SOLUTIONS & ANSWERS FOR JEE MAINS-2021
27th July Shift 1

[PHYSICS, CHEMISTRY & MATHEMATICS]

PART – A – PHYSICS

Section A

Q.1 Two identical tennis balls each having mass 'm' and charge 'q' are suspended from a fixed point by threads of length 'l'. What is the equilibrium separation when each thread makes a small angle 'θ' with the vertical ?

Options

1. $x = \left(\dfrac{q^2 l^2}{2\pi\varepsilon_0 m^2 g^2}\right)^{1/3}$

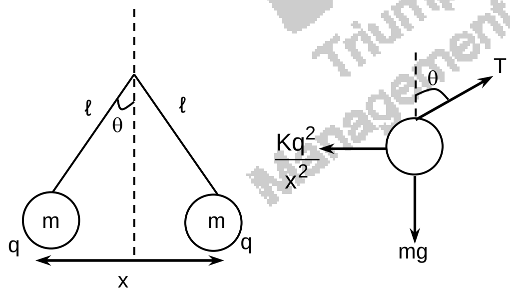
2. $x = \left(\dfrac{q^2 l}{2\pi\varepsilon_0 m g}\right)^{1/3}$

3. $x = \left(\dfrac{q^2 l}{2\pi\varepsilon_0 m g}\right)^{1/2}$

4. $x = \left(\dfrac{q^2 l^2}{2\pi\varepsilon_0 m^2 g}\right)^{1/3}$

Ans: $x = \left[\dfrac{q^2\ell}{2\pi\varepsilon_0 mg}\right]^{1/3}$

Sol:



$T\cos\theta = mg$

$T\sin\theta = \dfrac{Kq^2}{x^2} \Rightarrow \tan\theta = \dfrac{Kq^2}{x^2 mg}$

$\text{Tan}\,\theta \approx \sin\theta \approx \dfrac{x}{2\ell}$

$\therefore \dfrac{x}{2\ell} = \dfrac{Kq^2}{x^2 mg} \Rightarrow x = \left[\dfrac{q^2\ell}{2\pi\varepsilon_0 mg}\right]^{1/3}$

Q.2 A 0.07 H inductor and a 12 Ω resistor are connected in series to a 220 V, 50 Hz ac source. The approximate current in the circuit and the phase angle between current and source voltage are respectively. [Take π as $\frac{22}{7}$]

Options

1. $8.8\ \text{A and } \tan^{-1}\left(\frac{11}{6}\right)$
2. $8.8\ \text{A and } \tan^{-1}\left(\frac{6}{11}\right)$
3. $0.88\ \text{A and } \tan^{-1}\left(\frac{11}{6}\right)$
4. $88\ \text{A and } \tan^{-1}\left(\frac{11}{6}\right)$

Ans: $8.8\ \text{A and } \tan^{-1}\left(\frac{11}{6}\right)$

Sol: $X_L = L\omega = 2 \times \frac{22}{7} \times 50 \times 0.07 = 22\ \Omega$

$R = 12\ \Omega$

$\phi = \tan^{-1}\left(\frac{X_L}{R}\right) = \tan^{-1}\left(\frac{22}{12}\right) = \tan^{-1}\left(\frac{11}{6}\right)$

$z = \sqrt{X_L^2 + R^2} = 22.059$

$I = \frac{e}{Z} = \frac{220}{25.059} = 8.77\ \text{A}$

Q.3 **Assertion A :** If in five complete rotations of the circular scale, the distance travelled on main scale of the screw gauge is 5 mm and there are 50 total divisions on circular scale, then least count is 0.001 cm.

Reason R : $\text{Least Count} = \frac{\text{Pitch}}{\text{Total divisions on circular scale}}$

In the light of the above statements, choose the most appropriate answer from the options given below :

Options

1. Both **A** and **R** are correct and **R** is the correct explanation of **A**.
2. Both **A** and **R** are correct and **R** is NOT the correct explanation of **A**.
3. **A** is correct but **R** is not correct.
4. **A** is not correct but **R** is correct.

Ans: **A** is not correct but **R** is correct

Sol: $LC = \frac{\text{Pitch}}{\text{No. of circular scale division}}$

Distance travelled in 5 revolution = 5 mm

$\therefore$ distance travelled in 1 revolution = 1 mm

$\therefore LC = \frac{1}{50} = 0.02$

Q.4 In Young's double slit experiment, if the source of light changes from orange to blue then :

Options
1. the distance between consecutive fringes will decrease.
2. the distance between consecutive fringes will increase.
3. the intensity of the minima will increase.
4. the central bright fringe will become a dark fringe.

Ans: distance between consecutive fringes will decrease

Sol: $\beta = \frac{D}{d}\lambda$

As λ decreases, β also decreases

Q.5 A body takes 4 min. to cool from 61°C to 59°C. If the temperature of the surroundings is 30°C, the time taken by the body to cool from 51°C to 49°C is :

Options
1. 6 min.
2. 8 min.
3. 4 min.
4. 3 min.

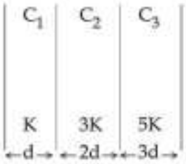
Ans: 6 min

Sol: $\frac{61-59}{4} = K\left[\frac{61+59}{2} - 30\right]$---------------(1)

$\frac{51-49}{t} = K\left[\frac{51+49}{2} - 30\right]$---------------(2)

$\frac{(1)}{(2)} \Rightarrow \frac{t}{4} = \frac{30}{20} \Rightarrow t = \frac{4\times 30}{20} = 6\,\text{min}$

Q.6 In the reported figure, a capacitor is formed by placing a compound dielectric between the plates of parallel plate capacitor. The expression for the capacity of the said capacitor will be : (Given area of plate = A)



Options

1. $\frac{9}{6}\frac{K\varepsilon_0 A}{d}$

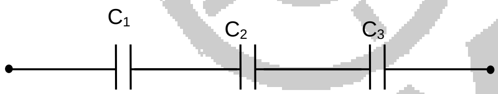
2. $\frac{15}{34}\frac{K\varepsilon_0 A}{d}$

3. $\frac{25}{6}\frac{K\varepsilon_0 A}{d}$

4. $\frac{15}{6}\frac{K\varepsilon_0 A}{d}$

Ans: $\frac{15}{34}\frac{K\varepsilon_0 A}{d}$

Sol:



$$C_1 = \frac{A\varepsilon_0 K}{d} \qquad C_2 = \frac{3A\varepsilon_0 K}{2d} \qquad C_3 = \frac{5A\varepsilon_0 K}{3d}$$

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} = \frac{d}{A\varepsilon_0 K} + \frac{2d}{3A\varepsilon_0 K} + \frac{3d}{5A\varepsilon_0 K}$$

$$= \frac{d}{A\varepsilon_0 K}\left[1 + \frac{2}{3} + \frac{3}{5}\right] = \frac{34}{15}\frac{d}{A\varepsilon_0 K}$$

$$= C_{eq} = \frac{15}{34}\frac{A\varepsilon_0 K}{d}$$

Q.7

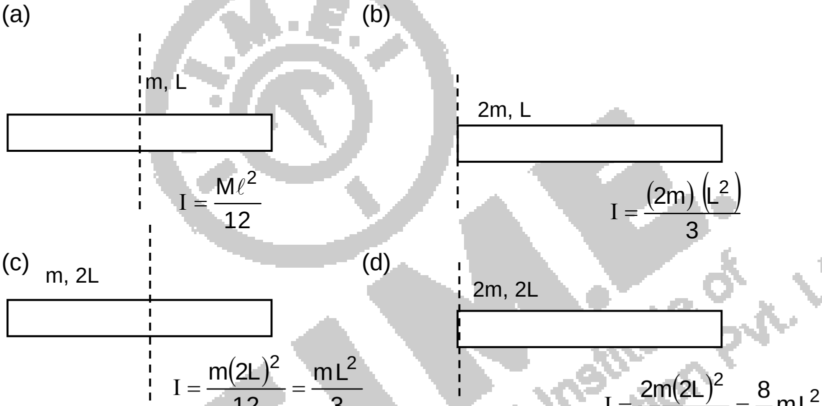
	List - I		List - II
(a)	MI of the rod (length L, Mass M, about an axis $\perp$ to the rod passing through the midpoint)	(i)	$8\ ML^2/3$
(b)	MI of the rod (length L, Mass 2M, about an axis $\perp$ to the rod passing through one of its end)	(ii)	$ML^2/3$
(c)	MI of the rod (length 2L, Mass M, about an axis $\perp$ to the rod passing through its midpoint)	(iii)	$ML^2/12$
(d)	MI of the rod (length 2L, Mass 2M, about an axis $\perp$ to the rod passing through one of its end)	(iv)	$2\ ML^2/3$

Choose the **correct** answer from the options given below :

Options
1. (a)-(iii), (b)-(iv), (c)-(i), (d)-(ii)
2. (a)-(iii), (b)-(iv), (c)-(ii), (d)-(i)
3. (a)-(ii), (b)-(iii), (c)-(i), (d)-(iv)
4. (a)-(ii), (b)-(i), (c)-(iii), (d)-(iv)

Ans: (a)-(iii), (b)-(iv), (c)-(ii), (d)-(i)

Sol: (a) (b)



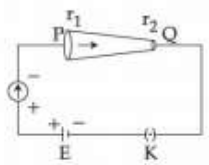
(a) $I = \dfrac{M\ell^2}{12}$

(b) $I = \dfrac{(2m)\left(L^2\right)}{3}$

(c) $I = \dfrac{m(2L)^2}{12} = \dfrac{mL^2}{3}$

(d) $I = \dfrac{2m(2L)^2}{3} = \dfrac{8}{3}mL^2$

Q.8 In the given figure, a battery of emf E is connected across a conductor PQ of length 'l' and different area of cross-sections having radii r_1 and r_2 ($r_2 < r_1$).



Choose the correct option as one moves from P to Q :

Options
1. All of these
2. Electron current decreases.
3. Drift velocity of electron increases.
4. Electric field decreases.

Ans: Drift velocity of electron increases

Sol: $i = nAV_de$ and $V_d = \dfrac{eE}{m}\tau\left(V_d \propto E\right)$

As A decreases, V_d increases [as r decreases E increases and hence V_d increases]

Q.9 If 'f' denotes the ratio of the number of nuclei decayed (N_d) to the number of nuclei at $t=0$ (N_0) then for a collection of radioactive nuclei, the rate of change of 'f' with respect to time is given as :

[λ is the radioactive decay constant]

Options
1. $-\lambda(1-e^{-\lambda t})$
2. $-\lambda e^{-\lambda t}$
3. $\lambda e^{-\lambda t}$
4. $\lambda(1-e^{-\lambda t})$

Ans: $\lambda e^{-\lambda t}$

Sol: $N = N_0 e^{-\lambda t}$

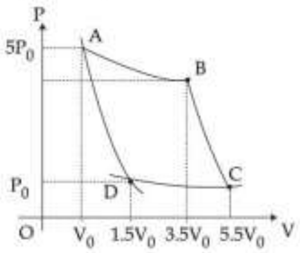
Decayed Nuclei, $Nd = N - N_0$

$= N_0 (1 - e^{-\lambda t})$

$f = \frac{Nd}{N_0} = (1 - e^{-\lambda t})$

$\Rightarrow \frac{df}{dt} = \lambda e^{-\lambda t}$

Q.10 In the reported figure, there is a cyclic process ABCDA on a sample of 1 mol of a diatomic gas. The temperature of the gas during the process A → B and C → D are T_1 and T_2 ($T_1 > T_2$) respectively.



Choose the correct option out of the following for work done if processes BC and DA are adiabatic.

Options
1. $W_{BC} + W_{DA} > 0$
2. $W_{AB} < W_{CD}$
3. $W_{AD} = W_{BC}$
4. $W_{AB} = W_{DC}$

Ans: $W_{AD} = W_{BC}$

Sol: Work done in adiabatic process = $\frac{-nR}{\gamma - 1}(T_f - T_i)$

$\therefore W_{AD} = \frac{-nR}{\gamma - 1}(T_2 - T_1)$

And $W_{BC} = \frac{-nR}{\gamma - 1}(T_2 - T_1)$

$\therefore W_{AD} = W_{BC}$

Q.11 The relative permittivity of distilled water is 81. The velocity of light in it will be : (Given $\mu_r = 1$)

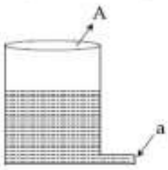
Options

1. 5.33×10^7 m/s
2. 4.33×10^7 m/s
3. 2.33×10^7 m/s
4. 3.33×10^7 m/s

Ans: $3.33 \times 10^7 \text{ m/s}$

Sol: $V = \dfrac{C}{\sqrt{\mu_r \varepsilon_r}} = 3.33 \times 10^7 \text{ m/s}$

Q.12 A light cylindrical vessel is kept on a horizontal surface. Area of base is A. A hole of cross-sectional area 'a' is made just at its bottom side. The minimum coefficient of friction necessary to prevent sliding the vessel due to the impact force of the emerging liquid is (a << A) :



Options

1. $\dfrac{a}{A}$
2. None of these
3. $\dfrac{2a}{A}$
4. $\dfrac{A}{2a}$

Ans: $\dfrac{2a}{A}$

Sol: For no sliding, $f \geq \rho a V^2$

$\mu m g \geq \rho a V^2$

$\mu \rho A h g = \rho a 2 g h$

$\Rightarrow \mu \geq \dfrac{2a}{A}$

Q.13 A particle starts executing simple harmonic motion (SHM) of amplitude 'a' and total energy E. At any instant, its kinetic energy is $\frac{3E}{4}$ then its displacement 'y' is given by :

Options

1. $y = \frac{a}{2}$
2. $y = \frac{a\sqrt{3}}{2}$
3. $y = \frac{a}{\sqrt{2}}$
4. $y = a$

Ans: $y = \frac{a}{2}$

Sol: $E = \frac{1}{2}m\omega^2A^2 = \frac{1}{2}KA^2$

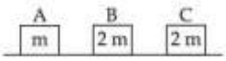
$KE = \frac{1}{2}m\omega^2\left(A^2 - y^2\right)$

$\frac{3}{4}E = \frac{1}{2}K\left(A^2 - y^2\right)$

$\frac{3}{4} \times \frac{1}{2}KA^2 = \frac{1}{2}K\left(A^2 - y^2\right) \Rightarrow y^2 = a^2 - \frac{3}{4}a^2$

$\Rightarrow y = \frac{a}{2}$

Q.14 Three objects A, B and C are kept in a straight line on a frictionless horizontal surface. The masses of A, B and C are m, 2 m and 2 m respectively. A moves towards B with a speed of 9 m/s and makes an elastic collision with it. Thereafter B makes a completely inelastic collision with C. All motions occur along same straight line. The final speed of C is :

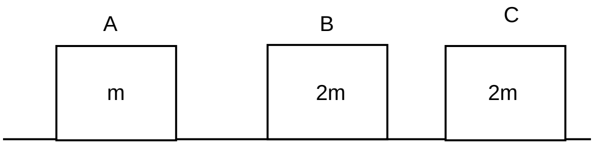


Options

1. 6 m/s
2. 3 m/s
3. 4 m/s
4. 9 m/s

Ans: 3 m / s

Sol:



Collision between A and B $\Rightarrow$

$9\ m = mV_A + 2mV_B$

$9 = V_A + 2V_B$ -----------------(1)

$9 = V_B - V_A$ -------------------(2) $\left[\text{from } C = \frac{V_2 - V_1}{u_1 - u_2}\right]$

Solving (1) and (2), $V_B = 6$ m/s

Collision between B and C $\Rightarrow$ 2 m V_B = 4m V_C

$\rightarrow V_C = 3$ m/s

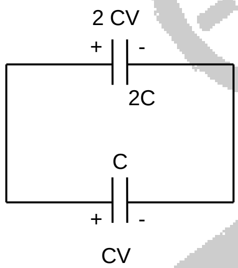
Q.15 Two capacitors of capacities 2C and C are joined in parallel and charged up to potential V. The battery is removed and the capacitor of capacity C is filled completely with a medium of dielectric constant K. The potential difference across the capacitors will now be :

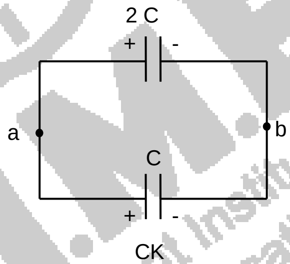
Options

1. $\frac{V}{K}$
2. $\frac{3V}{K}$
3. $\frac{V}{K+2}$
4. $\frac{3V}{K+2}$

Ans: $\frac{3V}{K+2}$

Sol:



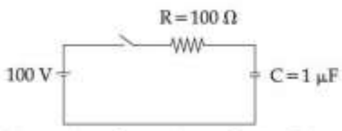


According to law of conservation of charge,

$2CV + CV = 2C \cdot V_{ab} + CK\, V_{ab}$

$$V_{ab} = \frac{3CV}{2C + CK} = \frac{3V}{2+K}$$

Q.16



A capacitor of capacitance C = 1 μF is suddenly connected to a battery of 100 volt through a resistance R = 100 Ω. The time taken for the capacitor to be charged to get 50 V is :

[Take ln 2 = 0.69]

Options

1. 0.30×10^{-4} s
2. 3.33×10^{-4} s
3. 1.44×10^{-4} s
4. 0.69×10^{-4} s

Ans: 0.69×10^{-4} s

Sol: $V = V_0 e^{-t/RC}$

$50 = 100\,e^{-t/RC}$

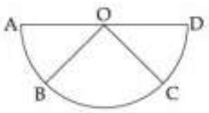
t = RC ℓn 2 = 100 × 10^{-6} × ℓn 2 = 10^{-4} × 0.693 = 69.3 μs

Q.17 **Assertion A :** If A, B, C, D are four points on a semi-circular arc with centre at 'O' such that $|\overrightarrow{AB}| = |\overrightarrow{BC}| = |\overrightarrow{CD}|$, then

$\overrightarrow{AB} + \overrightarrow{AC} + \overrightarrow{AD} = 4\overrightarrow{AO} + \overrightarrow{OB} + \overrightarrow{OC}$

Reason R : Polygon law of vector addition yields

$\overrightarrow{AB} + \overrightarrow{BC} + \overrightarrow{CD} = \overrightarrow{AD} = 2\overrightarrow{AO}$



In the light of the above statements, choose the **most appropriate** answer from the options given below :

Options
1. **A** is not correct but **R** is correct.
2. Both **A** and **R** are correct and **R** is the correct explanation of **A**.
3. **A** is correct but **R** is not correct.
4. Both **A** and **R** are correct but **R** is not the correct explanation of **A**.

Ans: Both **A** and **R** are correct **R** is not the correct explanation of **A**

Sol: $\overrightarrow{OA} + \overrightarrow{AB} = \overrightarrow{OB}$ --------(1)

$\overrightarrow{OB} + \overrightarrow{BC} = \overrightarrow{OC}$ --------(2)

$\overrightarrow{OC} + \overrightarrow{CD} = \overrightarrow{OD}$ -------(3)

$\overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA}$

$\overrightarrow{AC} = \overrightarrow{OC} - \overrightarrow{OA}$

$\overrightarrow{AD} = \overrightarrow{OD} - \overrightarrow{OA} = -\overrightarrow{OA} - \overrightarrow{OA}$

$\overrightarrow{AB} + \overrightarrow{AC} + \overrightarrow{AD} = \overrightarrow{OB} + \overrightarrow{OC} - 4\overrightarrow{OA}$

$= 4\overrightarrow{AO} + \overrightarrow{OB} + \overrightarrow{OC}$ --------(4)

$\overrightarrow{AB} + \overrightarrow{BC} + \overrightarrow{CD} = \overrightarrow{OB} + \overrightarrow{OC} + \overrightarrow{OD} - \overrightarrow{OA} - \overrightarrow{OB} - \overrightarrow{OC}$

$= -\overrightarrow{OA} - \overrightarrow{OA} = -2\overrightarrow{OA} = 2\overrightarrow{AO}$

Q.18 The number of molecules in one litre of an ideal gas at 300 K and 2 atmospheric pressure with mean kinetic energy 2×10^{-9} J per molecule is :

Options
1. 1.5×10^{11}
2. 3×10^{11}
3. 0.75×10^{11}
4. 6×10^{11}

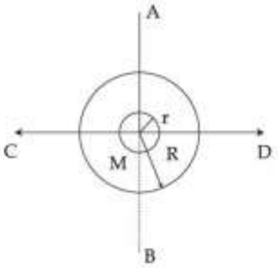
Ans: 1.5×10^{11}

Sol: $KE = \frac{3}{2}KT$

$$PV = \frac{N}{N_A} RT$$

$$N = \frac{PV}{KT} = 1.5 \times 10^{11}$$

Q.19 The figure shows two solid discs with radius R and r respectively. If mass per unit area is same for both, what is the ratio of MI of bigger disc around axis AB (which is $\perp$ to the plane of the disc and passing through its centre) to MI of smaller disc around one of its diameters lying on its plane ? Given 'M' is the mass of the larger disc. (MI stands for moment of inertia)



Options

1. $R^2 : r^2$
2. $2r^4 : R^4$
3. $2R^2 : r^2$
4. $2R^4 : r^4$

Ans: $2R^4 : r^4$

Sol: Ratio of moment of inertia = $\dfrac{\frac{1}{2}MR^2}{\frac{1}{4}mr^2} = \dfrac{2\sigma\pi R^2 \times R^2}{\sigma\pi r^2 \times r^2} = \dfrac{2R^4}{r^4}$

Q.20 A ball is thrown up with a certain velocity so that it reaches a height 'h'. Find the ratio of the two different times of the ball reaching $\frac{h}{3}$ in both the directions.

Options

1. $\dfrac{\sqrt{3}-1}{\sqrt{3}+1}$
2. $\dfrac{1}{3}$
3. $\dfrac{\sqrt{3}-\sqrt{2}}{\sqrt{3}+\sqrt{2}}$
4. $\dfrac{\sqrt{2}-1}{\sqrt{2}+1}$

Ans: $\dfrac{\sqrt{3}-\sqrt{2}}{\sqrt{3}+\sqrt{2}}$

Sol: $u = \sqrt{2gh}$ $\quad S = \dfrac{h}{3}$ $\quad a - g$

$$\frac{h}{3} = \sqrt{2gh}\,t + \frac{1}{2}(-g)t^2$$

$$t^2\left(\frac{g}{2}\right) - \sqrt{2gh}\,t + \frac{h}{3} = 0$$

$$\Rightarrow t_1, t_2 = \frac{\sqrt{2gh} \pm \sqrt{2gh - \frac{4g}{2}\frac{h}{3}}}{g}$$

$$\frac{t_1}{t_2} = \frac{\sqrt{2gh} - \sqrt{\frac{4gh}{3}}}{\sqrt{2gh} + \sqrt{\frac{4gh}{3}}} = \frac{\sqrt{3} - \sqrt{2}}{\sqrt{3} + \sqrt{2}}$$

Section B

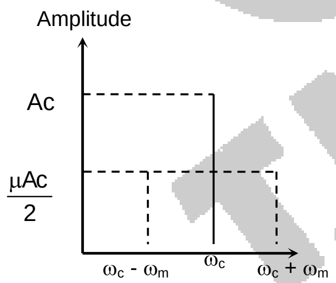
Q.1 The amplitude of upper and lower side bands of A.M. wave where a carrier signal with frequency 11.21 MHz, peak voltage 15 V is amplitude modulated by a 7.7 kHz sine wave of 5 V amplitude are $\frac{a}{10}$V and $\frac{b}{10}$V respectively. Then the value of $\frac{a}{b}$ is ________.

Given **2**
Answer :

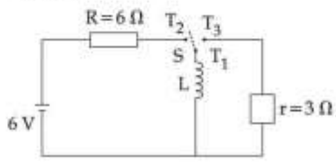
Ans: 1.00

Sol: $\frac{a}{10} = \frac{b}{10} = \frac{\mu A_C}{2}$

$\Rightarrow \frac{a}{b} = 1$



Q.2 Consider an electrical circuit containing a two way switch 'S'. Initially S is open and then T_1 is connected to T_2. As the current in $R = 6\ \Omega$ attains a maximum value of steady state level, T_1 is disconnected from T_2 and immediately connected to T_3. Potential drop across $r = 3\ \Omega$ resistor immediately after T_1 is connected to T_3 is ________ V. (Round off to the Nearest Integer)



Given --
Answer :

Ans: 3.00

Sol: When T_1 and T_2 are connected, the steady state current in the conductor is $\frac{6}{6} = 1A$.

When T_1 and T_3 are connected, the current through the inductor remains same

$\therefore V_{across}$ 3 Ω = I_r = 1 × 3 = 3V

Q.3 A radioactive sample has an average life of 30 ms and is decaying. A capacitor of capacitance 200 μF is first charged and later connected with resistor 'R'. If the ratio of charge on capacitor to the activity of radioactive sample is fixed with respect to time then the value of 'R' should be ________ Ω.

Given --
Answer :

Ans: 150.00

Sol: $q = q_0\ e^{-t/RC}$ --------(1)

$A = A_0\ e^{-\lambda t}$------------------(2)

$\frac{(1)}{(2)} \Rightarrow \frac{q}{A} = \frac{q_0}{A_0}\frac{e^{-t/RC}}{e^{-\lambda t}} \Rightarrow -\lambda t = \frac{-t}{RC}$

$RC = \frac{1}{\lambda}$

$R = \frac{30\times10^{-3}}{200\times10^{-6}} = 150\Omega$

Q.4 A stone of mass 20 g is projected from a rubber catapult of length 0.1 m and area of cross section 10^{-6} m^2 stretched by an amount 0.04 m. The velocity of the projected stone is ________ m/s.

(Young's modulus of rubber = 0.5×10^9 N/m^2)

Given **1000**
Answer :

Ans: 20.00

Sol: By energy conservation, $\frac{1}{2}\frac{YA}{L}x^2 = \frac{1}{2}mv^2$

$\frac{0.5\times10^9\times10^{-6}\times(0.04)^2}{0.01} = \frac{20}{1000}v^2$

$v^2 = 400$

$v = \sqrt{400} = 20\,m/s$

Q.5 In Bohr's atomic model, the electron is assumed to revolve in a circular orbit of radius 0.5 Å. If the speed of electron is 2.2×10^6 m/s, then the current associated with the electron will be ________ $\times10^{-2}$ mA. [Take π as $\frac{22}{7}$]

Given --
Answer :

Ans: 112.00

Sol: $I = \frac{e}{T} = \frac{e}{2\pi/\omega} = \frac{e\omega}{2\pi} = \frac{eV}{2\pi r} = \frac{1.6\times10^{-19}\times2.2\times10^6\times7}{2\times22\times0.5\times10^{-10}} = 1.12\,mA$ = 112 × 10^{-2} mA

Q.6 A particle of mass 9.1×10^{-31} kg travels in a medium with a speed of 10^6 m/s and a photon of a radiation of linear momentum 10^{-27} kg m/s travels in vacuum. The wavelength of photon is ________ times the wavelength of the particle.

Given **910**
Answer :

Ans: 910.00

Sol: For photon, $\lambda_1 = \frac{h}{p} = \frac{6.6\times10^{-34}}{10^{-27}}$

For particle, $\lambda_2 = \frac{h}{mV} = \frac{6.6\times10^{-34}}{9.1\times10^{-31}\times10^{6}}$

$\frac{\lambda_1}{\lambda_2} = 910$

Q.7 A transistor is connected in common emitter circuit configuration, the collector supply voltage is 10 V and the voltage drop across a resistor of 1000 Ω in the collector circuit is 0.6 V. If the current gain factor (β) is 24, then the base current is ________ μA. (Round off to the Nearest Integer)

Given --
Answer :

Ans: 25.00

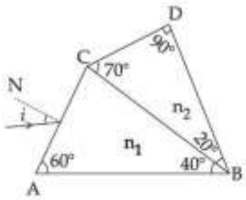
Sol: $I_C = \frac{0.6}{1000}$

$\beta = \frac{I_C}{I_B} \Rightarrow I_B = \frac{0.6}{1000\times24} = 25\,\mu A$

Q.8 A prism of refractive index n_1 and another prism of refractive index n_2 are stuck together (as shown in the figure). n_1 and n_2 depend on λ, the wavelength of light, according to the relation

$$n_1 = 1.2 + \frac{10.8\times10^{-14}}{\lambda^2} \text{ and } n_2 = 1.45 + \frac{1.8\times10^{-14}}{\lambda^2}$$

The wavelength for which rays incident at any angle on the interface BC pass through without bending at that interface will be ________ nm.



Given --
Answer :

Ans: 600.00

Sol: For no. bending, $n_1 = n_2$

$1.2 + \frac{10.8\times10^{-14}}{\lambda^2} = 1.45 + \frac{1.8\times10^{-4}}{\lambda^2}$

$9\times10^{-14} = 25\,\lambda^2 \Rightarrow \lambda = 6\times10^{-7} = 600$ nm

Q.9 In a uniform magnetic field, the magnetic needle has a magnetic moment 9.85×10^{-2} A/m^2 and moment of inertia 5×10^{-6} kg m^2. If it performs 10 complete oscillations in 5 seconds then the magnitude of the magnetic field is ________ mT. [Take π^2 as 9.85]

Given --
Answer :

Ans: 8.00

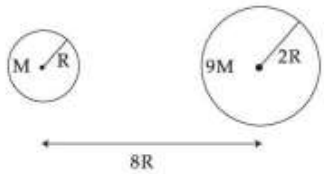
Sol: $T = 2\pi\sqrt{\frac{I}{MB}}$

$$\frac{1}{2} = 2\pi\sqrt{\frac{5\times10^{-6}}{9.85\times10^{-2}\times B}} = 2\pi\sqrt{\frac{5\times10^{-6}}{\pi^2\times B\times10^{-3}}}$$

$\Rightarrow B = 8 \times 10^{-3}T = 8$ mT

Q.10 Suppose two planets (spherical in shape) of radii R and 2 R, but mass M and 9 M respectively have a centre to centre separation 8 R as shown in the figure. A satellite of mass 'm' is projected from the surface of the planet of mass 'M' directly towards the centre of the second planet. The minimum speed 'v' required for the satellite to reach the surface of the second planet is $\sqrt{\frac{a}{7}\frac{GM}{R}}$ then the value of 'a' is ________.

[Given : The two planets are fixed in their position]



Given **49**
Answer :

Ans: 4.00

Sol: Let the net gravitational field become O at a distance x from planet A.

$$\frac{GM}{x^2} = \frac{G\times9M}{(8R-x)^2}$$

$(3x)^2 = (8R – x)^2 \Rightarrow x = 2R$

Now the particle should be projected such that it curers a minimum distance of 2R

$$\frac{1}{2}mv^2 - \frac{GmM}{R} - \frac{G(9M)m)}{7R} = -\frac{GMm}{2R} - \frac{G(9M)m}{6R}$$

$$\frac{1}{2}v^2 = \frac{2}{7}\frac{GM}{R}$$

$$\Rightarrow v = \sqrt{\frac{4}{7}\frac{GM}{R}} \Rightarrow a = 4$$

PART – B – CHEMISTRY

Section A

Q.1 Given below are two statements : One is labelled as **Assertion A** and the other is labelled as **Reason R**.

Assertion A : Lithium halides are some what covalent in nature.

Reason R : Lithium possess high polarisation capability.

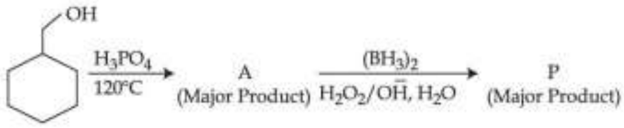
In the light of the above statements, choose the **most appropriate** answer from the options given below :

Options
1. **A** is false but **R** is true
2. **A** is true but **R** is false
3. Both **A** and **R** are true and **R** is the correct explanation of **A**
4. Both **A** and **R** are true but **R** is NOT the correct explanation of **A**

Ans: Both **A** and **R** are true and **R** is the correct explanation of **A**

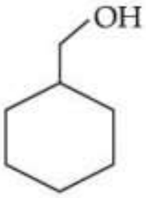
Sol: It is because of small size and high polarisability nature of Li^+ ion, lithium halides are covalent in nature.

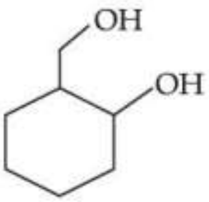
Q.2

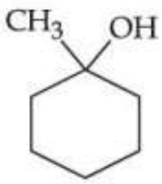


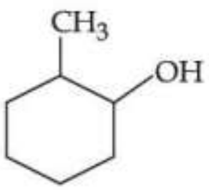
Consider the above reaction and identify the Product P :

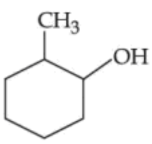
Options

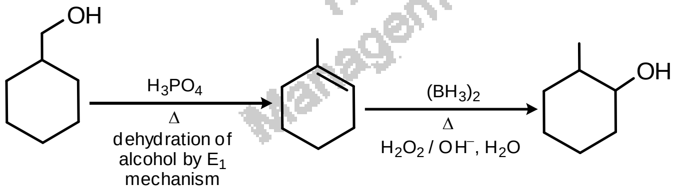
1. 

2. 

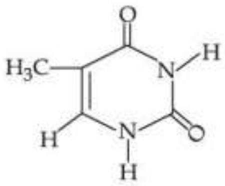
3. 

4. 

Ans: 

Sol: 

Q.3



(A)

The compound 'A' is a complementary base of __________ in DNA strands.

Options
1. Cytosine
2. Adenine
3. Uracil
4. Guanine

Ans: Adenine

Sol: The compound (A) is thymine and the complimentary base of thymine is adenine

Q.4 Given below are two statements :

Statement I : Rutherford's gold foil experiment cannot explain the line spectrum of hydrogen atom.

Statement II : Bohr's model of hydrogen atom contradicts Heisenberg's uncertainty principle.

In the light of the above statements, choose the **most appropriate** answer from the options given below :

Options
1. **Statement I** is false but **statement II** is true.
2. Both **statement I** and **statement II** are false.
3. Both **statement I** and **statement II** are true.
4. **Statement I** is true but **statement II** is false.

Ans: Both statement I and statement II are false

Sol: Both statements I and II are true

Q.5 Staggered and eclipsed conformers of ethane are :

Options
1. Enantiomers
2. Rotamers
3. Polymers
4. Mirror images

Ans: Rotamers

Sol: Staggered and eclipsed conformers of ethane are rotational isomers or rotamers

Q.6 The oxidation states of 'P' in $H_4P_2O_7$, $H_4P_2O_5$ and $H_4P_2O_6$, respectively, are :

Options
1. 5, 3 and 4
2. 6, 4 and 5
3. 7, 5 and 6
4. 5, 4 and 3

Ans: 5, 3 and 4

Sol: The oxidation state of P in

$\overset{+1}{H_4}\overset{x}{P_2}\overset{-2}{O_7}$ $\quad +4+2x-14=0$

$x=+5$

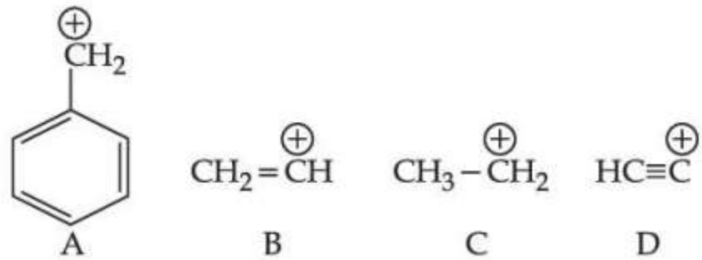
$\overset{+1}{H_4}\overset{x}{P_2}\overset{-2}{O_5}$ $\quad +4+2x-10=0$

$x=+3$

$\overset{+1}{H_4}\overset{x}{P_2}\overset{-2}{O_6}$ $\quad +4+2x-12=0$

$x=+4$

Q.7



The correct order of stability of given carbocations is :

Options
1. A > C > B > D
2. D > B > A > C
3. D > B > C > A
4. C > A > D > B

Ans: A > C > B > D

Sol: Benzyl carbocation is the most stable due to resonance stabilisation

Q.8 The product obtained from the electrolytic oxidation of acidified sulphate solutions, is :

Options
1. HO_2SOSO_2H
2. HO_3SOSO_3H
3. HO_3SOOSO_3H
4. HSO_4^-

Ans: HO_3SOOSO_3H

Sol: Electrolysis of acidified sulphate solution gives $H_2S_2O_8$

Q.9 Which one among the following chemical tests is used to distinguish monosaccharide from disaccharide ?

Options
1. Seliwanoff's test
2. Tollen's test
3. Iodine test
4. Barfoed test

Ans: Barfoed's test

Sol: Barfoed's test recognizes monosaccharides from disaccharides

Q.10 The parameters of the unit cell of a substance are $a = 2.5$, $b = 3.0$, $c = 4.0$, $\alpha = 90°$, $\beta = 120°$, $\gamma = 90°$. The crystal system of the substance is :

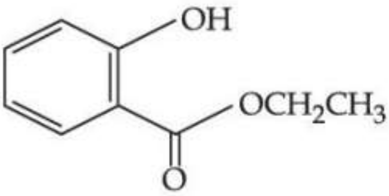
Options
1. Monoclinic
2. Hexagonal
3. Triclinic
4. Orthorhombic

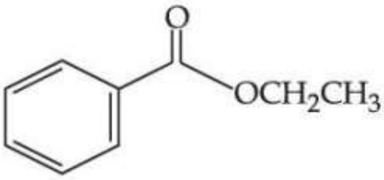
Ans: Monoclinic

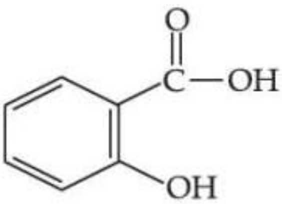
Sol: For monoclinic system, $a \neq b \neq c$ and $\alpha = \gamma = 90°$ and $\beta \neq 90°$

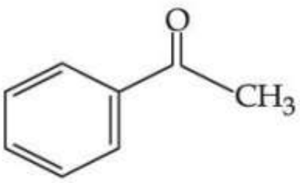
Q.11 Which one of the following compounds will give orange precipitate when treated with 2,4-dinitrophenyl hydrazine ?

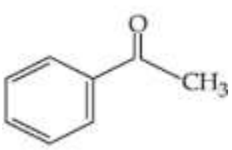
Options

1. 

2. 

3. 

4. 

Ans: 

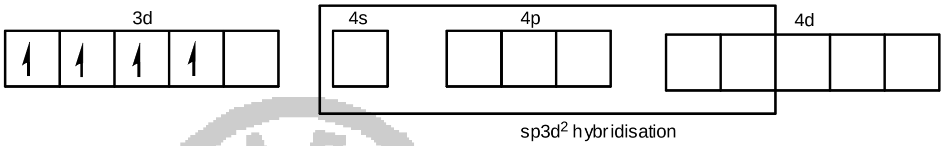
Sol: Aldehydes and ketones will give orange precipitate when treated with 2,4-dinitrophenyl hydrazine. Option (4) is acetophenone and hence it gives ppt with DNP

Q.12 The type of hybridisation and magnetic property of the complex $[MnCl_6]^{3-}$, respectively, are :

Options
1. sp^3d^2 and paramagnetic
2. sp^3d^2 and diamagnetic
3. d^2sp^3 and diamagnetic
4. d^2sp^3 and paramagnetic

Ans: sp^3d^2 and paramagnetic

Sol: In $[MnCl_6]^{3-}$, the central ion Mn^{3+} undergoes sp^3d^2 hybridisation



Since it contains 4 unpaired electrons, it is paramagnetic

Q.13 The statement that is INCORRECT about Ellingham diagram is :

Options
1. provides idea about free energy change.
2. provides idea about reduction of metal oxide.
3. provides idea about the reaction rate.
4. provides idea about changes in the phases during the reaction.

Ans: provides ideal about reduction of metal oxide

Sol: Ellingham diagram does not say anything about the kinetics of reduction process

Q.14 Presence of which reagent will affect the reversibility of the following reaction, and change it to a irreversible reaction :

$$CH_4 + I_2 \xrightarrow[\text{Reversible}]{h\nu} CH_3 - I + HI$$

Options
1. Concentrated HIO_3
2. HOCl
3. Liquid NH_3
4. dilute HNO_2

Ans: Concentrated HNO_3

Sol: Iodination of alkane is very slow and a reversible reaction. It can be carried out in presence of oxidizing agents like HNO_3 or HIO_3 and made irreversible
$HIO_3 + 5I \rightarrow 3I_2 + 3H_2O$

Q.15 For a reaction of order n, the unit of the rate constant is :

Options
1. $mol^{1-n}\,L^{1-n}\,s^{-1}$
2. $mol^{1-n}\,L^{n-1}\,s^{-1}$
3. $mol^{1-n}\,L^{2n}\,s^{-1}$
4. $mol^{1-n}\,L^{1-n}\,s$

Ans: $mol^{1-n}\,L^{n-1}\,s^{-1}$

Sol: For a general 'n'th order reaction, the unit of rate constant is given by $mol^{1-n}\,L^{n-1}\,s^{-1}$

Q.16 Which one of the following statements is **NOT** correct ?

Options
1. Eutrophication leads to increase in the oxygen level in water
2. The dissolved oxygen concentration below 6 ppm inhibits fish growth
3. Eutrophication indicates that water body is polluted
4. Eutrophication leads to anaerobic conditions

Ans: Eutrophication leads to increase in oxygen level of water

Sol: Eutrophication leads to decrease in oxygen level of water

Q.17 Match **List - I** with **List - II** :

	List - I (Drug)		**List - II (Class of Drug)**
(a)	Furacin	(i)	Antibiotic
(b)	Arsphenamine	(ii)	Tranquilizers
(c)	Dimetone	(iii)	Antiseptic
(d)	Valium	(iv)	Synthetic antihistamines

Choose the **most appropriate** match :

Options
1. (a)-(iii), (b)-(iv), (c)-(ii), (d)-(i)
2. (a)-(i), (b)-(iii), (c)-(iv), (d)-(ii)
3. (a)-(iii), (b)-(i), (c)-(iv), (d)-(ii)
4. (a)-(ii), (b)-(i), (c)-(iii), (d)-(iv)

Ans: (a)-(iii), (b)-(i), (c)-(iv), (d)-(ii)

Sol: Furacin – Antiseptic
Arsphenamine – Antibiotic
Dimetane – Synthetic antihistamine
Valium – Tranquilizer

Q.18 Given below are two statements :

Statement I : Aniline is less basic than acetamide.

Statement II : In aniline, the lone pair of electrons on nitrogen atom is delocalised over benzene ring due to resonance and hence less available to a proton.

Choose the **most appropriate** option :

Options
1. Both **statement I** and **statement II** are true.
2. **Statement I** is false but **statement II** is true.
3. Both **statement I** and **statement II** are false.
4. **Statement I** is true but **statement II** is false.

Ans: Statement I is false but statement II is true

Sol: Aniline is more basic than acetamide. In acetamide, during protonation, +ve charge is developed on N which is near to an e^- withdrawing C=O group getting destablished. Hence protonation takes place more faster in aniline

Q.19 The number of geometrical isomers found in the metal complexes $[PtCl_2(NH_3)_2]$, $[Ni(CO)_4]$, $[Ru(H_2O)_3Cl_3]$ and $[CoCl_2(NH_3)_4]^+$ respectively, are :

Options
1. 2, 0, 2, 2
2. 2, 1, 2, 2
3. 1, 1, 1, 1
4. 2, 1, 2, 1

Ans: 2, 0, 2, 2

Sol: $[PtCl_2(NH_3)_2]$ exhibits cis-trans isomerism
$[Ni(CO_4)$ is a tetrahedral complex and hence no geometrical isomers
$[Ru(H_2O)_3Cl_3]$ exhibits facial and merdional isomers
$[CoCl_2(NH_3)_4Cl_3]$ exhibits cis and trans isomerism

Q.20 Match **List - I** with **List - II** :

	List - I		**List - II**
(a)	NaOH	(i)	Acidic
(b)	$Be(OH)_2$	(ii)	Basic
(c)	$Ca(OH)_2$	(iii)	Amphoteric
(d)	$B(OH)_3$		
(e)	$Al(OH)_3$		

Choose the **most appropriate** answer from the options given below :

Options
1. (a)-(ii), (b)-(iii), (c)-(ii), (d)-(i), (e)-(iii)
2. (a)-(ii), (b)-(ii), (c)-(iii), (d)-(ii), (e)-(iii)
3. (a)-(ii), (b)-(ii), (c)-(iii), (d)-(i), (e)-(iii)
4. (a)-(ii), (b)-(i), (c)-(ii), (d)-(iii), (e)-(iii)

Ans: (a)-(ii), (b)-(iii), (c)-(ii), (d)-(i), (e)-(iii)

Sol: NaOH – basic
$Be(OH)_2$ – amphoteric
$Ca(OH)_2$ – basic
$B(OH)_3$ – acidic
$Al(OH)_3$ – amphoteric

Section B

Q.1 1.46 g of a biopolymer dissolved in a 100 mL water at 300 K exerted an osmotic pressure of 2.42×10^{-3} bar.

The molar mass of the biopolymer is _________ $\times 10^4$ g mol^{-1}. (Round off to the Nearest Integer)

[Use : $R = 0.083$ L bar mol^{-1} K^{-1}]

Given **15**
Answer :

Ans: 15

Sol: $\pi V = \frac{W_B RT}{M_B}$

$\therefore$ Molecular mass of polymer $= \frac{1.46 \times 0.083 \times 300}{2.42 \times 10^{-3} \times 0.1} = 1.5 \times 10^3 = 15 \times 10^4$ g mol^{-1}

Q.2 The difference between bond orders of CO and $NO^{\oplus}$ is $\frac{x}{2}$ where $x =$ _________. (Round off to the Nearest Integer)

Given **0**
Answer :

Ans: 0

Sol: Bond order of CO = 3
Bond order of NO^+ = 3

Difference $= 3 - 3 = 0 = \frac{x}{2}$

Q.3 An organic compound is subjected to chlorination to get compound A using 5.0 g of chlorine. When 0.5 g of compound A is reacted with $AgNO_3$ [Carius Method], the percentage of chlorine in compound A is _________ when it forms 0.3849 g of AgCl. (Round off to the Nearest Integer)

(Atomic masses of Ag and Cl are 107.87 and 35.5 respectively)

Given --
Answer :

Ans: 19

Sol: % of chorine in the compound $= \frac{35.5}{143.5} \times \frac{\text{Wt. of AgCl}}{\text{Wt. of OC}} \times 100$

% of chlorine $= \frac{35.5 \times 0.3849 \times 100}{143.5 \times 0.5} = 19.04\%$

Q.4 The conductivity of a weak acid HA of concentration 0.001 mol L^{-1} is 2.0×10^{-5} S cm^{-1}. If $\Lambda_m^{\circ}(HA) = 190$ S cm^2 mol^{-1}, the ionization constant (K_a) of HA is equal to _________ $\times 10^{-6}$. (Round off to the Nearest Integer)

Given --
Answer :

Ans: 12

Sol: $\Lambda_m^c = \frac{\kappa \times 1000}{M} = \frac{2\times10^{-5}\times10^3}{10^{-3}} = 20 \text{ S cm}^2 \text{ mol}^{-1}$

$$\alpha = \frac{\Lambda_m^c}{\Lambda_m^\circ} = \frac{20}{190} = 0.105$$

$$K_a = \frac{C\alpha^2}{1-\alpha} = \frac{10^{-3}\times(0.105)^2}{1-0.105}$$

$$= \frac{1.1\times10^{-5}}{0.895} = 1.23\times10^{-5} = 12.3\times10^{-6}$$

Q.5 In gaseous triethyl amine the "$-C-N-C-$" bond angle is _________ degree.

Given **120**
Answer :

Ans: 108

Sol: In gaseous triethylamine, the –C–N–C– bond angle is 108°

Q.6 CO_2 gas adsorbs on charcoal following Freundlich adsorption isotherm. For a given amount of charcoal, the mass of CO_2 adsorbed becomes 64 times when the pressure of CO_2 is doubled. The value of **n** in the Freundlich isotherm equation is _________ $\times10^{-2}$. (Round off to the Nearest Integer)

Given **17**
Answer :

Ans: 17

Sol: $\frac{x}{m} = KP^{1/n}$ ---- (1)

When $P \to 2P$, $\frac{x}{m} \to 64\,x/m$

i.e., $64\,x/m = K(2P)^{1/n}$ ---- (2)

(2) ÷ (1) ⇒

$$\frac{64\,x/m}{x/m} = \frac{K\,2^{1/n}P^{1/n}}{KP^{1/n}}$$

$$64 = 2^{1/n}$$

$2^6 = 2^{1/n}$ i.e., $\frac{1}{n} = 6$

$$\therefore n = \frac{1}{6} = 0.167 = 16.7\times10^{-2}$$

Q.7 The number of geometrical isomers possible in triamminetrinitrocobalt (III) is X and in trioxalatochromate (III) is Y. Then the value of X+Y is _________.

Given --
Answer :

Ans: 2

Sol: Triamminetrinitrocobalt(III) is $[Co(NH_3)(NO_2)_3]$
It exihibits facial and meridional isomers
∴ No. of geometrical isomers = 2 = X
Trioxalatochromate (III) is $[Cr(ox)_3]^{3-}$
It doesn't exhibit geometrical isomerism ∴ Y = 0
∴ X + Y = 2 + 0 = 2

Q.8 The density of NaOH solution is 1.2 g cm^{-3}. The molality of this solution is ________m. (Round off to the Nearest Integer)
[Use : Atomic masses : Na : 23.0 u O : 16.0 u H : 1.0 u
Density of H_2O : 1.0 g cm^{-3}]

Given **5**
Answer :

Ans: 5

Sol: Density of NaOH solution = 1.2 g cm^{-3}
Volume of solution = 1 L = 1000 cm^3
Mass of solution = 1.2 × 1000 = 1200 g
Mass of solvent (water) = 1 kg = 1000 g
∴ Mass of NaOH (solute) = 1200 − 1000 = 200 g

Number of moles of NaOH $= \frac{200}{40} = 5$

∴ Molality (M) $= \frac{n_{moles}}{\text{Wt of solvent in kg}} = \frac{5}{1} = 5\,M$

Q.9 $PCl_5 \rightleftharpoons PCl_3 + Cl_2 \quad K_c = 1.844$
3.0 moles of PCl_5 is introduced in a 1 L closed reaction vessel at 380 K. The number of moles of PCl_5 at equilibrium is ________ $\times 10^{-3}$. (Round off to the Nearest Integer)

Given --
Answer :

Ans: 1396

Sol:

	PCl_5	$\rightleftharpoons$	PCl_3	+ Cl_2
Initial	3		0	0
At eqbm	3-α		α	α

$K_c = \frac{[PCl_3][Cl_2]}{[PCl_5]} = \frac{\alpha^2}{3-\alpha}$

1.844 (3 − α) = α^2
5.532 − 1.844α = α^2
i.e., $\alpha^2 + 1.844\,\alpha - 5.532 = 0$

$\alpha = \frac{-1.844 \pm \sqrt{(1.844)^2 + 4 \times 5.532}}{2} \cong 1.604$

∴ No. of moles of PCl_5 at eqbm = 3 − α = 3 − 1.604 = 1.396 = 1396 × 10^{-3}

Q.10 For water at 100°C and 1 bar,
$\Delta_{vap}H - \Delta_{vap}U$ = ________ $\times 10^2$ J mol^{-1}. (Round off to the Nearest Integer)
[Use : R = 8.31 J mol^{-1} K^{-1}]
[Assume volume of $H_2O(l)$ is much smaller than volume of $H_2O(g)$. Assume $H_2O(g)$ can be treated as an ideal gas]

Given --
Answer :

Ans: 31

Sol: ΔH − ΔU = Δn RT
= 1 × 8.31 × 373
= 3099.6 J mol^{-1}
= 30.99 × 10^2 J mol^{-1}

PART – C – MATHEMATICS

Section A

Q.1 If the coefficients of x^7 in $\left(x^2+\frac{1}{bx}\right)^{11}$ and x^{-7} in $\left(x-\frac{1}{bx^2}\right)^{11}$, $b \neq 0$, are equal, then the value of b is equal to :

Options
1. 2
2. 1
3. -1
4. -2

Ans: 1

Sol: $\left(x^2+\frac{1}{bx}\right)^{11}$

$$T_{r+1} = {}^{11}C_r \left(x^2\right)^{11-r}\left(\frac{1}{bx}\right)^r$$

Coeff of $x^7 \Rightarrow 22-3r=7$

$15=3r \Rightarrow r=5$

Now, $\left(x-\frac{1}{bx^2}\right)^{11}$

$$T_{r+1} = {}^{11}C_r x^{11-r}\frac{1}{\left(bx^2\right)^r}$$

Coeff of $x^{-7} \Rightarrow 11-3r=-7$

$18=3r \Rightarrow r=6$

$${}^{11}C_5\frac{1}{b^5} = {}^{11}C_6\frac{1}{b^6}$$

$b^5(b-1)=0$

$\Rightarrow b=1, b\neq 0$

Q.2 A ray of light through (2, 1) is reflected at a point P on the y-axis and then passes through the point (5, 3). If this reflected ray is the directrix of an ellipse with eccentricity $\frac{1}{3}$ and the distance of the nearer focus from this directrix is $\frac{8}{\sqrt{53}}$, then the equation of the other directrix can be :

Options
1. $11x-7y-8=0$ or $11x+7y+15=0$
2. $2x-7y-39=0$ or $2x-7y-7=0$
3. $11x+7y+8=0$ or $11x+7y-15=0$
4. $2x-7y+29=0$ or $2x-7y-7=0$

Ans: 2x – 7y + 29 = 0 or 2x – 7y – 7 = 0

Sol: Equation of reflected ray

$y - 3) = \frac{2}{7}(x - 3)$

$7y - 21 = 2x - 10$

$2x - 7y + 11 = 0$

Then the equation of the directrix is $2x - 7y + \lambda = 0$

Distance of directrix from focus is

$\frac{a}{e} - ae = \frac{8}{\sqrt{53}}$

$a = \frac{3}{\sqrt{53}}$

Distance between two directrices $= \frac{2a}{e}$

$= 2 \times 3 \times \frac{3}{\sqrt{53}} = \frac{18}{\sqrt{53}}$

$\left|\frac{\lambda - 11}{\sqrt{53}}\right| = \frac{18}{\sqrt{53}}$

$\lambda = 29, -7$

$2x - 7y - 7 = 0$ or $2x - 7y + 29 = 0$

Q.3 The compound statement $(P \vee Q) \wedge (\sim P) \Rightarrow Q$ is equivalent to :

Options
1. $\sim(P \Rightarrow Q) \Leftrightarrow P \wedge \sim Q$
2. $\sim(P \Rightarrow Q)$
3. $P \vee Q$
4. $P \wedge \sim Q$

Ans: $(P \vee Q) \wedge (\sim P) \Rightarrow Q$

Sol:

P	Q	$P \vee Q$	$P \Rightarrow Q$	$\sim(P \Rightarrow Q)$	$\sim P$	$\sim Q$	$P \wedge \sim Q$	$\sim(P \Rightarrow Q) \Leftrightarrow P \wedge \sim Q$	$(P \vee Q) \wedge (\sim P)$	$(P \vee Q) \wedge (\sim P) \Rightarrow \sim Q$
T	T	T	T	F	F	F	F	T	F	T
T	F	T	F	T	F	T	T	T	F	T
F	T	T	T	F	T	F	F	T	T	T
F	F	F	T	F	T	T	F	T	F	T

Q.4 If the area of the bounded region

$R = \left\{(x, y) : \max\{0, \log_e x\} \le y \le 2^x, \frac{1}{2} \le x \le 2\right\}$

is, $\alpha(\log_e 2)^{-1} + \beta(\log_e 2) + \gamma$, then the value of $(\alpha + \beta - 2\gamma)^2$ is equal to :

Options
1. 8
2. 1
3. 4
4. 2

Ans: 2

Sol: Area bounded = $\int_{\frac{1}{2}}^{1} 2^x dx + \int_{1}^{2} (2^x - \ln x) dx$

$$= \left[\frac{4}{\ln 2} - \frac{\sqrt{2}}{\ln 2}\right] = [(2\ln 2 - 2) - (-1)] = \frac{4-\sqrt{2}}{\ln 2} - 2\ln 2 + 1$$

On comparing with $\frac{\alpha}{\ln 2} + \beta \ln 2 + \gamma$

$\alpha = 4 - \sqrt{2}, \beta = -2, \gamma = 1$

So, $(\alpha + \beta - 2\gamma)^2 = 2$

Q.5

Let $y = y(x)$ be solution of the differential equation $\log_e\left(\frac{dy}{dx}\right) = 3x + 4y$, with $y(0) = 0$.

If $y\left(-\frac{2}{3}\log_e 2\right) = \alpha \log_e 2$, then the value of α is equal to :

Options

1. $\frac{1}{4}$
2. 2
3. $-\frac{1}{4}$
4. $-\frac{1}{2}$

Ans: $-\frac{1}{4}$

Sol: $\log_e\left(\frac{dy}{dx}\right) = 3x + 4y, y(0) = 0$

$$\frac{dy}{dx} = e^{3x}.e^{4y}$$

$$\int e^{-4}dy = \int e^{3x}dx$$

$$\frac{e^{-4y}}{-4} = \frac{e^{3x}}{3} + C$$

$y(0) = 0$

$$-\frac{1}{4} = \frac{1}{3} + C$$

$$C = -\frac{7}{12}$$

$$\frac{e^{-4y}}{-4} = \frac{e^{3x}}{3} - \frac{7}{12}$$

Put $x = -\frac{2}{3}\log_e 2$

$$\frac{e^{-4y}}{-4} = \frac{e^{3\left(-\frac{2}{3}\right)\log_e 2}}{3} - \frac{7}{12}$$

$$y = -\frac{1}{4}\log_e 2$$

$$\Rightarrow \alpha = -\frac{1}{4}$$

Q.6 Let the plane passing through the point $(-1, 0, -2)$ and perpendicular to each of the planes $2x+y-z=2$ and $x-y-z=3$ be $ax+by+cz+8=0$. Then the value of $a+b+c$ is equal to :

Options
1. 3
2. 4
3. 5
4. 8

Ans: 4

Sol: Equation of plane is ax + by + cz + 8 = 0

Since it passes through (–1,0,–2)

$-a - 2c + 8 = 0$ ... (1)

ax + by + cz + 8 is perpendicular to 2x + y – z = 2 and x – y – z = 3

Therefore $2a + b - c = 0$ .... (2)

$a - b - c = 0.$ (3)

Solving (1) , (2) and (3) , we get

$a = 2, b = -1, c = 3$

Therefore, $a + b + c = 4$

Q.7 Let P and Q be two distinct points on a circle which has center at C(2, 3) and which passes through origin O. If OC is perpendicular to both the line segments CP and CQ, then the set{P, Q} is equal to :

Options
1. $\{(2+2\sqrt{2}, 3+\sqrt{5}), (2-2\sqrt{2}, 3-\sqrt{5})\}$
2. $\{(-1, 5), (5, 1)\}$
3. $\{(4, 0), (0, 6)\}$
4. $\{(2+2\sqrt{2}, 3-\sqrt{5}), (2-2\sqrt{2}, 3+\sqrt{5})\}$

Ans: $(-1,5)\&(5,1)$

Sol: Given circle is $(x-2)^2 + (y-3)^3 = 13$

Equation of OC is $y = \frac{3}{2}x$

Equation of the line perpendicular to the OC and passing through (2,3) is 3y +2x = 13

Coordinates of P,Q $\Rightarrow \left(2 \pm \sqrt{13}\cos\theta, 3 \pm \sqrt{13}\sin\theta\right)$

$$\Rightarrow \left(2 \pm \sqrt{13}\left(\frac{-3}{\sqrt{13}}\right), 3 \pm \sqrt{13}\left(\frac{2}{\sqrt{13}}\right)\right)$$

$$\Rightarrow (-1,5)\&(5,1)$$

Q.8 If the mean and variance of the following data :

6, 10, 7, 13, a, 12, b, 12

are 9 and $\frac{37}{4}$ respectively, then $(a-b)^2$ is equal to :

Options 1. 16

2. 12

3. 24

4. 32

Ans: 16

Sol: $9 = \frac{10+13+6+7+a+12+b+12}{8}$

$\Rightarrow$ a + b = 12……… (i)

$\sum x_i^2 = 100+169+36+49+a^2+144+b^2+144$

$= a^2 + b^2 + 642$

$\sigma^2 = \frac{\sum x_i^2}{8} - (\bar{x})^2 = \frac{37}{4}$

$\frac{a^2+b^2+642}{8} - 81 = \frac{37}{4}$

$\Rightarrow a^2 + b^2 = 80 \Rightarrow a^2 + (12-a)^2 = 80 \Rightarrow a^2 - 12a + 32 = 0$

$\Rightarrow$ a = 4 or a = 8

b = 8 or b = 4

So, $(a-b)^2 = 16$

Q.9 Let $f : \mathbf{R} \to \mathbf{R}$ be a function such that $f(2) = 4$ and $f'(2) = 1$. Then, the value of $\lim_{x \to 2} \frac{x^2 f(2) - 4f(x)}{x-2}$ is equal to :

Options 1. 12

2. 8

3. 16

4. 4

Ans: 12

Sol: $\lim_{x\to 2} \frac{x^2 f(2) - 4f(x)}{x-2} = \frac{0}{0}$

Applying L Hospitals rule, $\lim_{x\to 2} \frac{2xf(2) - 4f'(x)}{1}$

$= 2\cdot(2)\cdot f(2) - 4f'(2) = 16 - 4 = 12$

Q.10 Let

A = {(x, y) ∈ **R** × **R** | $2x^2+2y^2-2x-2y=1$},

B = {(x, y) ∈ **R** × **R** | $4x^2+4y^2-16y+7=0$} and

C = {(x, y) ∈ **R** × **R** | $x^2+y^2-4x-2y+5 \le r^2$}.

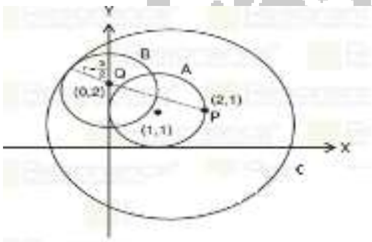
Then the minimum value of |r| such that $A \cup B \subseteq C$ is equal to :

Options

1. $\dfrac{3+\sqrt{10}}{2}$
2. $1+\sqrt{5}$
3. $\dfrac{2+\sqrt{10}}{2}$
4. $\dfrac{3+2\sqrt{5}}{2}$

Ans: $\frac{3+2\sqrt{5}}{2}$

Sol:



Let centre of circle C is P
Centre of circle B is Q
For A U B $\subseteq$ C

$r \ge PQ + \dfrac{3}{2}$

$r \ge \sqrt{2^2+1^2} + \dfrac{3}{2}$

$r \ge \dfrac{3+2\sqrt{5}}{2} \Rightarrow r_{min} = \dfrac{3+2\sqrt{5}}{2}$

Q.11 Let α, β be two roots of the equation $x^2 + (20)^{\frac{1}{4}} x + (5)^{\frac{1}{2}} = 0$. Then $\alpha^8 + \beta^8$ is equal to :

Options

1. 50
2. 160
3. 100
4. 10

Ans: 50

Sol: $\left(x^2+\sqrt{5}\right)^2=\sqrt{20}x^2$

Squaring both sides

$x^4+5+2\sqrt{5}x^2=\sqrt{20}x^2=2\sqrt{5}x^2$

$$\left.\begin{aligned}x^4&=-5\\x^8&=25\end{aligned}\right\}$$

$\alpha^8=25$, $\beta^8=25$

$\alpha^8+\beta^8=50$

Q.12

If $\sin\theta+\cos\theta=\frac{1}{2}$, then $16(\sin(2\theta)+\cos(4\theta)+\sin(6\theta))$ is equal to :

Options
1. -23
2. 23
3. -27
4. 27

Ans: -23

Sol: $(\sin\theta+\cos\theta)^2=\frac{1}{4}$

$\sin 2\theta=\frac{-3}{4}$

$16(\sin 2\theta+\cos 4\theta+\sin 6\theta)=16(\sin 2\theta+1-2\sin^2 2\theta+3\sin 2\theta-4\sin^3 2\theta)$

$=-23$

Q.13

Let $\vec{a}=\hat{i}+\hat{j}+2\hat{k}$ and $\vec{b}=-\hat{i}+2\hat{j}+3\hat{k}$. Then the vector product

$\left(\vec{a}+\vec{b}\right)\times\left(\left(\vec{a}\times\left(\left(\vec{a}-\vec{b}\right)\times\vec{b}\right)\right)\times\vec{b}\right)$ is equal to :

Options
1. $7\left(30\hat{i}-5\hat{j}+7\hat{k}\right)$
2. $5\left(30\hat{i}-5\hat{j}+7\hat{k}\right)$
3. $5\left(34\hat{i}-5\hat{j}+3\hat{k}\right)$
4. $7\left(34\hat{i}-5\hat{j}+3\hat{k}\right)$

Ans: $7\left(34\hat{i}-5\hat{j}+3\hat{k}\right)$

Sol: $\vec{a}=\hat{i}+\hat{j}+2\hat{k}$, $\vec{b}=-\hat{i}+2\hat{j}+3\hat{k}$

$\vec{a}+\vec{b}=3\hat{j}+5\hat{k}, \vec{a}-\vec{b}=2\hat{i}-\hat{j}-\hat{k}$

$$\left(\vec{a}-\vec{b}\right)\times\vec{b}=\begin{vmatrix}\hat{i}&\hat{j}&\hat{k}\\2&-1&-1\\-1&2&3\end{vmatrix}=-\hat{i}-5\hat{j}+3\hat{k}$$

$$\vec{a}\times\left(\left(\vec{a}-\vec{b}\right)\times\vec{b}\right)=\begin{vmatrix}\hat{i}&\hat{j}&\hat{k}\\1&1&2\\-1&-5&3\end{vmatrix}=13\hat{i}-5\hat{j}-4\hat{k}$$

$$\left(\vec{a}\times\left(\left(\vec{a}-\vec{b}\right)\times\vec{b}\right)\right)\times\vec{b}=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 13 & -5 & -4\\ -1 & 2 & 3\end{vmatrix}=-7\hat{i}-35\hat{j}+21\hat{k}$$

$$\left(\vec{a}+\vec{b}\right)\times\left(\left(\vec{a}\times\left(\left(\vec{a}-\vec{b}\right)\times\vec{b}\right)\right)\times\vec{b}\right)=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 0 & 3 & 5\\ -7 & -35 & 21\end{vmatrix}$$

$$=7\left(34\hat{i}-5\hat{j}+3\hat{k}\right)$$

Q.14

The value of $\lim\limits_{n\to\infty}\frac{1}{n}\sum\limits_{j=1}^{n}\frac{(2j-1)+8n}{(2j-1)+4n}$ is equal to :

Options

1. $2-\log_e\left(\frac{2}{3}\right)$
2. $1+2\log_e\left(\frac{3}{2}\right)$
3. $5+\log_e\left(\frac{3}{2}\right)$
4. $3+2\log_e\left(\frac{2}{3}\right)$

Ans: $2-\log_e\left(\frac{2}{3}\right)$

Sol: $\lim\limits_{n\to\infty}\frac{1}{n}\sum\limits_{j=1}^{n}\frac{\left(2\frac{j}{n}-\frac{1}{n}+8\right)}{\left(2\frac{j}{n}-\frac{1}{n}+4\right)}$

$=\int_0^1\frac{2x+8}{2x+4}dx$

$=1+4.\frac{1}{2}\left[\ln(2x+4)\right]_0^1$

$=1+2\ln\left(\frac{3}{2}\right)$

Q.15 Let $A = \begin{bmatrix} 1 & 2 \\ -1 & 4 \end{bmatrix}$. If $A^{-1} = \alpha I + \beta A$, $\alpha, \beta \in \mathbf{R}$, I is a 2×2 identity matrix, then $4(\alpha - \beta)$ is equal to :

Options
1. 2
2. $\frac{8}{3}$
3. 4
4. 5

Ans: 4

Sol: **A=** $\begin{bmatrix} 1 & 2 \\ -1 & 4 \end{bmatrix}$

Characteristic equation is given by

$\Rightarrow A^2 - 5A + 6I = 0$

Multiply both side by A^{-1}

$A - 5I + 6A^{-1} = 0$

$A^{-1} = \frac{1}{6}(5I - A) = \frac{5}{6}I - \frac{1}{6}A$ .....(1)

$A^{-1} = \alpha I + \beta A$ ......(2)

Comparing equation (1) and (2) we get

$\alpha = \frac{5}{6}, \beta = -\frac{1}{6}$

$4(\alpha - \beta) = 4\left(\frac{5}{6} + \frac{1}{6}\right) = 4$

Q.16 The value of the definite integral

$$\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \frac{dx}{(1 + e^{x\cos x})(\sin^4 x + \cos^4 x)}$$

is equal to :

Options
1. $-\frac{\pi}{2}$
2. $\frac{\pi}{\sqrt{2}}$
3. $-\frac{\pi}{4}$
4. $\frac{\pi}{2\sqrt{2}}$

Ans: $\frac{\pi}{2\sqrt{2}}$

Sol: $I = \int\limits_{-\pi/4}^{\pi/4} \frac{e^{x\cos x}}{\left(1+e^{x\cos x}\right)\left(\sin^4 x+\cos^4 x\right)}$

$2I = \int\limits_{-\pi/4}^{\pi/4} \frac{1}{\left(\sin^4 x+\cos^4 x\right)}$

$2I = \int\limits_{-\pi/4}^{\pi/4} \frac{1}{1-\frac{1}{2}\sin^2 2x} dx$

$2I = \int_{-\pi/4}^{\pi/4} \frac{2\sec^2 2x}{\sec^2 2x+1} dx$

$2I = \frac{4}{2}\int\limits_0^{\infty} \frac{dt}{2+t^2} = \left(\frac{1}{\sqrt{2}}\tan^{-1}\frac{t}{\sqrt{2}}\right)_0^{\infty}$

$I = \frac{\pi}{2\sqrt{2}}$

Q.17

Let $f : \left(-\frac{\pi}{4}, \frac{\pi}{4}\right) \to \mathbf{R}$ be defined as

$$f(x) = \begin{cases} (1+|\sin x|)^{\frac{3a}{|\sin x|}} & , -\frac{\pi}{4} < x < 0 \\ b & , x = 0 \\ e^{\cot 4x/\cot 2x} & , 0 < x < \frac{\pi}{4} \end{cases}$$

If f is continuous at $x=0$, then the value of $6a+b^2$ is equal to :

Options
1. $1+e$
2. $1-e$
3. $e-1$
4. e

Ans: 1+e

Sol: RHL = $f(0^+) = \lim\limits_{x\to 0^+} e^{\frac{\cot 4x}{\cot 2x}} = \lim\limits_{x\to 0^+} e^{\frac{\tan 2x}{\tan 4x}} = e^{\frac{1}{2}}$

LHL = $f(0^-)$= $\lim\limits_{x\to 0^-} (1-\sin x)^{\frac{3a}{-\sin x}}$

$e^{\lim\limits_{x\to 0^-}\frac{-3a}{\sin x}(-\sin x)} = e^{3a}$

Since f(x) is continuous,

$\Rightarrow e^{\frac{1}{2}} = e^{3a} = b \Rightarrow a = \frac{1}{6}$ & $b = e^{\frac{1}{2}}$

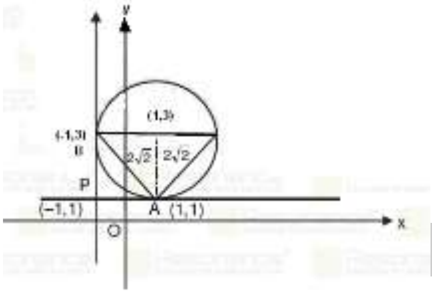
$\Rightarrow 6a+b^2 = 1+e$

Q.18 Two tangents are drawn from the point P(−1, 1) to the circle $x^2+y^2-2x-6y+6=0$. If these tangents touch the circle at points A and B, and if D is a point on the circle such that length of the segments AB and AD are equal, then the area of the triangle ABD is equal to :

Options
1. 2
2. $(3\sqrt{2}+2)$
3. 4
4. $3(\sqrt{2}-1)$

Ans: 4

Sol:



$$\Delta PAB=\frac{1}{2}\times 4\times 2=4$$

Q.19 The probability that a randomly selected 2-digit number belongs to the set {n ϵ **N** : (2^n-2) is a multiple of 3} is equal to :

Options
1. $\frac{1}{3}$
2. $\frac{1}{6}$
3. $\frac{2}{3}$
4. $\frac{1}{2}$

Ans: $\frac{1}{2}$

Sol: Total number of cases = $^{90}C_1 = 90$

Now $2^n - 2 = (3-1)^n - 2$

$= {}^nC_0\, 3^n - {}^nC_1\, 3^{n-1} + \ldots\ldots.. + (-1)^{n-1}\, {}^nC_{n-1}\, 3 + (-1)^n\, {}^nC_n - 2$

$= 3(3^{n-1} - n.3^{n-2} + \ldots\ldots.. + (-1)^{n-1}.n) + (-1)^n - 2$

Therefore $(2^n - 2)$ is multiple of 3 only when n is odd

Number of favourable cases = 45,

Required probability= $\frac{45}{90}=\frac{1}{2}$

Q.20 Let **C** be the set of all complex numbers. Let

$S_1 = \{z \in \mathbf{C} \mid |z-3-2i|^2 = 8\}$,

$S_2 = \{z \in \mathbf{C} \mid \mathrm{Re}(z) \geq 5\}$ and

$S_3 = \{z \in \mathbf{C} \mid |z - \overline{z}| \geq 8\}$.

Then the number of elements in $S_1 \cap S_2 \cap S_3$ is equal to :

Options
1. 1
2. Infinite
3. 2
4. 0

Ans: 1

Sol: Let z = x+iy

S_3 is y| ≥ 4
S_2 is x ≥ 5
S_1 is $(x-3)^2 + (y-2)^2 = 8$
⇒ There is exactly one point (5,4) in $S_1 \cap S_2 \cap S_3$

Section B

Q.1 Let $F : [3, 5] \to \mathbf{R}$ be a twice differentiable function on (3, 5) such that

$F(x) = e^{-x} \int_3^x (3t^2 + 2t + 4F'(t))dt$.

If $F'(4) = \dfrac{\alpha e^{\beta} - 224}{(e^{\beta} - 4)^2}$, then $\alpha + \beta$ is equal to _________.

Given -- Answer :

Ans: 16.00

Sol: Put x = 3 ⇒ F(3) = 0

$e^{-x}\left[t^3 + t^2 + 4F(t)\right]_3^x = F(x)$

$e^{-x}(x^3 + x^2 + 4F(x) - (27 + 9 + 4F(3)) = F(x)$

$\Rightarrow F(x) = e^{-x}(x^3 + x^2 - 36 + 4F(x))$

$\Rightarrow e^x F(x) = x^3 + x^2 - 36 + 4F(x)$

$$F(x) = \frac{x^3 + x^2 - 36}{(e^x - 4)}$$

$$F'(x) = \frac{12e^4 - 224}{(e^4 - 4)^2}$$

Hence α = 12 and β = 4

α + β = 16

Q.2 Let the domain of the function

$$f(x) = \log_4 \left(\log_5\left(\log_3\left(18x - x^2 - 77\right)\right)\right) \text{ be (a, b).}$$

Then the value of the integral

$$\int_a^b \frac{\sin^3 x}{(\sin^3 x + \sin^3(a + b - x))} dx$$

is equal to ________.

Given -2
Answer :

Ans: 1.00

Sol: $\log_5 \log_3 (18x - x^2 - 77) > 0$

$18x - x^2 - 77 > 3$

$x^2 - 18x + 80 < 0$

$x \in (8, 10)$

$\therefore$ a = 8 and b = 10

$$\therefore I = \int_a^b \frac{\sin^3 x dx}{\sin^3 x + \sin^3(a+b-x)} = \int_8^{10} \frac{\sin^3 x dx}{\sin^3 x + \sin^3(18-x)}$$

$$2I = \int_8^{10} 1 dx = 2$$

$\therefore I = 1$

Q.3 If $y = y(x)$, $y \in \left[0, \frac{\pi}{2}\right)$ is the solution of the differential equation

$\sec y \frac{dy}{dx} - \sin(x+y) - \sin(x-y) = 0$, with $y(0) = 0$, then $5y'\left(\frac{\pi}{2}\right)$ is equal to ________.

Given 2
Answer :

Ans: 2.00

Sol: $\sec y \frac{dy}{dx} = 2\sin(x)\cos(y)$ ....(1)

$\int \sec^2 y dy = \int 2 \sin x dx$

tany = – 2cosx + c

When x = 0, y = 0 $\Rightarrow$ c = 2

tany = – 2cosx + 2 ....(2)

$\because \sec^2 y \frac{dy}{dx} = 2\sin x$

$(1 + \tan^2 y)\frac{dy}{dx} = 2\sin x$ ....(3)

From, (2) & (3)

$$\frac{dy}{dx} = \frac{2\sin x}{1 + (2 - 2\cos x)^2}$$

$$y'\left(\frac{\pi}{2}\right) = \frac{2}{1+(2)^2} = \frac{2}{5} \Rightarrow 5y'\left(\frac{\pi}{2}\right) = 2$$

Q.4

If $\log_3 2$, $\log_3(2^x-5)$, $\log_3\left(2^x-\frac{7}{2}\right)$ are in an arithmetic progression, then the value of x is equal to ________.

Given --
Answer :

Ans: 3.00

Sol: $2\log_3(2^x - 5) = \log_3 2 + \log_3\left(2^x - \frac{7}{2}\right)$

$\Rightarrow \log_3(2^x - 5)^2 = \log_3 \left(\mathbf{2}\left(2^x - \frac{7}{2}\right)\right)$

$\Rightarrow (2^x)^2 - 10(2^x) + 25 = 2(2^x) - 7$

$\Rightarrow (2^x)^2 - 12(2^x) + 32 = 0$

$\Rightarrow 2^x = 4$ or 8

$\Rightarrow x = 2$ or 3

But, when $x = 2$, $\log(2^x - 5)$ is not defined

$\Rightarrow x = 3$

Q.5

$$\text{Let } f(x) = \begin{vmatrix} \sin^2 x & -2+\cos^2 x & \cos 2x \\ 2+\sin^2 x & \cos^2 x & \cos 2x \\ \sin^2 x & \cos^2 x & 1+\cos 2x \end{vmatrix}, x \in [0, \pi].$$

Then the maximum value of $f(x)$ is equal to ________.

Given **6**
Answer :

Ans: 6.00

Sol: $R_1 \to R_1 - R_2$

$R_2 \to R_2 - R_3$

$$= \begin{vmatrix} -2 & -2 & 0 \\ 2 & 0 & -1 \\ \sin^2 x & \cos^2 x & 1+\cos 2x \end{vmatrix}$$

Expanding along 1st row, we get

$= 4 + 2\cos 2x$

Maximum value of $\cos 2x$ is 1

Therefore, maximum value of $f(x) = 6$

Q.6 For real numbers α and β, consider the following system of linear equations :

$x+y-z=2,\ x+2y+\alpha z=1,\ 2x-y+z=\beta.$

If the system has infinite solutions, then $\alpha+\beta$ is equal to ________.

Given **5**
Answer :

Ans: 5.00

Sol: By, Cramer's rule, for infinite number of solutions

$\Delta = \Delta_1 = \Delta_2 = \Delta_3 = 0$

$$\Delta = \begin{vmatrix} 1 & 1 & -1 \\ 1 & 2 & 2 \\ 2 & -1 & 1 \end{vmatrix} = 0$$

$1(2+\alpha) - 1(1-2\alpha) - 1(-1-4) = 0$

$3\alpha = -6$

$$\alpha = -2$$

$$\Delta 1 = \begin{vmatrix} 1 & 1 & 2 \\ 1 & 2 & 1 \\ 2 & -1 & \beta \end{vmatrix} = 0$$

$1(2\beta+1) - 1(\beta-2) + 2(-1-4) = 0$

$$\beta = 7$$

$$\therefore \alpha + \beta = 5$$

Q.7 Let $\vec{a} = \hat{i} + \hat{j} + \hat{k}$, $\vec{b}$ and $\vec{c} = \hat{j} - \hat{k}$ be three vectors such that $\vec{a} \times \vec{b} = \vec{c}$ and $\vec{a} \cdot \vec{b} = 1$. If the length of projection vector of the vector $\vec{b}$ on the vector $\vec{a} \times \vec{c}$ is l, then the value of $3l^2$ is equal to ________.

Given --
Answer :

Ans: 2.00

Sol: $\vec{a} \times \vec{b} = \vec{c}$

Taking dot product with $\vec{c}$

$(\vec{a} \times \vec{b}) \cdot \vec{c} = |\vec{c}|^2 = 2$

Length of projection of $\vec{b}$ on $(\vec{a} \times \vec{c})$ is ℓ

$$\ell = \frac{|\vec{b} \cdot (\vec{a} \times \vec{c})|}{|\vec{a} \times \vec{c}|}$$

$$\ell = \frac{2}{\sqrt{6}}$$

$3\ell^2 = 2$

Q.8 Let $f : [0, 3] \to \mathbf{R}$ be defined by

$f(x) = \min\{x - [x], 1 + [x] - x\}$

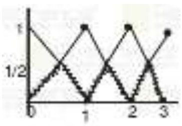
where $[x]$ is the greatest integer less than or equal to x.

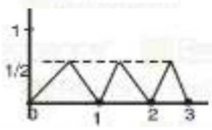
Let P denote the set containing all $x \in [0, 3]$ where f is discontinuous, and Q denote the set containing all $x \in (0, 3)$ where f is not differentiable. Then the sum of number of elements in P and Q is equal to ________.

Given --
Answer :

Ans: 5.00

Sol: f(x) = min {{x}, 1–{x}}





From figure, number of discontinuities is 0 and non-differentiable points is 5.

So, n(P) = 0, n(Q) = 5

n(P) + n(Q) = 5

Q.9 Let S = {1, 2, 3, 4, 5, 6, 7}. Then the number of possible functions $f : S \to S$ such that $f(m \cdot n) = f(m) \cdot f(n)$ for every m, n $\in$ S and m·n $\in$ S is equal to ________.

Given **10**
Answer :

Ans: 490.00

Sol: Putting

$m = 1,\quad$ then $f(n) = f(1)\, f(n)$

$\Rightarrow f(1) = 1$

When $m = n = 2 \Rightarrow f(4) = f(2)\, f(2)$

$$= \begin{cases} f(2)=1 & \Rightarrow \quad f(4)=1 \\ f(2)=2 & \Rightarrow \quad f(4)=4 \\ f(2)=3 & \Rightarrow \quad f(4)=9 \notin s \end{cases}$$

Hence f(2) can be 1 or 2

Put m = 2, n= 3

$$\Rightarrow f(6) = f(2) f(3) = \begin{cases} \text{when } f(2)=1, & f(3)=1,2,3,\ldots\ldots.. 7 \\ & f(6)=1,2,3,\ldots\ldots.. 7 \quad f(4)=4 \\ f(2)= & \text{then} \quad\quad f(3)=1,2,3 \end{cases}$$

Also f(5) & f(7) may take any value from {1,2,3,..0…..7}

So total number of such functions = $1 \times 1 \times 7 \times 1 \times 7 \times 1 \times 7 + 1 \times 1 \times 3 \times 1 \times 7 \times 1 \times 7$

= 49(7+3) = 490

Q.10 Let a plane P pass through the point (3, 7, −7) and contain the line,

$\frac{x-2}{-3} = \frac{y-3}{2} = \frac{z+2}{1}$. If distance of the plane P from the origin is d, then d^2 is equal to

__________.

Given --
Answer :

Ans: 3.00

Sol: Equation of plane a(x–3)+b(y–7)+c(z+7) = 0 ……. (1)

where a,b,c are the direction ratio of the normal to the plane

Given line is $\frac{x-3}{-3} = \frac{y-3}{2} = \frac{z+2}{1}$ ……. (2)

Since plane (1) contains the given line then

–3a + 2b + c = 0 ……….(3)

Also point (2,3,–2) on line (2i) lies in plane (1)

a(2–3) + b(3–7) + c(–2+7) = 0

–a – 4b +5c = 0 ………(4)

Solving (1) and (4), we get a=14k, b=14k, c=14k

Substituting in (1), we get x+y+z – 3 = 0

This is equation of plane

Now distance of the plane from the origin is

$$d = \left|\frac{-3}{\sqrt{1+1+1}}\right| = \frac{3}{\sqrt{3}} = \sqrt{3}$$

$d^2=3$