}{\cos\theta} \times \frac{2a}{\sin\theta} = \frac{2ab}{\sin 2\theta} \geq 2ab$

$\Rightarrow k = 2$

Q.9 Let $\vec{a} = \hat{i} + 5\hat{j} + \alpha\hat{k}$, $\vec{b} = \hat{i} + 3\hat{j} + \beta\hat{k}$ and $\vec{c} = -\hat{i} + 2\hat{j} - 3\hat{k}$ be three vectors such that, $|\vec{b} \times \vec{c}| = 5\sqrt{3}$ and $\vec{a}$ is perpendicular to $\vec{b}$. Then the greatest amongst the values of $|\vec{a}|^2$ is __________.

Given **51**
Answer :

Ans: 90

Sol: Given $\vec{a}$ and $\vec{b}$ are perpendicular $\Rightarrow \vec{a}.\vec{b} = 0$

$1 + 15 + \alpha\beta = 0 \Rightarrow \alpha\beta = -16$ ----(1)

Also, $|\vec{b} \times \vec{c}| = 5\sqrt{3} \Rightarrow (10 + \beta^2)14 - (5 - 3\beta)^2 = 75$

$\Rightarrow 5\beta^2 + 30\beta + 40 = 0$

Solving, we get $\beta = -4, -2$

$\Rightarrow \alpha = 4, 8$

$\Rightarrow |\vec{a}|^2 = (26 + \alpha^2) = 90$

Q.10 If $A = \{x \in \mathbf{R} : |x-2| > 1\}$, $B = \{x \in \mathbf{R} : \sqrt{x^2 - 3} > 1\}$, $C = \{x \in \mathbf{R} : |x - 4| \geq 2\}$ and $\mathbf{Z}$ is the set of all integers, then the number of subsets of the set $(A \cap B \cap C)^c \cap \mathbf{Z}$ is __________.

Given **--**
Answer :

Ans: 256

Sol: A = $(-\infty, 1) \cup (3, \infty)$

B = $(-\infty, -2) \cup (2, \infty)$

C = $(-\infty, 2) \cup (6, \infty)$

So, $A \cap B \cap C = (-\infty, -2) \cup (6, \infty)$

$Z \cap (A \cap B \cap C)^c = \{-2, -1, 0, 1, 2, 3, 4, 5\}$

Hence no. of its subsets = $2^8 = 256$