SOLUTIONS & ANSWERS FOR JEE MAINS-2021
25th July Shift 2

[PHYSICS, CHEMISTRY & MATHEMATICS]

PART – A – PHYSICS

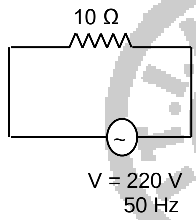
Section A

Q.1 A 10 Ω resistance is connected across 220 V - 50 Hz AC supply. The time taken by the current to change from its maximum value to the rms value is :

Options
1. 4.5 ms
2. 3.0 ms
3. 2.5 ms
4. 1.5 ms

Ans: 2.5 ms

Sol:



$i = i_0 \sin wt$ When $i_{rms} = \frac{i_0}{\sqrt{2}}$, $t = t_2$

When $i = i_0$, $t = t_1$ $\frac{i_0}{\sqrt{2}} = i_0 \sin wt$

$\therefore i_0 = i_0 \sin wt_1$ $\sin wt_2 = \frac{1}{\sqrt{2}}$

$\sin wt_i$ $wt_2 = \frac{\pi}{4}$

$wt_1 = \frac{\pi}{2}$ $t_2 = \frac{\pi}{4w}$

$t_1 = \frac{\pi}{2w}$

$\therefore$ Time taken by current from maximum value of rms value,

$t_2 - t_1 = \frac{\pi}{2w} - \frac{\pi}{4w} = \frac{\pi}{4w}$ $w = 2\pi f$

$\therefore t_2 - t_1 = \frac{\pi}{4 \times 2\pi f} = \frac{1}{8f} = \frac{1}{8 \times 50} = \frac{1}{400}$ sec

$\therefore t_2 - t_1 = 2.5$ ms

Q.2 Two vectors $\vec{X}$ and $\vec{Y}$ have equal magnitude. The magnitude of $(\vec{X} - \vec{Y})$ is n times the magnitude of $(\vec{X} + \vec{Y})$. The angle between $\vec{X}$ and $\vec{Y}$ is :

Options

1. $\cos^{-1}\left(\frac{n^2+1}{-n^2-1}\right)$
2. $\cos^{-1}\left(\frac{-n^2-1}{n^2-1}\right)$
3. $\cos^{-1}\left(\frac{n^2+1}{n^2-1}\right)$
4. $\cos^{-1}\left(\frac{n^2-1}{-n^2-1}\right)$

Ans: $\cos^{-1}\left(\frac{n^2-1}{-n^2-1}\right)$

Sol: $|x| = |y|$

$|x + y| = n\,|x - y|$

$\sqrt{x^2+y^2-2xy\cos\theta} = n\sqrt{(x^2+y^2+2xy\cos\theta)}$

Squaring both side

$x^2 + y^2 - 2xy\cos\theta = n^2(x^2 + y^2 + 2xy\cos\theta)$

$\therefore |x| = |y|$

$2x - 2x^2\cos\theta = n^2(2x^2 + 2x\cos\theta)$

$2x^2(1 - \cos\theta) = n^2 . 2x^2(1+\cos\theta)$

$1 - \cos\theta = n^2 + n^2\cos\theta$

$\cos\theta = \frac{1-n^2}{1+n^2}$

$\theta = \cos^{-1}\left(\frac{n^2-1}{-n^2-1}\right)$

Q.3 The relation between time t and distance x for a moving body is given as $t = mx^2 + nx$, where m and n are constants. The retardation of the motion is : (Where v stands for velocity)

Options

1. $2\,mnv^3$
2. $2\,mv^3$
3. $2\,n^2v^3$
4. $2\,nv^3$

Ans: $2mv^3$

Sol: $t = mx^2 + nx$

$\frac{1}{v} = \frac{dt}{dx} = \frac{d}{dx}(mx^2+nx) = 2mx+n$

$\therefore v = \frac{1}{2mx+n}$ $\qquad \frac{dv}{dx} = \frac{-2m}{(2mx+n)^2}$

$\therefore a = \frac{dv}{dt} = \frac{dv}{dx}\times\frac{dx}{dt} = \frac{-2m}{(2mx+n)^2}\times\frac{1}{(2mx+n)} = \frac{-2m}{(2mx+n)^3}$

$a = (-2m)\,v^3$

$|a| = 2mv^3$

Q.4 Consider a planet in some solar system which has a mass double the mass of earth and density equal to the average density of earth. If the weight of an object on earth is W, the weight of the same object on that planet will be :

Options

1. $\sqrt{2}$ W
2. $2^{\frac{1}{3}}$ W
3. W
4. 2 W

Ans: $2^{1/3}W$

Sol: Given density is same

$M_E = \frac{4}{3}\pi R_E^3 \rho$

For other planet $2M_E = \frac{4}{3}\pi R^3 \rho$

$\therefore R = 2^{1/3} R_E$

Now weight on earth $W_E = mg_E = \frac{GM_E m}{R_E^2}$

Weight on other planed $w = \frac{G \times 2M_E m}{R^2} = \frac{G \times 2M_E m}{\left(2^{1/3} R_E\right)^2}$

$\therefore w = 2^{1/3} W_E$

Q.5 When radiation of wavelength λ is incident on a metallic surface, the stopping potential of ejected photoelectrons is 4.8 V. If the same surface is illuminated by radiation of double the previous wavelength, then the stopping potential becomes 1.6 V. The threshold wavelength of the metal is :

Options

1. 8 λ
2. 6 λ
3. 2 λ
4. 4 λ

Ans: 4λ

Sol: Stopping potential = 4.8 eV = maximum K.E. of ejected electrons

$\therefore 4.8 = h\nu - \phi = \frac{hc}{\lambda} - \phi - - - - - (1)$

For double λ, stopping potential is 1.6 V

$1.6 = \frac{hc}{2\lambda} - \phi - - - - - (2)$

$(1) - (2) \Rightarrow 3.2 = \frac{hc}{\lambda} - \frac{hc}{2\lambda} = \frac{hc}{2\lambda}$

$\therefore \lambda = \frac{hc}{6.4}$

To find threshold λ put $\lambda = \frac{hc}{6.4}$ and φ = 1.6 V in (2) $\frac{hc}{\lambda_{th}} = 1.6$

$\lambda_{th} = \frac{hc}{1.6} = \left(\frac{hc}{6.4}\right) \times 4 = 4\lambda$

Q.6 The instantaneous velocity of a particle moving in a straight line is given as $v = \alpha t + \beta t^2$, where α and β are constants. The distance travelled by the particle between 1 s and 2 s is :

Options

1. $\frac{3}{2}\alpha + \frac{7}{3}\beta$
2. $3\alpha + 7\beta$
3. $\frac{\alpha}{2} + \frac{\beta}{3}$
4. $\frac{3}{2}\alpha + \frac{7}{2}\beta$

Ans: $\frac{3}{2}\alpha + \frac{7}{3}\beta$

Sol: $v = \alpha t + \beta t^2$

$\frac{ds}{dt} = \alpha t + \beta t^2 \Rightarrow ds = (\alpha t + \beta t^2)\, dt$

$\therefore \int_{s_1}^{s_2} ds = \int_{t=1}^{t=2} (\alpha t + \beta t^2)\, dt$

$[s]_{s_1}^{s_2} = \left[\frac{\alpha t^2}{2} + \frac{\beta t^3}{3}\right]_1^2$

$s_2 - s_1 = \frac{\alpha(4-1)}{2} + \frac{\beta(8-1)}{3} = \frac{3\alpha}{2} + \frac{7\beta}{3}$

Q.7 A balloon was moving upwards with a uniform velocity of 10 m/s. An object of finite mass is dropped from the balloon when it was at a height of 75 m from the ground level. The height of the balloon from the ground when object strikes the ground was around : (takes the value of g as 10 m/s^2)

Options

1. 125 m
2. 200 m
3. 250 m
4. 300 m

Ans: 125 m

Sol: Let t ne the time of the object to reach ground

$\therefore$ u + 10 m/s a = -10 m/s^2 g = -75 m

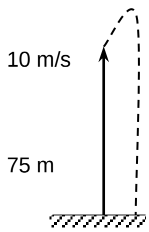
$s = ut + \frac{1}{2}at^2$

$-75 = 10t - \frac{1}{2} \times 10 \times t^2$

$\therefore$ T = 5 s

Now with in t = 5 s the height covered by balloon

H = 75 + ut = 75 + (10 × 5) = 125 m



Q.8 A force $\vec{F} = (40\hat{i} + 10\hat{j})$N acts on a body of mass 5 kg. If the body starts from rest, its position vector $\vec{r}$ at time $t = 10$ s, will be :

Options

1. $(100\hat{i} + 400\hat{j})$ m
2. $(100\hat{i} + 100\hat{j})$ m
3. $(400\hat{i} + 400\hat{j})$ m
4. $(400\hat{i} + 100\hat{j})$ m

Ans: $(400\hat{i} + 100\hat{j})$m

Sol: $\vec{F} = (40\hat{i} + 10\hat{j})$N $\quad F = ma = m\frac{dv}{dt}$

$\therefore \frac{dv}{dt} = \frac{F}{m} = \frac{40\hat{i} + 10\hat{j}}{5} = (8\hat{i} + 2\hat{j})$m / s

$\therefore \vec{dv} = (8\hat{i} + 2\hat{j})dt$

$v = \int (8\hat{i} + 2\hat{j})dt = (8t\hat{i} + 2t\hat{j}) + c$

Given t = 0 $\quad$ v = 0 $\Rightarrow$ c = 0

$v = 8t\hat{i} + 2t\hat{j} = \frac{dr}{dt}$

$r = \int (8t\hat{i} + 2t\hat{j})dt$

$= (8\hat{i} + 2\hat{j})\frac{t^2}{2} + c$

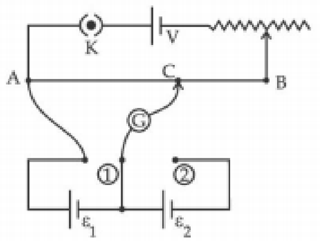
When t = 0 $\quad$ r = 0 $\Rightarrow$ c = 0

$\vec{r} = (8\hat{i} + 2\hat{j})\frac{t^2}{2}$

When t = 10 s

$\vec{r} = (400\hat{i} + 100\hat{j})$m

Q.9 In the given potentiometer circuit arrangement, the balancing length AC is measured to be 250 cm. When the galvanometer connection is shifted from point (1) to point (2) in the given diagram, the balancing length becomes 400 cm. The ratio of the emf of two cells, $\frac{\varepsilon_1}{\varepsilon_2}$ is :



Options

1. $\frac{3}{2}$
2. $\frac{4}{3}$
3. $\frac{5}{3}$
4. $\frac{8}{5}$

Ans: $\frac{5}{3}$

Sol: $E_1 \propto \ell_1$

$E_1 + E_2 \propto \ell_2$

$$\therefore \frac{E_1}{E_1+E_2} = \frac{\lambda_1}{\lambda_2} = \frac{250}{400} = \frac{5}{8}$$

$\therefore 8E_1 = 5\,(E_1 + E_2)$

$3E_1 = 5E_2$

$$\frac{E_1}{E_2} = \frac{5}{3}$$

Q.10 A heat engine has an efficiency of $\frac{1}{6}$. When the temperature of sink is reduced by 62°C, its efficiency get doubled. The temperature of the source is :

Options

1. 124°C
2. 37°C
3. 62°C
4. 99°C

Ans: 99°C

Sol: Efficiency $\eta = 1 - \frac{T_2}{T_1} = \frac{1}{6}$ – – – – – – (1)

If $T_2 = T_2 - 60 \Rightarrow \eta = 2\eta$

$$2\eta = 1 - \frac{T_2 - 62}{T_1} = \frac{2}{6} \text{ – – – – – – – – (2)}$$

$$\therefore \frac{2}{6} = 1 - \frac{T_2}{T_1} + \frac{62}{T_1}$$

$\frac{2}{6} = \frac{1}{6} + \frac{62}{T_1}$ $\left(\therefore 1 - \frac{T_2}{T_1} = \frac{1}{6}\right)$

$\frac{62}{T_1} = \frac{1}{6}$ $T_1 = 372\,k$

In °C = 372 – 273 = 99°C

Q.11 The force is given in terms of time t and displacement x by the equation

$$F = A\cos Bx + C\sin Dt$$

The dimensional formula of $\frac{AD}{B}$ is :

Options
1. $[M^1 L^1 T^{-2}]$
2. $[M^2 L^2 T^{-3}]$
3. $[M^0 L T^{-1}]$
4. $[M L^2 T^{-3}]$

Ans: $M L^2 T^{-3}$

Sol: F = A cos Bx + c sin Dt

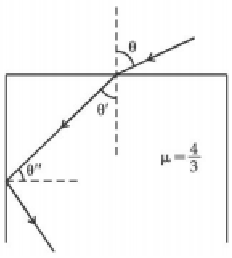
$[F] = [A] = [MLT^{-2}]$

$[Bx] = 1$ $\therefore [B] = [L^{-1}]$

$[Dt] = 1$ $\therefore [D] = [T^{-1}]$

$\therefore \left[\frac{AD}{B}\right] = \frac{[MLT^{-2}][T^{-1}]}{[L^{-1}]} = [ML^2T^{-3}]$

Q.12 A ray of light entering from air into a denser medium of refractive index $\frac{4}{3}$, as shown in figure. The light ray suffers total internal reflection at the adjacent surface as shown. The maximum value of angle θ should be equal to :



Options
1. $\sin^{-1}\frac{\sqrt{7}}{4}$
2. $\sin^{-1}\frac{\sqrt{5}}{3}$
3. $\sin^{-1}\frac{\sqrt{5}}{4}$
4. $\sin^{-1}\frac{\sqrt{7}}{3}$

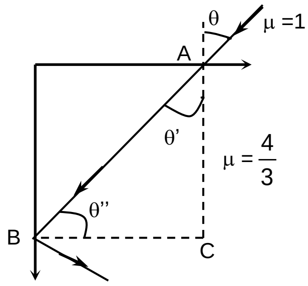
Ans: $\sin^{-1}\frac{\sqrt{7}}{3}$

Sol: At max angle θ, ray at point B goes in gazing emergence at point B (for TIR)

$n_1 \sin i = n_2 \sin r$

$\frac{4}{3} \sin \theta'' = 1 \times \sin 90°$

$\theta'' = \sin^{-1}\left(\frac{3}{4}\right)$



We know from ΔABC

$\theta' = \frac{\pi}{2} - \theta''$

Similarly at point A (for TIR)

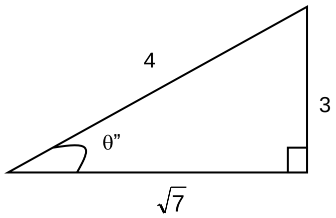
$1 \times \sin \theta = \frac{4}{3} \sin \theta'$

$\text{Sin } \theta = \frac{4}{3} \times \sin\left(\frac{\pi}{2} - \theta''\right)$

$= \frac{4}{3} \cos\left[\cos^{-1}\frac{\sqrt{7}}{4}\right]$

$\theta'' = \sin^{-1}\left(\frac{3}{4}\right)$

$\therefore \sin \theta'' = \frac{3}{4}$



$\therefore \sin\left(\frac{\pi}{2} \theta''\right) = \cos\theta$

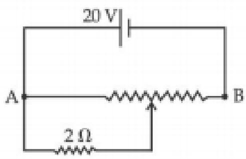
$4 \Rightarrow$ hypotenuse

$3 \Rightarrow$ opposite side

$\sin\theta = \frac{4}{3} \times \frac{\sqrt{7}}{4}$

$\theta = \sin^{-1}\left(\frac{\sqrt{7}}{3}\right)$

Q.13 The given potentiometer has its wire of resistance 10 Ω. When the sliding contact is in the middle of the potentiometer wire, the potential drop across 2 Ω resistor is :

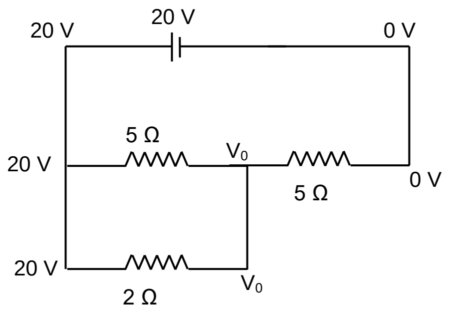


Options

1. $\frac{40}{11}$ V
2. $\frac{40}{9}$ V
3. 5 V
4. 10 V

Ans: $\frac{40}{9}$ V

Sol:



$$\frac{20-V_0}{5}+\frac{0-V_0}{5}+\frac{20-V_0}{2}=0$$

$$V_0=\frac{140}{9}\text{ volt}$$

Potential difference across 2Ω resistor is $20 - V_0$

$$\therefore\ 20 - \frac{140}{9}=\frac{40}{9}\text{ volt}$$

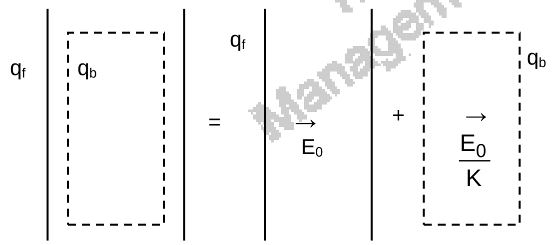
Q.14 If q_f is the free charge on the capacitor plates and q_b is the bound charge on the dielectric slab of dielectric constant k placed between the capacitor plates, then bound charge q_b can be expressed as :

Options

1. $q_b = q_f\left(1-\frac{1}{\sqrt{k}}\right)$
2. $q_b = q_f\left(1+\frac{1}{\sqrt{k}}\right)$
3. $q_b = q_f\left(1+\frac{1}{k}\right)$
4. $q_b = q_f\left(1-\frac{1}{k}\right)$

Ans: $q_b = q_f\left(1-\frac{1}{K}\right)$

Sol:



Due to free charge $E = E_0$

di-electric with di-electric constant K

$$E' = \frac{E_0}{K}$$

$$\therefore\ q_B = q_f\left(1-\frac{1}{K}\right)$$

Q.15 An electron moving with speed v and a photon moving with speed c, have same D-Broglie wavelength. The ratio of kinetic energy of electron to that of photon is :

Options

1. $\frac{v}{3c}$
2. $\frac{3c}{v}$
3. $\frac{2c}{v}$
4. $\frac{v}{2c}$

Ans: $\frac{v}{2c}$

Sol: Given $\lambda_e = \lambda_{ph}$

$$\frac{h}{P_e} = \frac{h}{P_{ph}} \Rightarrow P_e = P_{ph}$$

$$\therefore \sqrt{2mK_e} = \frac{E_{ph}}{c}$$

$$\therefore 2m\,K_e = \left(\frac{E_{ph}}{c}\right)^2 \Rightarrow \frac{K_e}{E_{ph}} = \frac{E_{ph}}{c^2}\left(\frac{1}{2m}\right)$$

$$= \frac{P_{ph}}{c}\left(\frac{1}{2m}\right)$$

$$= \frac{P_e}{c}\left(\frac{1}{2m}\right) = \frac{mv}{c}\left(\frac{1}{2m}\right)$$

$$= \frac{v}{2c}$$

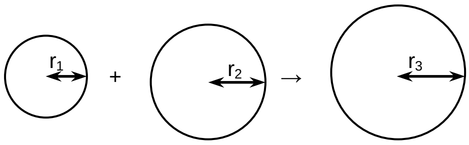
Q.16 Two spherical soap bubbles of radii r_1 and r_2 in vacuum combine under isothermal conditions. The resulting bubble has a radius equal to :

Options

1. $\frac{r_1 + r_2}{2}$
2. $\sqrt{r_1^2 + r_2^2}$
3. $\sqrt{r_1 r_2}$
4. $\frac{r_1 r_2}{r_1 + r_2}$

Ans: $\sqrt{r_1^2 + r_2^2}$

Sol:



No. of moles is conserved

$n_1 + n_2 = n_3$

$P_1V_1 + P_2V_2 = P_3V_3$

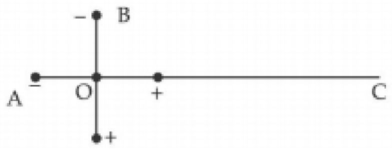
$$\frac{4s}{r_1}\left(\frac{4}{3}\pi r_1^3\right)+\frac{4s}{r_2}\left(\frac{4}{3}\pi r_2^3\right)=\frac{4s}{r_3}\left(\frac{4}{3}\pi r_3^3\right)$$

$$r_1^2 + r_2^2 = r_3^2$$

$$r_3 = \sqrt{r_1^2 + r_2^2}$$

Q.17 Two ideal electric dipoles A and B, having their dipole moment p_1 and p_2 respectively are placed on a plane with their centres at O as shown in the figure. At point C on the axis of dipole A, the resultant electric field is making an angle of 37° with the axis.

The ratio of the dipole moment of A and B, $\frac{P_1}{P_2}$ is : (take $\sin 37° = \frac{3}{5}$)

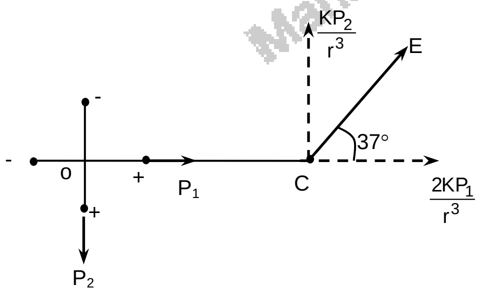


Options

1. $\frac{3}{2}$
2. $\frac{3}{8}$
3. $\frac{4}{3}$
4. $\frac{2}{3}$

Ans: $\frac{2}{3}$

Sol:



$$K = \frac{1}{4\pi\varepsilon_0}$$

$$\tan 37° = \frac{\frac{KP_2}{r^3}}{\frac{2KP_1}{r^3}} = \frac{3}{4}$$

$$\therefore \frac{P_2}{2P_1} = \frac{3}{4} \Rightarrow 4P_2 = 6P_1$$

$$\frac{P_1}{P_2} = \frac{2}{3}$$

Q.18 Two ions having same mass have charges in the ratio 1 : 2. They are projected normally in a uniform magnetic field with their speeds in the ratio 2 : 3. The ratio of the radii of their circular trajectories is :

Options
1. 1 : 4
2. 2 : 3
3. 3 : 1
4. 4 : 3

Ans: 4 : 3

Sol: $R = \frac{mv}{qB}$

$$\frac{R_1}{R_3} = \frac{\frac{mv_1}{q_1B}}{\frac{mv_2}{q_2B}} = \frac{V_1}{q_1} \times \frac{q_2}{V_2} = \frac{q_2}{q_1} \times \frac{V_1}{V_2}$$

$$\therefore \frac{R_1}{R_2} = \left(\frac{2}{1}\right) \times \left(\frac{2}{3}\right) = \frac{4}{3}$$

Q.19 In a simple harmonic oscillation, what fraction of total mechanical energy is in the form of kinetic energy, when the particle is midway between mean and extreme position.

Options
1. $\frac{3}{4}$
2. $\frac{1}{4}$
3. $\frac{1}{2}$
4. $\frac{1}{3}$

Ans: $\frac{3}{4}$

Sol: $K = \frac{1}{2} mw^2 (A^2 - x^2) = \frac{1}{2} mw^2 \left(A^2 - \frac{A^2}{4}\right) = \frac{1}{2} mw^2 \times \frac{3A^2}{4}$

$$K = \frac{3}{4}\left(\frac{1}{2} mw^2 A^2\right)$$

Q.20 A prism of refractive index μ and angle of prism A is placed in the position of minimum angle of deviation. If minimum angle of deviation is also A, then in terms of refractive index value of A is :

Options

1. $2\cos^{-1}\left(\frac{\mu}{2}\right)$

2. $\sin^{-1}\left(\frac{\mu}{2}\right)$

3. $\cos^{-1}\left(\frac{\mu}{2}\right)$

4. $\sin^{-1}\left(\sqrt{\frac{\mu-1}{2}}\right)$

Ans: $\cos^{-1}\left(\mu/2\right)$

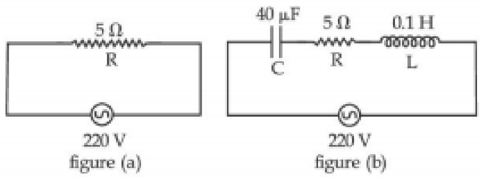
Sol: $\mu = \frac{\sin\left(A+D/2\right)}{\sin\left(A/2\right)}$ here D = A

$= \frac{\sin 2A/2}{\sin A/2} = \frac{\sin A}{\sin\left(A/2\right)} = 2\cos\frac{A}{2}$

$A = 2\cos^{-1}\left(\mu/2\right)$

Section B

Q.1 Two circuits are shown in the figure (a) & (b). At a frequency of __________ rad/s the average power dissipated in one cycle will be same in both the circuits.



Given --
Answer :

Ans: 500.00

Sol: From figure (a)

$P_{avg} = \frac{V_{rms}^2}{R}$

$\frac{V_{rms}^2}{z^2} \times R = \frac{V_{rms}^2}{R}$

$R^2 = z^2$

$25 = \left[\sqrt{(X_C - X_L)^2 + 5^2}\right]^2 = (X_C - X_L)^2 + 25$

$X_C = X_L \Rightarrow \frac{1}{wc} = wL$

$w^2 = \frac{1}{Lc} = \frac{10^6}{0.1 \times 40}$

w = 500

Q.2

The nuclear activity of a radioactive element becomes $\left(\frac{1}{8}\right)^{th}$ of its initial value in 30 years. The half-life of radioactive element is _________ years.

Given 10
Answer :

Ans: 10.00

Sol: $A = A_0 e^{-\lambda t}$

$\frac{A_0}{8} = A_0 e^{-\lambda t}$

$\therefore \lambda t = \ell n\, 8 = \ell n\, 2^3$

$\lambda t = 3\, \ell n\, 2$

$\frac{\lambda n 2}{\lambda} = \frac{t}{3} = \frac{30}{3} = 10$ years

Q.3 A system consists of two types of gas molecules A and B having same number density $2 \times 10^{25}/m^3$. The diameter of A and B are 10 Å and 5 Å respectively. They suffer collision at room temperature. The ratio of average distance covered by the molecule A to that of B between two successive collision is _________ $\times 10^{-2}$.

Given --
Answer :

Ans: 25.00

Sol: Mean free path $\lambda = \frac{1}{\sqrt{2}\pi d^2 n}$

$\frac{\lambda_1}{\lambda_2} = \frac{d_2^2 n_2}{d_1^2 n_1}$

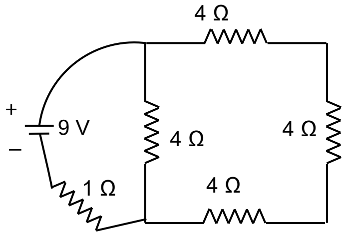
$\left(\frac{5}{10}\right)^2 = 0.25 = 25 \times 10^{-2}$

Q.4 A 16 Ω wire is bend to form a square loop. A 9 V supply having internal resistance of 1 Ω is connected across one of its sides. The potential drop across the diagonals of the square loop is ________ $\times 10^{-1}$ V.

Given --
Answer :

Ans: 45.00

Sol:



By KVL in outer loop

9 – 12i – 4i = 0

16i = 9

$8i = \frac{9}{2} = 4.5 = 45 \times 10^{-1}$

Q.5 A message signal of frequency 20 kHz and peak voltage of 20 volt is used to modulate a carrier wave of frequency 1 MHz and peak voltage of 20 volt. The modulation index will be ________.

Given --
Answer :

Ans: 1.00

Sol: Modulation index $\mu = \frac{A_m}{A_c} = \frac{20}{20} = 1$

Q.6 A light beam of wavelength 500 nm is incident on a metal having work function of 1.25 eV, placed in a magnetic field of intensity B. The electrons emitted perpendicular to the magnetic field B, with maximum kinetic energy are bent into circular arc of radius 30 cm. The value of B is ________ $\times 10^{-7}$ T.
Given $hc = 20 \times 10^{-26}$ J-m, mass of electron $= 9 \times 10^{-31}$ kg

Given --
Answer :

Ans: 125.00

Sol: By photo electric equation

$$\frac{hc}{\lambda} - \phi = K_{max}$$

$$K_{max} = \frac{1240}{500} - 1.25 \approx 1.25$$

$$r = \frac{\sqrt{2mK}}{eB} \quad B = \frac{\sqrt{2mK}}{er} = 125 \times 10^{-7}\ T$$

Q.7 From the given data, the amount of energy required to break the nucleus of aluminium $^{27}_{13}Al$ is ________ $x \times 10^{-3}$ J.
Mass of neutron = 1.00866 u
Mass of proton = 1.00726 u
Mass of Aluminium nucleus = 27.18846 u
(Assume 1 u corresponds to x J of energy)
(Round off to the nearest integer)

Given **124**
Answer :

Ans: 27.16

Sol: $\Delta m = [zm_p + (A - z)\, m_n] - M_{Al} = (13 \times 100726 + 14 \times 1.00866) - 27.18846$
$= 27.2156 - 27.18846 = 0.2716$ u
$E = mc^2 = 27.16\ x \times 10^{-3}$ J

Q.8 In a semiconductor, the number density of intrinsic charge carriers at 27°C is $1.5 \times 10^{16}/m^3$. If the semiconductor is doped with impurity atom, the hole density increases to $4.5 \times 10^{22}/m^3$. The electron density in the doped semiconductor is ________ $\times 10^9/m^3$.

Given --
Answer :

Ans: 5.00

Sol: $n_e\, n_b = n_i^2$

$$n_e = \frac{n_i^2}{n_h} = \frac{(1.5 \times 10^{16})^2}{4.5 \times 10^{22}} = 5 \times 10^9 / m^3$$

Q.9 A solid disc of radius 20 cm and mass 10 kg is rotating with an angular velocity of 600 rpm, about an axis normal to its circular plane and passing through its centre of mass. The retarding torque required to bring the disc at rest in 10 s is ________ $\pi \times 10^{-1}$ Nm.

Given --
Answer :

Ans: 4.00

Sol: $T = \frac{\Delta L}{\Delta t} = \frac{I(w_f - w_i)}{\Delta t} = \frac{\frac{MR^2}{2}(0-w)}{\Delta t} = \frac{10\times(20\times10^{-2})^2}{2}\times\frac{600\times\pi}{30\times10}$ = 0.4 π = 4π ×10^{-2}

Q.10 A force of $F = (5y + 20)\hat{j}$ N acts on a particle. The workdone by this force when the particle is moved from $y = 0$ m to $y = 10$ m is ________ J.

Given **450**
Answer :

Ans: 450.00

Sol: $F = (5y+20)\hat{j}$

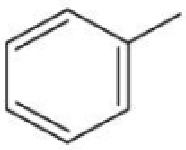
$$w = \int F\,dy = \int_0^{10}(5y+20)dy = \left(\frac{5y^2}{2}+20y\right)_0^{10} = \frac{5}{2}\times100 + 20\times10 = 450\text{ J}$$

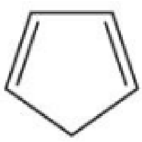
PART – B – CHEMISTRY

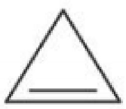
Section A

Q.1 Which among the following is the strongest acid ?

Options

1. 

2. 

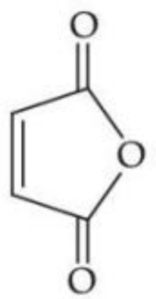
3. 

4. $CH_3CH_2CH_2CH_3$

Ans: 

Sol: Cyclopentadiene is strongest acid, because the conjugate based formed from cyclopentadienyl anion that is aromatic

Q.2



Maleic anhydride

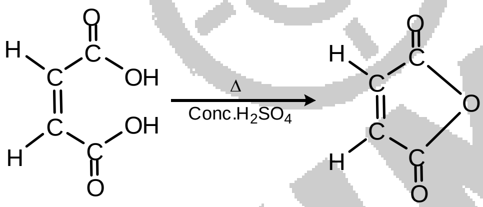
Maleic anhydride can be prepared by :

Options

1. Heating trans-but-2-enedioic acid
2. Treating trans-but-2-enedioic acid with alcohol and acid
3. Treating cis-but-2-enedioic acid with alcohol and acid
4. Heating cis-but-2-enedioic acid

Ans: Heating cis-but-2-enedioic acid

Sol: cis-but-2-enedioic acid



Maleic acid

Q.3 Which one of the following metal complexes is most stable ?

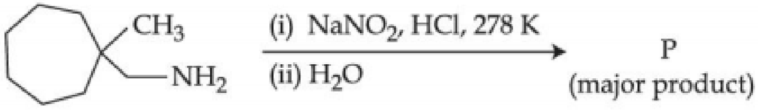
Options

1. $[Co(en)(NH_3)_4]Cl_2$
2. $[Co(NH_3)_6]Cl_2$
3. $[Co(en)_3]Cl_2$
4. $[Co(en)_2(NH_3)_2]Cl_2$

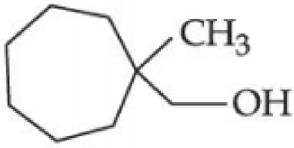
Ans: $[Co(en)_3]Cl_2$

Sol: Chelation increase the stability of complex. The number of chelating ligand increases the stability also increases.
the **complex** $[Co(en)_3]Cl_2$ is more stable one **due** to the presence of three chelating rings around the central metal ion in

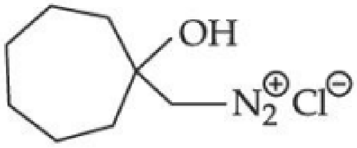
Q.4 What is the major product "P" of the following reaction ?

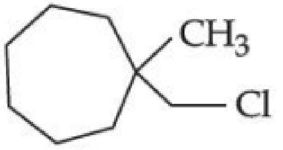


Options

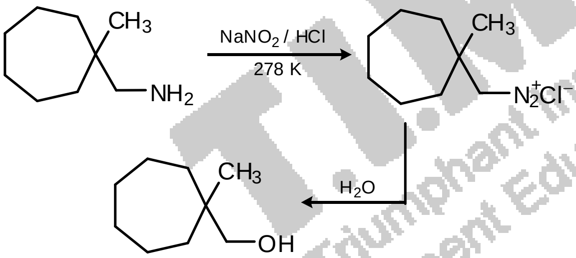
1. 

2. 

3. 

4. 

Ans: 

Sol: 

Q.5 Identify the process in which change in the oxidation state is five :

Options

1. $CrO_4^{2-} \rightarrow Cr^{3+}$
2. $Cr_2O_7^{2-} \rightarrow 2Cr^{3+}$
3. $C_2O_4^{2-} \rightarrow 2CO_2$
4. $MnO_4^{-} \rightarrow Mn^{2+}$

Ans: $MnO_4^{-} \longrightarrow Mn^{2+}$

Sol: $\overset{+7}{MnO_4^{-}} \longrightarrow Mn^{2+}$

Change in oxidation number – 5

Q.6 Match **List I** with **List II** :

List - I Example of Colloids		List - II Classification
(a) Cheese	(i)	dispersion of liquid in liquid
(b) Pumice stone	(ii)	dispersion of liquid in gas
(c) Hair cream	(iii)	dispersion of gas in solid
(d) Cloud	(iv)	dispersion of liquid in solid

Choose the most appropriate answer from the options given below :

Options
1. (a) - (iv), (b) - (i), (c) - (iii), (d) - (ii)
2. (a) - (iii), (b) - (iv), (c) - (i), (d) - (ii)
3. (a) - (iv), (b) - (iii), (c) - (ii), (d) - (i)
4. (a) - (iv), (b) - (iii), (c) - (i), (d) - (ii)

Ans: (a)-(iv), (b)-(iii), (c)-(i), (d)-(ii)

Sol: (a) Cheese – Dispersion of liquid in solid
(b) Pumic stone – Dispersion of gas in solid
(c) Hair cream – Dispersion of liquid in liquid
(d) Cloud – Dispersion of liquid in gas
(a)-(iv), (b)-(iii), (c)-(i), (d)-(ii)

Q.7 The spin only magnetic moments (in BM) for free Ti^{3+}, V^{2+} and Sc^{3+} ions respectively are (At. No. Sc : 21 ; Ti : 22 ; V : 23)

Options
1. 3.87, 1.73, 0
2. 1.73, 3.87, 0
3. 0, 3.87, 1.73
4. 1.73, 0, 3.87

Ans: 1.73, 3.87, 0

Sol: Magnetic moment $r = \sqrt{n(n+2)}$ BM

$Ti^{3+} - [Xe]d^1 = \sqrt{1(1+2)} = 1.73$

$V^{2+} - [Xe]d^3 = \sqrt{3(3+2)} = 3.87$

$Sc^{3+} - [Xe]d^0 = 0$

Q.8 Match **List I** with **List II** :

List - I Elements		List - II Properties
(a) Li	(i)	Poor water solubility of I^- salt
(b) Na	(ii)	Most abundant element in cell fluid
(c) K	(iii)	Bicarbonate salt used in fire extinguisher
(d) Cs	(iv)	Carbonate salt decomposes easily on heating

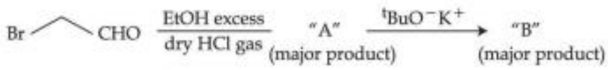
Choose the correct answer from the options given below :

Options
1. (a) - (iv), (b) - (iii), (c) - (ii), (d) - (i)
2. (a) - (i), (b) - (ii), (c) - (iii), (d) - (iv)
3. (a) - (iv), (b) - (ii), (c) - (iii), (d) - (i)
4. (a) - (i), (b) - (iii), (c) - (ii), (d) - (iv)

Ans: (a)-(iv), (b)-(iii), (c)-(ii), (d)-(i)

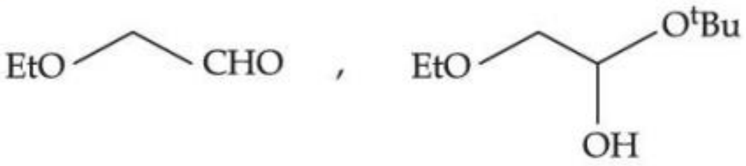
Sol: Li – Carbonate of Lithium easily undergo decomposition on heating
Na – $NaHCO_3$ used in the extinguisher
K – Most abundant element in cell fluid
Cs – Due to hydration enthalpy CsI poorly soluble in water

Q.9

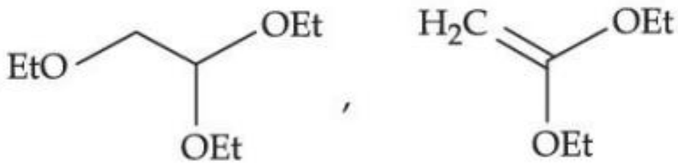


[where Et ⇒ $-C_2H_5$ ${}^tBu$ ⇒ $(CH_3)_3C-$]
Consider the above reaction sequence, Product "A" and Product "B" formed respectively are :

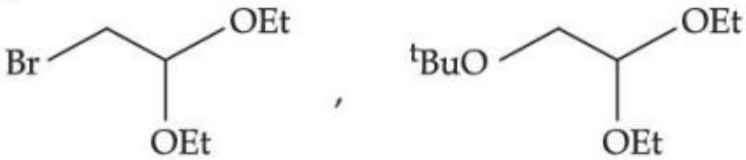
Options 1.



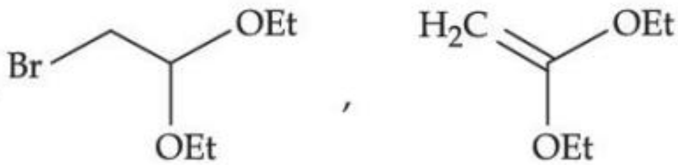
2.



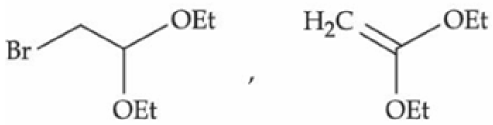
3.



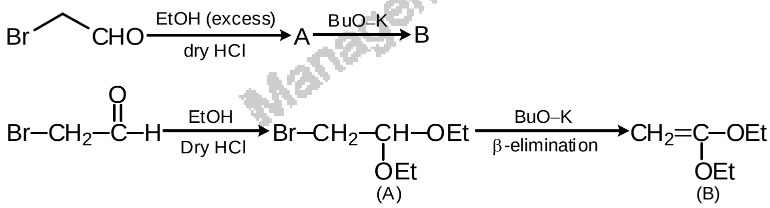
4.



Ans:



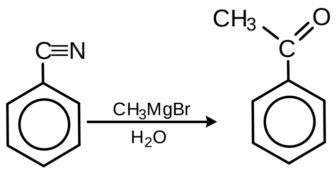
Sol:



Q.10 A reaction of benzonitrile with one equivalent CH_3MgBr followed by hydrolysis produces a yellow liquid "P". The compound "P" will give positive ________.

Options
1. Schiff's test
2. Ninhydrin's test
3. Iodoform test
4. Tollen's test

Ans: Iodoform test

Sol: 

Presence of ' $CH_3-\overset{O}{\overset{\|}{C}}-$ ' an benzene ring gives iodoform test

Q.11 Match **List I** with **List II** : (Both having metallurgical terms)

	List - I		**List - II**
(a)	Concentration of Ag ore	(i)	Reverberatory furnace
(b)	Blast furnace	(ii)	Pig iron
(c)	Blister copper	(iii)	Leaching with dilute NaCN solution
(d)	Froth floatation method	(iv)	Sulfide ores

Choose the correct answer from the options given below :

Options
1. (a) - (iii), (b) - (ii), (c) - (i), (d) - (iv)
2. (a) - (iii), (b) - (iv), (c) - (i), (d) - (ii)
3. (a) - (iv), (b) - (i), (c) - (iii), (d) - (ii)
4. (a) - (iv), (b) - (iii), (c) - (ii), (d) - (i)

Ans: (a)-(iii), (b)-(ii), (c)-(i), (d)-(iv)

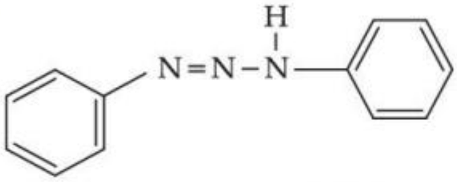
Sol:
(a) Concentration of Ag ore – leaching with dilute NaCl solution
(b) Blast furnace – Pig iron is formed
(c) Blister copper – Produced in Reveberatory furnace
(d) Froth floatation – Used of the concentration of sulphide or

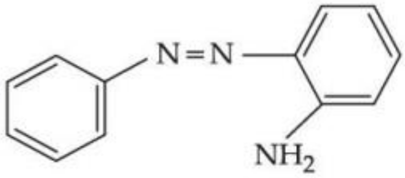
Q.12

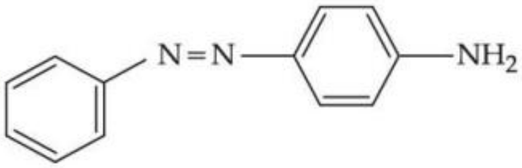
$$C_6H_5NO_2 \xrightarrow{Sn + HCl} \text{"A"} \xrightarrow[H^{\oplus}]{C_6H_5\overset{\oplus}{N_2}\overset{\ominus}{Cl}} \underset{\text{(Yellow coloured Compound)}}{P}$$

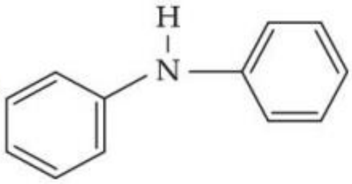
Consider the above reaction, the Product "P" is :

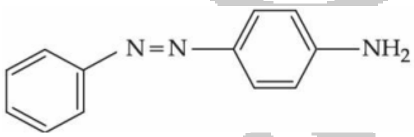
Options

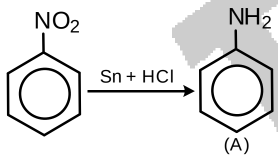
1. 

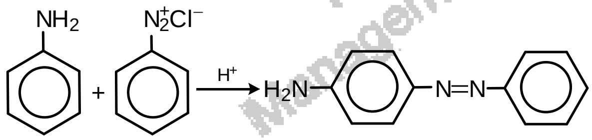
2. 

3. 

4. 

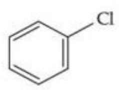
Ans: 

Sol: 

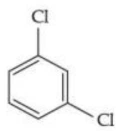


Q.13 The correct decreasing order of densities of the following compounds is :

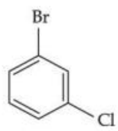
(A)



(B)



(C)



(D)

Options

1. (D) > (C) > (B) > (A)
2. (C) > (B) > (A) > (D)
3. (C) > (D) > (A) > (B)
4. (A) > (B) > (C) > (D)

Ans: (D) > (C) > (B) > (A)

Sol: D > C > B > A

Q.14 A biodegradable polyamide can be made from :

Options
1. Hexamethylene diamine and adipic acid
2. Glycine and aminocaproic acid
3. Glycine and isoprene
4. Styrene and caproic acid

Ans: Glycine and aminocaproic acid

Sol: Nylon 2 nylon 6 is biodegradable polyamide can be made from glycine and aminocaproic acid

Q.15 The ionic radii of F^- and O^{2-} respectively are 1.33 Å and 1.4 Å, while the covalent radius of N is 0.74 Å.
The correct statement for the ionic radius of N^{3-} from the following is :

Options
1. It is bigger than F^- and N, but smaller than of O^{2-}
2. It is bigger than O^{2-} and F^-
3. It is smaller than F^- and N
4. It is smaller than O^{2-} and F^-, but bigger than of N

Ans: It is bigger than O^{2-} and F^-

Sol: F^- , O^{2-} and N^{3-} are isoelectronic species with total number of electron equal to 10
For isoelectronic species higher the negative charge larger the size of the ion
$N^{3-} > O^{2-} > F^-$

Q.16 In the following the correct bond order sequence is :

Options
1. $O_2^+ > O_2^- > O_2^{2-} > O_2$
2. $O_2^+ > O_2 > O_2^- > O_2^{2-}$
3. $O_2 > O_2^- > O_2^{2-} > O_2^+$
4. $O_2^{2-} > O_2^+ > O_2^- > O_2$

Ans: $O_2^+ > O_2 > O_2^- > O_2^{2-}$

Sol: Bond order $O_2^+ = \frac{1}{2}[10-5] = 2.5$

Bond order $O_2 = \frac{1}{2}[10-6] = 2$

Bond order $O_2^{2-} = \frac{1}{2}[10-8] = 1$

Bond order $O_2^- = \frac{1}{2}[10-7] = 1.5$

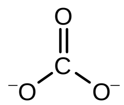
$O_2^+ > O_2 > O_2^- > O_2^{2-}$

Q.17 Identify the species having one π-bond and maximum number of canonical forms from the following :

Options
1. O_2
2. CO_3^{2-}
3. SO_2
4. SO_3

Ans: CO_3^{2-}

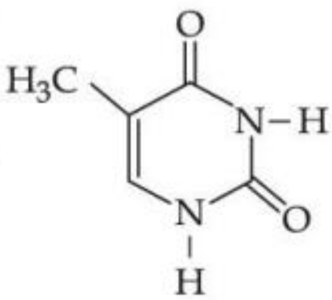
Sol:

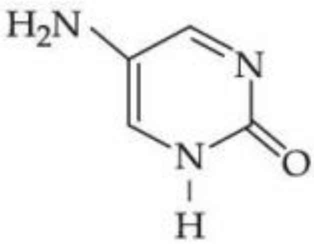


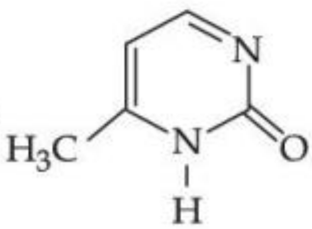
The species with π bond and that show resonance is CO_3^{2-}

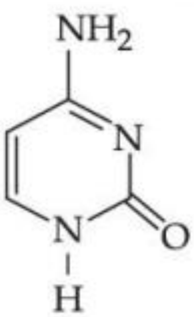
Q.18 Which one of the following is correct structure for cytosine ?

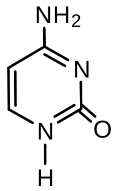
Options

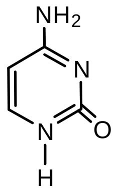
1. 

2. 

3. 

4. 

Ans: 

Sol: 

Q.19 Given below are two statements :

Statement I : Chlorofluoro carbons breakdown by radiation in the visible energy region and release chlorine gas in the atmosphere which then reacts with stratospheric ozone.

Statement II : Atmospheric ozone reacts with nitric oxide to give nitrogen and oxygen gases, which add to the atmosphere.

For the above statements choose the correct answer from the options given below :

Options
1. Both **statement I** and **II** are false
2. **Statement I** is correct but **statement II** is false
3. Both **statement I** and **II** are correct
4. **Statement I** is incorrect but **statement II** is true

Ans: Both statement I and II are false

Sol: Statement I is wrong
Chlorofluoro carbons break down by ultraviolet radiation and releases chlorine radicle which react with ozone and starts chain reaction
Statement II is wrong
Atmospheric ozone react with nitric oxide to produce NO_2 and oxygen

Q.20 Which one of the following metals forms interstitial hydride easily ?

Options
1. Co
2. Cr
3. Mn
4. Fe

Ans: Cr

Sol: Group 7, 8, 9 not form hydride among 'd' block elements referred as hydride gap
Among group 6 only chromium from hydride with formula CrH

Section B

Q.1 When 3.00 g of a substance 'X' is dissolved in 100 g of CCl_4, it raises the boiling point by 0.60 K. The molar mass of the substance 'X' is ________ g mol^{-1}. (Nearest integer)
[Given K_b for CCl_4 is 5.0 K kg mol^{-1}]

Given Answer : **250**

Ans: 250

Sol: $\Delta T_b = K_b \times m$

$$0.60 = 5 \times \frac{3 \times 1000}{M_{solute} \times 100}$$

$$M_{solute} = \frac{150}{0.6} = 250$$

Q.2 An LPG cylinder contains gas at a pressure of 300 kPa at 27°C. The cylinder can withstand the pressure of 1.2×10^6 Pa. The room in which the cylinder is kept catches fire. The minimum temperature at which the bursting of cylinder will take place is __________ °C. (Nearest integer)

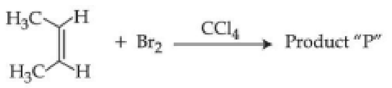
Given **108**
Answer :

Ans: 927

Sol: $\frac{P_1}{T_2} = \frac{P_2}{T_2}$

$$\frac{300 \times 10^3}{300} = \frac{1.2 \times 10^6}{T_2}$$

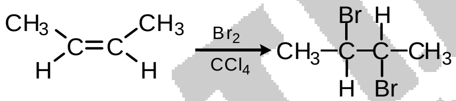
T_2 = 1200 K
= 927°C

Q.3 

Consider the above chemical reaction. The total number of stereoisomers possible for Product 'P' is __________.

Given --
Answer :

Ans: 2

Sol: 

Total number of product possible is 2

Q.4 0.8 g of an organic compound was analysed by Kjeldahl's method for the estimation of nitrogen. If the percentage of nitrogen in the compound was found to be 42%, then __________ mL of 1 M H_2SO_4 would have been neutralized by the ammonia evolved during the analysis.

Given --
Answer :

Ans: 12

Sol: $$\% \text{ N} = \frac{1.4 \times \text{Vol. acid} \times \text{Neutralized by } NH_3 \times \text{N acid}}{\text{Wt. of organic compound}}$$

$$42 = \frac{1.4 \times V_{H_2SO_4} \times 1N}{0.2}$$

$$V_{H_2SO_4} = \frac{42 \times 8 \times 10^{-1}}{14 \times 10^{-1}} = 12 \text{ mL}$$

Q.5 Number of electrons present in 4f orbital of Ho^{3+} ion is ________. (Given Atomic No. of Ho = 67)

Given -- Answer :

Ans: 10

Sol: Ho = [Xe] $4f^{11}\ 5d^0\ 6s^2$
Ho^{3+} = [Xe] $4f^{10}$
Number of electrons present in 4f orbital = 10

Q.6 For a chemical reaction A → B, it was found that concentration of B is increased by 0.2 mol L^{-1} in 30 min. The average rate of the reaction is ________ $\times 10^{-1}$ mol L^{-1} h^{-1}. (in nearest integer)

Given **4** Answer :

Ans: 4

Sol: A → B
1 0
After 30 minutes → 2 mol L^{-1}

$$\text{Average rate} = \frac{4[B]}{4t} = \frac{0.2}{30/60} = 0.4 \text{ mol L}^{-1}\text{ hr}^{-1} = 4 \times 10^{-1} \text{ mol L}^{-1}\text{ hr}^{-1}$$

Q.7 The number of significant figures in 0.00340 is ________.

Given **3** Answer :

Ans: 3

Sol: Number of significant figure 0.00340 is 3

Q.8 A system does 200 J of work and at the same time absorbs 150 J of heat. The magnitude of the change in internal energy is ________ J. (Nearest integer)

Given **50** Answer :

Ans: 50

Sol: $\Delta U = q + w$
$\Delta U = 150 - 200 = -50$ J
Magnitude 50

Q.9 An accelerated electron has a speed of 5×10^6 ms^{-1} with an uncertainty of 0.02%. The uncertainty in finding its location while in motion is $x \times 10^{-9}$ m. The value of x is ________. (Nearest integer)
[Use mass of electron $= 9.1 \times 10^{-31}$ kg, $h = 6.63 \times 10^{-34}$ Js, $\pi = 3.14$]

Given -- Answer :

Ans: 58

Sol:
$$\Delta x.\ m\Delta v = \frac{h}{4\pi}$$

$$\Delta x = \frac{6.63 \times 10^{-34}}{4 \times 3.14 \times 9.1 \times 10^{-31} \times 5 \times 10^{+6} \times \frac{0.02}{100}}$$

$\Rightarrow 0.5800 \times 10^{-7}$
$\Rightarrow 58 \times 10^{-9}$

Q.10 Assuming that $Ba(OH)_2$ is completely ionised in aqueous solution under the given conditions the concentration of H_3O^+ ions in 0.005 M aqueous solution of $Ba(OH)_2$ at 298 K is ________ $\times 10^{-12}$ mol L^{-1}. (Nearest integer)

Given --
Answer :

Ans: **1**

Sol: $Ba(OH)_2 \longrightarrow Ba^{2+} + \underset{2\times(0.005)=1\times10^{-2}}{\underset{\downarrow}{2OH^-}}$

At 298 K, $[H_3O^+] \times [OH^-] = 10^{-14}$

$$[H_3O^+] = \frac{10^{-14}}{10^{-2}} = 10^{-12} = 1\times10^{-12}$$

PART – C – MATHEMATICS

Section A

Q.1 If ${}^nP_r = {}^nP_{r+1}$ and ${}^nC_r = {}^nC_{r-1}$, then the value of r is equal to :

Options
1. 1
2. 4
3. 2
4. 3

Ans: 2

Sol:

$nP_r = nP_{r+1}$

$\Rightarrow \frac{n!}{(n-r)!} = \frac{n!}{[n-(r+1)]!}$

$\Rightarrow (n-r)! = (n-r-1)!$

$\Rightarrow (n-r) = 1 ....(1)$

$nC_r = nC_{r-1}$

$\Rightarrow \frac{n!}{r!(n-r)!} = \frac{n!}{(r-1)![n-r+1]!}$

$\Rightarrow r(r-1)!(n-r)! = (r-1)!(n-r+1)(n-r)!$

$\Rightarrow r(r-1)!(n-r)! = (r-1)!(n-r+1)!(n-r)!$

$\Rightarrow r = (n-r)+1 \Rightarrow r = 1+1 = 2$

Q.2 The number of distinct real roots of $\begin{vmatrix} \sin x & \cos x & \cos x \\ \cos x & \sin x & \cos x \\ \cos x & \cos x & \sin x \end{vmatrix} = 0$ in the interval $-\frac{\pi}{4} \leq x \leq \frac{\pi}{4}$ is :

Options
1. 1
2. 3
3. 4
4. 2

Ans: 1

Sol: $R_1 \to R_1 + R_2 + R_3$

$$\begin{vmatrix} \sin x + 2\cos x & \sin x + 2\cos x & \sin x + 2\cos x \\ \cos x & \sin x & \cos x \\ \cos x & \cos x & \sin x \end{vmatrix}$$

$$= (\sin x + 2\cos x)\begin{vmatrix} 1 & 1 & 1 \\ \cos x & \sin x & \cos x \\ \cos x & \cos x & \sin x \end{vmatrix}$$

$c_1 \to c_2 - c_1, c_3 \to c_3 - c_1$

$$\Rightarrow (\sin x + 2\cos x)\begin{vmatrix} 1 & 0 & 0 \\ \cos x & \sin x - \cos x & 0 \\ \cos x & 0 & \sin x - \cos x \end{vmatrix}$$

$\Rightarrow (\sin x + 2\cos x)(\sin x - \cos x)^2 = 0$

$\Rightarrow \tan x = -2$ or $\tan x = 1$

In $\left[-\frac{\pi}{4}, \frac{\pi}{4}\right]$

$\tan x = 1 \Rightarrow x = \frac{\pi}{4}$

$\tan x = -2 \Rightarrow x = \tan^{-1}(-2)$

$= \tan^{-1} 2 \notin \left[-\frac{\pi}{4}, \frac{\pi}{4}\right]$

Only one solution

Q.3

The value of the integral $\int_{-1}^{1} \log\left(x + \sqrt{x^2 + 1}\right)dx$ is :

Options
1. -1
2. 2
3. 1
4. 0

Ans: 0

Sol: Let f(x)= $f(x) = \log\left(x + \sqrt{x^2 + 1}\right)$

$\Rightarrow f(-x) = \log\left(-x + \sqrt{x^2 + 1}\right))$

$f(x) + f(-x) = \log\left(x + \sqrt{x^2 + 1}\right) + \log\left(-x + \sqrt{x^2 + 1}\right)$

$= \log\left[\left(\sqrt{x^2 + 1}\right)^2 - x^2\right] = \log(x^2 + 1 - x^2) = \log 1 = 0$

$\Rightarrow f(-x) = -f(x)$

Hence f(x)is an odd function

$\therefore \int_{-1}^{1} \log\left(x + \sqrt{x^2 + 1}\right) = 0$

Q.4 Let the equation of the pair of lines, $y = px$ and $y = qx$, can be written as $(y - px)(y - qx) = 0$. Then the equation of the pair of the angle bisectors of the lines $x^2 - 4xy - 5y^2 = 0$ is :

Options
1. $x^2 - 3xy - y^2 = 0$
2. $x^2 + 4xy - y^2 = 0$
3. $x^2 - 3xy + y^2 = 0$
4. $x^2 + 3xy - y^2 = 0$

Ans: $x^2 + 3xy - y^2 = 0$

Sol: The Combined equation of angular bisection of the lines $ax^2 + 2bxy + by^2 = 0$ is

$$\frac{x^2 - y^2}{a - b} = \frac{xy}{h}$$

a=1 , h=-2 , b=-5

$$\Rightarrow \frac{x^2 - y^2}{1 - (-5)} = \frac{xy}{-2} \Rightarrow \frac{x^2 - y^2}{6} = \frac{xy}{-2}$$

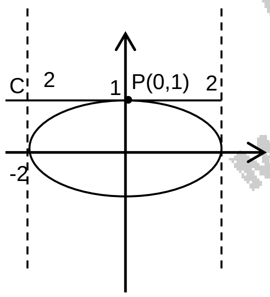
$$\frac{x^2 - y^2}{3} = -xy \Rightarrow x^2 - y^2 = -3xy \Rightarrow x^2 + 3xy - y^2 = 0$$

Q.5 If a tangent to the ellipse $x^2 + 4y^2 = 4$ meets the tangents at the extremities of its major axis at B and C, then the circle with BC as diameter passes through the point :

Options
1. $(1, 1)$
2. $(\sqrt{2}, 0)$
3. $(\sqrt{3}, 0)$
4. $(-1, 1)$

Ans: $(\sqrt{3}, 0)$

Sol: Consider a special case.



$$\frac{x^2}{4} - \frac{y^2}{1} = 1$$

The equation of the circle with BC as diameter is $(x - 0)^2 + (y - 1)^2 = 2^2$

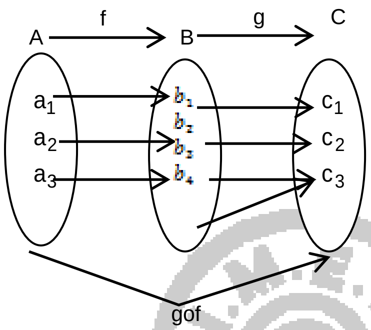
$\Rightarrow x^2 + (y - 1)^2 = 4$ which passes through $(\sqrt{3}, 0)$ and never passes through the other given points

Q.6 Consider functions $f : A \to B$ and $g : B \to C$ ($A, B, C \subseteq \mathbf{R}$) such that $(gof)^{-1}$ exists, then :

Options
1. f is onto and g is one-one
2. f and g both are onto
3. f is one-one and g is onto
4. f and g both are one-one

Ans: f is one-one and g is onto

Sol: $(gof)^{-1}$ exists iff gof is one one and onto



Clearly f is one-one and g is onto

Q.7 Let a, b and c be distinct positive numbers. If the vectors $a\hat{i} + a\hat{j} + c\hat{k}$, $\hat{i} + \hat{k}$ and $c\hat{i} + c\hat{j} + b\hat{k}$ are co-planar, then c is equal to :

Options
1. $\dfrac{2}{\dfrac{1}{a} + \dfrac{1}{b}}$
2. $\dfrac{1}{a} + \dfrac{1}{b}$
3. $\dfrac{a + b}{2}$
4. $\sqrt{ab}$

Ans: $\sqrt{ab}$

Sol: $[\vec{a}\ \vec{b}\ \vec{c}] = 0$

$$\Rightarrow \begin{vmatrix} a & a & c \\ 1 & 0 & 1 \\ c & c & b \end{vmatrix} = 0$$

$$\Rightarrow a(0 - c) - a(b - c) + c(c - 0) = 0$$

$$\Rightarrow -ac - ab + ac + c^2 = 0$$

$$\Rightarrow c^2 = ab \Rightarrow c = \sqrt{ab}$$

Q.8 The number of real solutions of the equation, $x^2 - |x| - 12 = 0$ is :

Options
1. 2
2. 1
3. 3
4. 4

Ans: 2

Sol: $|x|^2 - |x| - 12 = 0 \quad (\because x^2 = |x|^2)$

$\Rightarrow (|x| - 4)(|x| + 3) = 0$

$\Rightarrow |x| = 4 \quad (\because |x| \neq -3)$

$\Rightarrow x = \pm 2$

Q.9 Consider the statement "The match will be played only if the weather is good and ground is not wet". Select the correct negation from the following :

Options
1. The match will not be played or weather is good and ground is not wet.
2. The match will not be played and weather is not good and ground is wet.
3. If the match will not be played, then either weather is not good or ground is wet.
4. The match will be played and weather is not good or ground is wet.

Ans: The match will be played and weather is not good or ground is wet

Sol: The given statement is equivalent to p

$p \Rightarrow (q \wedge r)$

$\sim[p \Rightarrow (q \wedge r)] = \sim[\sim p \vee (q \wedge r)] \quad \because x \Rightarrow y = \sim x \vee y$

$= \sim(\sim p) \wedge \sim(q \wedge r) = p \wedge (\sim q \vee \sim r)]$

Q.10 If $|\vec{a}| = 2$, $|\vec{b}| = 5$ and $|\vec{a} \times \vec{b}| = 8$, then $|\vec{a} \cdot \vec{b}|$ is equal to :

Options
1. 4
2. 6
3. 5
4. 3

Ans: 6

Sol: $(\vec{a} \times \vec{b})^2 + (\vec{a}.\vec{b})^2 = |\vec{a}|^2 |\vec{b}|^2$

$\Rightarrow 64 + (\vec{a}.\vec{b})^2 = (2 \times 5)^2 = 100$

$\Rightarrow (\vec{a}.\vec{b})^2 = 36 \Rightarrow |\vec{a}.\vec{b}| = 6$

Q.11

If $[x]$ be the greatest integer less than or equal to x, then $\sum_{n=8}^{100}\left[\frac{(-1)^n n}{2}\right]$ is equal to :

Options 1. -2

2. 4

3. 0

4. 2

Ans: 4

Sol: $\sum_{n=8}^{100}\left[\frac{(-1)^n n}{2}\right]$

$=\left[\frac{8}{2}\right]+\left[\frac{-9}{2}\right]+\left[\frac{10}{2}\right]+\left[\frac{-11}{2}\right]+\left[\frac{12}{2}\right]-\left[\frac{13}{2}\right]+........+\left[\frac{100}{2}\right]$

$=\left\{\left[\frac{8}{2}\right]+\left[\frac{10}{2}\right]+\left[\frac{12}{2}\right]+.....+\left[\frac{100}{2}\right]\right\}+\{[-4.5]+[-5.5]+[-6.5]+....+[-49.5]\}$

=(4+5+6+…….+50)+(-5-6-7-…….-50)

$=\left[\frac{50(50+1)}{2}-(1+2+3)\right]-\left[\frac{50(50+1)}{2}-(1+2+3+4)\right]$

=-6+10=4

Q.12

If the greatest value of the term independent of 'x' in the expansion of $\left(x\sin\alpha+a\frac{\cos\alpha}{x}\right)^{10}$ is $\frac{10!}{(5!)^2}$, then the value of 'a' is equal to :

Options 1. -2

2. 1

3. 2

4. -1

Ans: 2

Sol: $\left(x\sin\alpha+\frac{a\cos\alpha}{x}\right)^{10}$

$T_{r+1}=10C_r(x\sin\alpha)^{10-r}\left(\frac{a\cos\alpha}{x}\right)^r=10C_r(\sin\alpha)^{10-r}(a\cos\alpha)^r.x^{10-2r}$

$r=5\Rightarrow T_{r+1}=10C_5(\sin\alpha)^5.a^5.(\cos\alpha)^5$

$=\frac{10!}{5!(10-5)!}.\frac{1}{2^5}(2\sin\alpha\cos\alpha)^5.a^5$

$\max=\frac{10!}{(5!)^2}.\frac{1}{32}\times a^5==\frac{10!}{(5!)^2}\Rightarrow a=2$

Q.13 The first of the two samples in a group has 100 items with mean 15 and standard deviation 3. If the whole group has 250 items with mean 15.6 and standard deviation $\sqrt{13.44}$, then the standard deviation of the second sample is :

Options
1. 6
2. 8
3. 4
4. 5

Ans: 4

Sol: $n_1=100, n_2=150,\ \bar{x}_1=15, \sigma_1=3$

$\bar{x}_{(combined)}=15.6,\ \sigma_{(combined)}=\sqrt{13.44}$

$\bar{x}_c=\dfrac{n_1\bar{x}_1+n_2\bar{x}_2}{n_1+n_2} \Rightarrow (100)(15)+(150)(\bar{x}_2)=250\times 15.6$

$\Rightarrow 150.\bar{x}_2 \Rightarrow 2400 \Rightarrow \bar{x}_2=16$

$d_1=|\bar{x}_1-\bar{x}_c|=|15-15.6|=0.6$

$d_2=|\bar{x}_2-\bar{x}_c|=|16-15.6|=0.4$

$$\sigma_c=\sqrt{\frac{n_1(\sigma_1{}^2+d_1{}^2)+n_2(\sigma_2{}^2+d_2{}^2)}{n_1+n_2}}$$

$$\Rightarrow 13.44=\frac{[100(9+0.36)+150(\sigma_2{}^2+0.16)}{250}$$

$\Rightarrow 936+150\sigma_2{}^2+24=3360$

$150\sigma_2{}^2=2400$

$\Rightarrow \sigma_2{}^2=16 \Rightarrow \sigma_2=4$

Q.14

The lowest integer which is greater than $\left(1+\dfrac{1}{10^{100}}\right)^{10^{100}}$ is ________.

Options
1. 1
2. 3
3. 2
4. 4

Ans: 3

Sol: we know that $\lim\limits_{x\to 0}(1+x)^{1/x}=e$

Let f(x)= $(1+x)^{1/x}$

$f(1)=2, f(\frac{1}{2})=\left(1+\frac{1}{2}\right)^2=2.25$ ...

As $x\to 0,\ f(x)=2.718....$

$$\therefore \left[1+\frac{1}{(10^{100})}\right]^{(10^{100})}\approx 2.718$$

$\therefore \text{Ans}=3$

Q.15

If $f(x) = \begin{cases} \int_0^x (5 + |1 - t|)dt, & x > 2 \\ 5x + 1, & x \leq 2 \end{cases}$, then

Options

1. $f(x)$ is not differentiable at $x=1$
2. $f(x)$ is continuous but not differentiable at $x=2$
3. $f(x)$ is everywhere differentiable
4. $f(x)$ is not continuous at $x=2$

Ans: f(x) is continuous but not differentiable at x=2

Sol: $\text{if } x > 2, f(x) = \int_0^x 5dx + \int_0^x |1-t|\,dt$

$$= 5x + \int_0^x |t-1|\,dt = 5x + \int_0^1 |t-1|\,dt + \int_1^x |t-1|\,dt$$

$$= 5x - \int_0^1 |t-1|\,dt + \int_1^x |t-1|\,dt$$

$$= 5x - \left[\frac{t^2}{2} - t\right]_0^1 + \left[\frac{t^2}{2} - t\right]_1^x = 5x - \left[\left(\frac{1}{2} - 1\right) - 0\right] + \left[\left(\frac{x^2}{2} - x\right) - \left(\frac{1}{2} - 1\right)\right]$$

$$= 5x + \frac{1}{2} + \frac{x^2}{2} - x + \frac{1}{2} = \frac{x^2}{2} + 4x + 1$$

$$\therefore f(x) = \begin{cases} \frac{x^2}{2} + 4x + 1 & \text{if } x > 2 \\ 5x + 1 & \text{if } x \leq 2 \end{cases}$$

$$\lim_{x \to 2^+} f(x) = \lim_{x \to 2^+} \left(\frac{x^2}{2} + 4x + 1\right) = 2 + 8 + 1 = 11$$

$$\lim_{x \to 2^-} f(x) = \lim_{x \to 2^-} (5x + 1) = 11$$

f(2)=11

$\therefore$ f(x) is continuous at x=2

$$\text{If } x > 2, \frac{dy}{dx} = x + 4 \Rightarrow \left(\frac{dy}{dx}\right)_{x=2^+} = 6$$

$$If\, x < 2, \frac{dy}{dx} = 5 \Rightarrow \left(\frac{dy}{dx}\right)_{x=2^-} = 5$$

$\therefore$ L.D $\neq$ R.D

$\Rightarrow$ Not differentiable at x=2

Q.16 Let X be a random variable such that the probability function of a distribution is given by

$P(X=0)=\frac{1}{2},\ P(X=j)=\frac{1}{3^j}\ (j=1,2,3,\ldots,\infty)$ Then the mean of the distribution and P(X is positive and even) respectively are :

Options

1. $\frac{3}{4}$ and $\frac{1}{9}$
2. $\frac{3}{4}$ and $\frac{1}{8}$
3. $\frac{3}{4}$ and $\frac{1}{16}$
4. $\frac{3}{8}$ and $\frac{1}{8}$

Ans: $\frac{3}{4}$ and $\frac{1}{8}$

Sol: $P(x=0)=\frac{1}{2}, P(x=j)=\frac{1}{3^j}\ (j=1,,2,3,\ldots\ldots\infty)$

$$E(x)=\sum p_i x_i=(0)\left(\frac{1}{2}\right)+(1)\left(\frac{1}{3}\right)+2\left(\frac{1}{3^2}\right)+3\left(\frac{1}{3^3}\right)+4\left(\frac{1}{3^4}\right)+\ldots..$$

$$S=\frac{1}{3}+\frac{2}{3^2}+\frac{3}{3^3}+\ldots\ldots..\text{to}\infty$$

$$\frac{1}{3}S=0+\frac{1}{3^2}+\frac{2}{3^3}+\ldots\ldots..\text{to}\infty$$

$$\frac{2}{3}S=\frac{1}{3}+\frac{1}{3^2}+\frac{1}{3^3}+\ldots\ldots..\text{to}\infty=\frac{1/3}{1-\frac{1}{3}}=\frac{1/3}{2/3}=\frac{1}{2}$$

$$\Rightarrow E(x)=\frac{3}{2}\times\frac{1}{2}=\frac{3}{4}$$

P(x is positive and even)=P(x=2,4,6,8,….)

$$=\frac{1}{3^2}+\frac{1}{3^4}+\frac{1}{3^6}\ldots\ldots..to\infty=\frac{1/3^2}{1-\frac{1}{3^2}}=\frac{1/9}{8/9}=\frac{1}{8}$$

Q.17

The value of $\cot\frac{\pi}{24}$ is :

Options

1. $3\sqrt{2}-\sqrt{3}-\sqrt{6}$
2. $\sqrt{2}-\sqrt{3}-2+\sqrt{6}$
3. $\sqrt{2}+\sqrt{3}+2-\sqrt{6}$
4. $\sqrt{2}+\sqrt{3}+2+\sqrt{6}$

Ans: $\sqrt{2}+\sqrt{3}+\sqrt{2}+\sqrt{6}$

Sol: $\cot\theta=\frac{2\cos^2\theta}{2\sin\theta\cos\theta}=\frac{1+\cos 2\theta}{\sin 2\theta}$

$$\therefore \cot\frac{\pi}{24} = \frac{1+\cos\frac{\pi}{12}}{\sin\frac{\pi}{12}} = \frac{1+\cos 15^o}{\sin 15^o} = \frac{1+\frac{\sqrt{3}+1}{2\sqrt{2}}}{\left(\frac{\sqrt{3}-1}{2\sqrt{2}}\right)}$$

$$= \frac{2\sqrt{2}+\sqrt{3}+1}{\left(\sqrt{3}-1\right)} = \frac{\left(2\sqrt{2}+\sqrt{3}+1\right)\left(\sqrt{3}+1\right)}{2}$$

$$= \frac{2\sqrt{6}+2\sqrt{2}+3+\sqrt{3}+\sqrt{3}+1}{2} = \frac{2\sqrt{6}+2\sqrt{2}+2\sqrt{3}+4}{2} = \sqrt{6}+\sqrt{2}+\sqrt{3}+2$$

Q.18 Let $y = y(x)$ be the solution of the differential equation $x dy = (y + x^3 \cos x) dx$ with $y(\pi) = 0$, then $y\left(\frac{\pi}{2}\right)$ is equal to :

Options

1. $\frac{\pi^2}{4} + \frac{\pi}{2}$
2. $\frac{\pi^2}{4} - \frac{\pi}{2}$
3. $\frac{\pi^2}{2} + \frac{\pi}{4}$
4. $\frac{\pi^2}{2} - \frac{\pi}{4}$

Ans: $\frac{\pi^2}{4} + \frac{\pi}{2}$

Sol: $x dy = \left(y + x^3 \cos x\right) dx$

$\Rightarrow x\frac{dy}{dx} - y = x^3 \cos x$

$\Rightarrow \frac{x\frac{dy}{dx} - y}{x^2} = x\cos x \Rightarrow \frac{d}{dx}\left(\frac{y}{x}\right) = x\cos x$

$\frac{y}{x} = \int x\cos x = x\int \cos x dx - \int 1.\sin x dx = x\sin x + \cos x + c$

$\Rightarrow y = x(x\sin x + \cos x + c)$

$y(\pi) = 0 \Rightarrow 0 = \pi(0 - 1 + c) \Rightarrow c = 1$

$\therefore y = x^2 \sin x + x\cos x + x$

$y\left(\frac{\pi}{2}\right) = \frac{\pi^2}{4} + \frac{\pi}{2}$

Q.19 The sum of all those terms which are rational numbers in the expansion of $\left(2^{1/3} + 3^{1/4}\right)^{12}$ is :

Options

1. 43
2. 35
3. 27
4. 89

Ans: 43

Sol: $\left(2^{1/3}+3^{1/4}\right)^{12}$

$T_{r+1}=12C_r\left(2^{1/3}\right)^{12-r}.\left(3^{1/4}\right)^r=12C_r\,2^{\left(4-\frac{r}{3}\right)}3^{r/4}$

r must be a multiple of 3 and 4

$\Rightarrow r=0, r=12$

$\therefore$ Sum of the rational terms $=12C_0\,2^4 3^0+12C_{12}\,2^0 3^3$

=16+27=43

Q.20

If $P=\begin{bmatrix}1 & 0\\ 1/2 & 1\end{bmatrix}$, then P^{50} is :

Options

1. $\begin{bmatrix}1 & 0\\ 50 & 1\end{bmatrix}$
2. $\begin{bmatrix}1 & 25\\ 0 & 1\end{bmatrix}$
3. $\begin{bmatrix}1 & 50\\ 0 & 1\end{bmatrix}$
4. $\begin{bmatrix}1 & 0\\ 25 & 1\end{bmatrix}$

Ans: $\begin{bmatrix}1 & 0\\ 25 & 1\end{bmatrix}$

Sol: $p=\begin{bmatrix}1 & 0\\ \frac{1}{2} & 1\end{bmatrix}$

$p^2=\begin{bmatrix}1 & 0\\ \frac{1}{2} & 1\end{bmatrix}\begin{bmatrix}1 & 0\\ \frac{1}{2} & 1\end{bmatrix}=\begin{bmatrix}1 & 0\\ 1 & 1\end{bmatrix}$

$p^3=p^2.p=\begin{bmatrix}1 & 0\\ 1.5 & 1\end{bmatrix}$

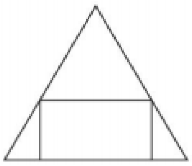
$p^4=p^2.p^2=\begin{bmatrix}1 & 0\\ 2 & 1\end{bmatrix}$

$a_{21}\, in p^{50} is 0.5+(50-1)(0.5)=25$

$\therefore p^{50}=\begin{bmatrix}1 & 0\\ 25 & 1\end{bmatrix}$

Section B

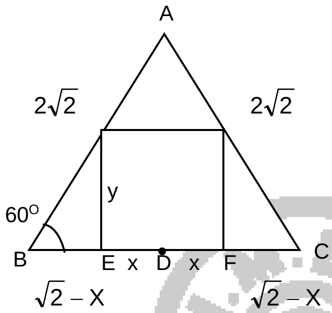
Q.1 If a rectangle is inscribed in an equilateral triangle of side length $2\sqrt{2}$ as shown in the figure, then the square of the largest area of such a rectangle is _________.



Given --
Answer :

Ans: 3.00

Sol:



$$\tan 60 = \frac{y}{\sqrt{2}-x} = \sqrt{3} \Rightarrow y = \sqrt{6} - \sqrt{3}x$$

$$\text{Area,A=2xy=2x}\left(\sqrt{6}-\sqrt{3}x\right) = 2\sqrt{6}x - 2\sqrt{3}x^2$$

$$\frac{dA}{dx} = 2\sqrt{6} - 4\sqrt{3}x = 0 \Rightarrow x = \frac{2\sqrt{6}}{4\sqrt{3}} = \frac{\sqrt{2}.\sqrt{3}}{2\sqrt{3}} = \frac{1}{\sqrt{2}}$$

$$\frac{d^2A}{dx^2} < 0 \Rightarrow A \text{ is maximum when } x = \frac{1}{\sqrt{2}}$$

$$\therefore A_{max} = 2.\frac{1}{\sqrt{2}}\left[\sqrt{6} - \frac{\sqrt{3}}{\sqrt{2}}\right] = \frac{\sqrt{2}\left(\sqrt{12}-\sqrt{3}\right)}{\sqrt{2}} = 2\sqrt{3} - \sqrt{3} = \sqrt{3}$$

$$\Rightarrow A^2 = 3$$

Q.2 Let $n \in \mathbf{N}$ and $[x]$ denote the greatest integer less than or equal to x. If the sum of $(n+1)$ terms ${}^nC_0, 3\cdot{}^nC_1, 5\cdot{}^nC_2, 7\cdot{}^nC_3, \ldots$ is equal to $2^{100}\cdot 101$, then $2\left[\frac{n-1}{2}\right]$ is equal to _________.

Given --
Answer :

Ans: 98.00

Sol: $1.{}^nC_0 + 3{}^nC_1 + 5{}^nC_2 + 7{}^nC_3 + \ldots\ldots.. + (n+1)^{th}\text{term} = 2^{100}.101$

We know that $a_0.{}^nC_0 + a_1{}^nC_1 + a_2{}^nC_2 + \ldots\ldots.a_n{}^nC_n = (a_0 + a_n)2^{n-1}$

If $a_0., a_1, a_2, a_3, \ldots\ldots.a_n$ are in A.P

$$(n+1)^{th}\text{term} = [1 + (n+1-1)2]^nC_n = (2n+1)^nC_n$$

$$\therefore \text{sum} = [1 + (2n+1)].2^{n-1} = (n+1)2^n = 101.2^{100}$$

$$\Rightarrow n = 100$$

$$2\left[\frac{n-1}{2}\right] = 2\left[\frac{99}{2}\right] = 2 \times [49.5] = 2 \times 49 = 98$$

Q.3 Let a curve $y = f(x)$ pass through the point $(2, (\log_e 2)^2)$ and have slope $\frac{2y}{x \log_e x}$ for all positive real value of x. Then the value of $f(e)$ is equal to ________.

Given --
Answer :

Ans: 1.00

Sol: $\frac{dy}{dx} = \frac{2y}{x \log x}$

$\Rightarrow \int \frac{dy}{y} = 2\int \frac{1}{x \log x}$

$\log y = 2.\log(\log x) + \log C$

$\Rightarrow \log y = \log\left|(\log x)^2.C\right|$

$\Rightarrow y = C.(\log x)^2$ passes through $\left(2, (\log 2)^2\right) \Rightarrow (\log 2)^2 = C.(\log 2)^2 \Rightarrow C = 1$

$\therefore y = (\log x)^2$

$f(e) = (\log e)^2 = 1$

Q.4 If the lines $\frac{x-k}{1} = \frac{y-2}{2} = \frac{z-3}{3}$ and $\frac{x+1}{3} = \frac{y+2}{2} = \frac{z+3}{1}$ are co-planar, then the value of k is ________.

Given --
Answer :

Ans: 1.00

Sol: $\begin{vmatrix} x_2 - x_1 & y_2 - y_1 & z_2 - z_1 \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix} = 0$

$\Rightarrow \begin{vmatrix} k+1 & 4 & 6 \\ 1 & 2 & 3 \\ 3 & 2 & 1 \end{vmatrix} = 0$

$\Rightarrow$ (k+1)(2-6)-4(1-9)+6(2-6)=0

-4k-4+32-24=0

-4k+4=0

k=1

Q.5 The equation of a circle is $\text{Re}(z^2) + 2(\text{Im}(z))^2 + 2\text{Re}(z) = 0$, where $z = x + iy$. A line which passes through the center of the given circle and the vertex of the parabola, $x^2 - 6x - y + 13 = 0$, has y-intercept equal to ________.

Given **3**
Answer :

Ans: 1.00

Sol: Let z=x+iy

$z^2 = \left(x^2 - y^2\right) + i(2xy)$

$\text{Re}\left(z^2\right) + 2(\text{In}(z))^2 + 2\,\text{Re}(z) = 0$

$\Rightarrow x^2 - y^2 + 2y^2 + 2x = 0$

$\Rightarrow x^2 + y^2 + 2x = 0$

$2g = 2, 2f = 0 \Rightarrow$ centre : $(-g, -f) = (-1, 0)$

Parabola : $x^2 - 6x - y + 13 = 0$

$\Rightarrow (x-3)^2 - 9 - y + 13 = 0$

$(x-3)^2 = y - 4$

$\Rightarrow (x-3)^2 = 4\left(\frac{1}{4}\right)(y-4)$

Vertex:(3,4)

Line: $y - y_1 = m(x - x_1)$

$\Rightarrow y - 0 = \left[\frac{4-0}{3-(-1)}\right](x+1)$

$\Rightarrow y = x + 1$

$\Rightarrow -x + y = 1$

$\Rightarrow \frac{-x}{1} + \frac{y}{1} = 1$

y intercept=1

Q.6 A fair coin is tossed n-times such that the probability of getting at least one head is at least 0.9. Then the minimum value of n is __________.

Given **2**
Answer :

Ans: 4.00

Sol: The required probability

$= 1 - \left(\frac{1}{2}\right)^n \geq 0.9$

$\Rightarrow \frac{1}{2^n} \leq 0.1$

$\Rightarrow 2^n \geq 10$

$\Rightarrow n = 4,5,6......$

Least value of n is 4

Q.7 If the co-efficients of x^7 and x^8 in the expansion of $\left(2 + \frac{x}{3}\right)^n$ are equal, then the value of n is equal to __________.

Given **14**
Answer :

Ans: 55.00

Sol: $\left(2 + \frac{x}{3}\right)^n$

$T_{r+1} = {}^nC_r 2^{n-r}.\left(\frac{x}{3}\right)^r = {}^nC_r 2^{n-r}.\frac{1}{3^r}.x^r$

Coefficient of $x^7 = {}^nC_7.2^{n-7}\frac{1}{3^7}$

Coefficient of $x^8 = {}^nC_8.2^{n-8}\frac{1}{3^8}$

$\Rightarrow {}^nC_7.\frac{2^n}{2^7}\frac{1}{3^7} = {}^nC_8.\frac{2^n}{2^8}\frac{1}{3^8}$

$\Rightarrow {}^nC_7 = {}^nC_8.\frac{1}{6}$

$\Rightarrow \frac{{}^nC_8}{{}^nC_7} = 6 \Rightarrow \frac{n-7}{8} = 6 \Rightarrow n - 7 = 48$

n=55

Q.8 If $a+b+c=1$, $ab+bc+ca=2$ and $abc=3$, then the value of $a^4+b^4+c^4$ is equal to _________.

Given **18**
Answer :

Ans: 13.00

Sol: a,b,c are the roots of the equation

$x^3 - x^2 + 2x - 3 = 0 ....(1)$

Substituting a,b,c in (1) and adding

$(a^3 + b^3 + c^3) - (a^2 + b^2 + c^2) + 2(a + b + c) - 9 = 0 .....(2)$

where $(a^2 + b^2 + c^2) = (a + b + c)^2 - 2(ab + bc + ac) = 1 - 2(2) = -3$

$\therefore (a^3 + b^3 + c^3) - (-3) + 2(1) - 9 = 0 \Rightarrow a^3 + b^3 + c^3 = 4$

$(1) \Rightarrow x^4 - x^3 + 2x^2 - 3x = 0$

$\Rightarrow (a^4 + b^4 + c^4) - 4 + 2(-3) - 3(1) = 0$

$\Rightarrow a^4 + b^4 + c^4 = 13$

Q.9 If $(\vec{a} + 3\vec{b})$ is perpendicular to $(7\vec{a} - 5\vec{b})$ and $(\vec{a} - 4\vec{b})$ is perpendicular to $(7\vec{a} - 2\vec{b})$, then the angle between $\vec{a}$ and $\vec{b}$ (in degrees) is _________.

Given --
Answer :

Ans: 60.00

Sol: $(\vec{a} + \vec{b}).(7\vec{a} - 5\vec{b}) = 0$

$\Rightarrow 7\vec{a}^2 - 5\vec{a}.\vec{b} + 21\vec{a}.\vec{b} - 15\vec{b}^2 = 0$

$\Rightarrow 7\vec{a}^2 + 16\vec{a}.\vec{b} - 15\vec{b}^2 = 0......(1)$

$(\vec{a} - 4\vec{b})(7\vec{a} - 2.\vec{b}) = 0$

$7\vec{a}^2 - 2\vec{a}.\vec{b} - 28\vec{a}.\vec{b} + 8.\vec{b}^2 = 0$

$\Rightarrow 7\vec{a}^2 - 30\vec{a}.\vec{b} + 8.\vec{b}^2 = 0........(2)$

$(1) - (2) \Rightarrow 46\vec{a}.\vec{b} - 23.\vec{b}^2 = 0$

$2\vec{a}.\vec{b} = .\vec{b}^2$

$\Rightarrow 2|\vec{a}||.\vec{b}|\cos\theta = |\vec{b}|^2 \Rightarrow |\vec{b}| = 2|\vec{a}|\cos\theta.$

$\Rightarrow 7\vec{a}^2 + 16|\vec{a}||\vec{b}|\cos\theta - 15.\vec{b}^2 = 0$

$7\vec{a}^2 + 32\vec{a}^2\cos^2\theta - 15.4\vec{a}^2\cos^2\theta = 0$

$\Rightarrow 7 + 32\cos^2\theta - 60\cos^2\theta = 0$

$\Rightarrow 28\cos^2\theta = 7$

$\cos^2\theta = \frac{1}{4}$

$\cos\theta = \frac{1}{2}$

$\theta = 60^o$

Q.10 Consider the function
$$f(x) = \frac{P(x)}{\sin(x-2)}, \quad x \neq 2$$
$$= 7 \quad , \quad x = 2$$

where $P(x)$ is a polynomial such that $P''(x)$ is always a constant and $P(3)=9$. If $f(x)$ is continuous at $x=2$, then $P(5)$ is equal to _________.

Given --
Answer :

Ans: 39.00

Sol: $f(x)=\begin{cases}\dfrac{p(x)}{\sin(x-2)} & \text{if } x\neq 2\\ 7 & \text{if } x=2\end{cases}$

P''(x) is a constant $\Rightarrow P(x)=ax^2+bx+c$

$P(3)=9a+3b+c=9.....(1)$

$$\lim_{x\to 2} f(x)=\lim_{x\to 2}\left[\frac{ax^2+bx+c}{\sin(x-2)}\right]$$

Since denominator=0 when x=2,

P(2)=0 $\Rightarrow 4a+2b+c=0......(2)$

$$\lim_{x\to 2} f(x)=\lim_{x\to 2}\left(\frac{2ax+b}{\cos(x-2)}\right)$$

$=4a+b=7.......(3)$

$(4)-(2)\Rightarrow 5a+b=9........(4)$

$(4)-(3)\Rightarrow a=2$

$(3)\Rightarrow 8+b=7\Rightarrow b=-1$

$(2)\Rightarrow 8+-2+c=0$

$\Rightarrow c=-6$

$\therefore P(5)=25a+5b+c$

=50-5-6=39