SOLUTIONS & ANSWERS FOR JEE MAINS-2021
31st August Shift 1
[PHYSICS, CHEMISTRY & MATHEMATICS]

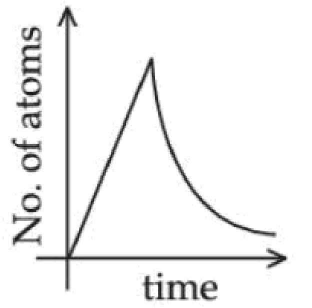
PART – A – PHYSICS

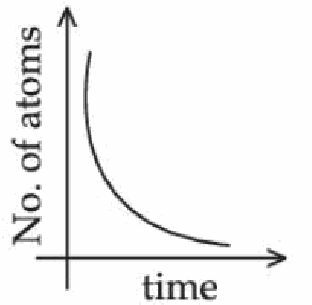
Section A

Q.1 A sample of a radioactive nucleus A disintegrates to another radioactive nucleus B, which in turn disintegrates to some other stable nucleus C. Plot of a graph showing the variation of number of atoms of nucleus B vesus time is :

(Assume that at $t = 0$, there are no B atoms in the sample)

Options

1.


2.


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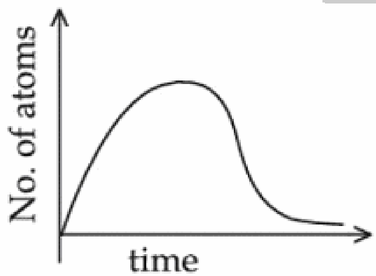

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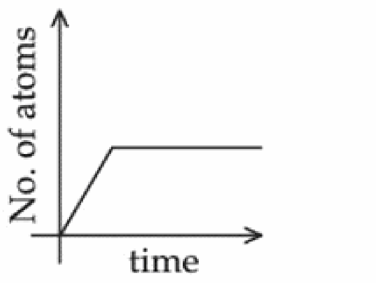
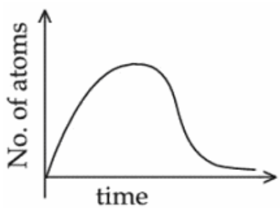


Ans: 

Sol: A → B → C (stable)

Initially no. of atoms of B = 0 after t = 0, no. of atoms of B will starts increasing and reaches maximum value when rate of decay of B = rate of formation of B.

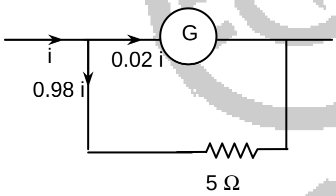
After that maximum value, no. of atoms will starts decreasing as growth and decay both are exponential functions, so best possible graph is (3).

Q.2 Consider a galvanometer shunted with 5 Ω resistance and 2% of current passes through it. What is the resistance of the given galvanometer ?

Options
1. 300 Ω
2. 226 Ω
3. 344 Ω
4. 245 Ω

Ans: 245 Ω

Sol:



$0.02i\, R_g = 0.98\, i \times 5$ R_g : Resistance of the Galvanometer

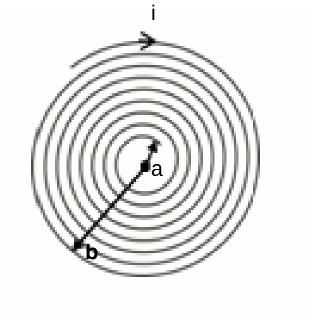
$R_g = 245\ \Omega$

Q.3 A coil having N turns is wound tightly in the form of a spiral with inner and outer radii 'a' and 'b' respectively. Find the magnetic field at centre, when a current I passes through coil :

Options
1. $\dfrac{\mu_0 I}{4(a-b)}\left[\dfrac{1}{a}-\dfrac{1}{b}\right]$
2. $\dfrac{\mu_0 I}{8}\left[\dfrac{a+b}{a-b}\right]$
3. $\dfrac{\mu_0 I}{8}\left(\dfrac{a-b}{a+b}\right)$
4. $\dfrac{\mu_0 IN}{2(b-a)}\log_e\left(\dfrac{b}{a}\right)$

Ans: $\frac{\mu_0 IN}{2(b-a)}\log_e\left(\frac{b}{a}\right)$

Sol:



No. of turns in dx width = $\frac{N}{b-a}dx$

$$\int dB = \int_a^b \left(\frac{N}{b-a}\right)dx\,\frac{\mu_0 i}{2x}$$

$$B = \frac{N\mu_0 i}{2(b-a)}\ell n\left(\frac{b}{a}\right) \qquad (l_n = \log_e)$$

Q.4 In an ac circuit, an inductor, a capacitor and a resistor are connected in series with $X_L = R = X_C$. Impedance of this circuit is :

Options

1. $R\sqrt{2}$
2. $2R^2$
3. R
4. Zero

Ans: R

Sol: $Z = \sqrt{(X_L - X_C)^2 + R^2} = R \qquad \because X_L = X_C$

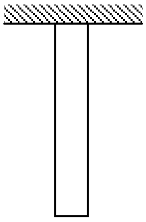
Q.5 A uniform heavy rod of weight 10 kg ms^{-2}, cross-sectional area 100 cm^2 and length 20 cm is hanging from a fixed support. Young modulus of the material of the rod is 2×10^{11} Nm^{-2}. Neglecting the lateral contraction, find the elongation of rod due to its own weight :

Options

1. 4×10^{-8} m
2. 5×10^{-8} m
3. 5×10^{-10} m
4. 2×10^{-9} m

Ans: 5×10^{-10} m

Sol:



We know, $\Delta\ell = \frac{WL}{2AY}$

$$\Delta\ell = \frac{10\times1}{2\times5}\times100\times10^{-4}\times2\times10^{11}$$

$$\Delta\ell = \frac{1}{2}\times10^{-19} = 5\times10^{-10}\text{ m}$$

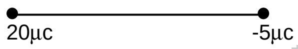
Q.6 Two particles A and B having charges 20 μC and −5 μC respectively are held fixed with a separation of 5 cm. At what position a third charged particle should be placed so that it does not experience a net electric force ?

Options

1. At midpoint between two charges
2. At 5 cm from 20 μC on the left side of system
3. At 5 cm from −5 μC on the right side
4. At 1.25 cm from a −5 μC between two charges

Ans: At 5 cm from –5μC on the right side

Sol:



Null point is possible only right side of -5μC

$$E_N = +\frac{k(-5\mu C)}{x^2} + \frac{k(20\mu C)}{(5+x)^2} = 0 \quad \left(\therefore k = \frac{1}{4\pi\varepsilon_0}\right)$$

x = 5 cm

Q.7 Match **List - I** with **List - II**.

	List - I		**List - II**
(a)	Torque	(i)	MLT^{-1}
(b)	Impulse	(ii)	MT^{-2}
(c)	Tension	(iii)	ML^2T^{-2}
(d)	Surface Tension	(iv)	MLT^{-2}

Choose the **most appropriate** answer from the option given below :

Options

1. (a)-(iii), (b)-(iv), (c)-(i), (d)-(ii)
2. (a)-(iii), (b)-(i), (c)-(iv), (d)-(ii)
3. (a)-(i), (b)-(iii), (c)-(iv), (d)-(ii)
4. (a)-(ii), (b)-(i), (c)-(iv), (d)-(iii)

Ans: (a)-(iii), (b)-(i), (c)-(iv), (d)-(ii)

Sol: torque = $\tau \rightarrow ML^2T^{-2}$ (III)
Impulse I $\Rightarrow MLT^{-1}$ (I)
Tension force $\Rightarrow MLT^{-2}$ (IV)
Surface tension $\Rightarrow MT^{-2}$ (II)

Q.8 Which of the following equations is dimensionally incorrect ?
Where t = time, h = height, s = surface tension, θ = angle, ρ = density, a, r = radius, g = acceleration due to gravity, v = volume, p = pressure, W = work done, Γ = torque, ϵ = permittivity, E = electric field, J = current density, L = length.

Options

1. $v = \dfrac{\pi p a^4}{8 \eta L}$
2. $W = \Gamma\theta$
3. $J = \epsilon \dfrac{\partial E}{\partial t}$
4. $h = \dfrac{2s\cos\theta}{\rho r g}$

Ans: $v = \dfrac{\pi \rho a^4}{8\eta L}$

Sol: (i) $\dfrac{\pi \rho a^4}{8\eta L} = \dfrac{dv}{dt}$ = volumetric flow rate

(ii) $h\rho g = \dfrac{2s}{r}\cos\theta$

(iii) RHS $\Rightarrow \varepsilon \times \dfrac{1}{4\pi\varepsilon_0}\dfrac{a}{r^2} \times \dfrac{1}{\varepsilon} = \dfrac{q}{t} \times \dfrac{1}{r^2} = \dfrac{I}{L^2} = IL^{-2}$

LHS

$R = \dfrac{I}{A} = IL^{-2}$

(iv) $W = \tau\theta$

Q.9 A reversible engine has an efficiency of $\frac{1}{4}$. If the temperature of the sink is reduced by 58°C, its efficiency becomes double. Calculate the temperature of the sink :

Options

1. 180.4°C
2. 382°C
3. 280°C
4. 174°C

Ans: 174°C

Sol: T_2 = sink temperature

$$\eta = 1 - \frac{T_2}{T_1}$$

$$\frac{1}{4} = 1 - \frac{T_2}{T_1}$$

$$\frac{T_2}{T_1} = \frac{3}{4} \text{-------(i)}$$

$$\frac{1}{2} = 1 - \frac{T_2 - 58}{T_1}$$

$$\frac{T_2}{T_1} = \frac{58}{T_1} = \frac{1}{2}$$

$$\frac{3}{4} = \frac{58}{T_1} + \frac{1}{2}$$

$$\frac{1}{4} = \frac{58}{T_1} \Rightarrow T_1 = 232$$

$$T_2 = \frac{3}{4} \times 232$$

T_2 = 174°C

Q.10 A small square loop of side 'a' and one turn is placed inside a larger square loop of side b and one turn (b > > a). The two loops are coplanar with their centres coinciding. If a current I is passed in the square loop of side 'b', then the coefficient of mutual inductance between the two loops is :

Options

1. $\frac{\mu_0}{4\pi} \frac{8\sqrt{2}}{a}$

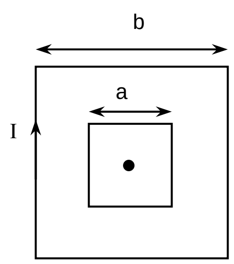
2. $\frac{\mu_0}{4\pi} 8\sqrt{2} \frac{a^2}{b}$

3. $\frac{\mu_0}{4\pi} \frac{8\sqrt{2}}{b}$

4. $\frac{\mu_0}{4\pi} 8\sqrt{2} \frac{b^2}{a}$

Ans: $\frac{\mu_0}{4\pi} 8\sqrt{2} \frac{a^2}{b}$

Sol:



$$B = \left[\frac{\mu_0}{4\pi}\frac{I}{b/2}\times 2\sin 45\right]\times 4$$

$$\phi = 2\sqrt{2}\,\frac{\mu_0}{\pi}\frac{I}{b}\times a^2$$

$$\therefore M = \frac{\phi}{I} = \frac{2\sqrt{2}\mu_0 a^2}{\pi b} = \frac{\mu_0}{4\pi}8\sqrt{2}\,\frac{a^2}{b}$$

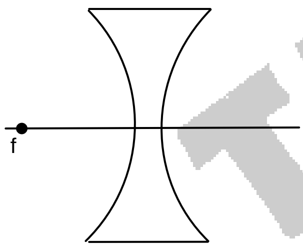
Q.11 An object is placed at the focus of concave lens having focal length f. What is the magnification and distance of the image from the optical centre of the lens ?

Options

1. Very high, ∞
2. $\frac{1}{2}, \frac{f}{2}$
3. $\frac{1}{4}, \frac{f}{4}$
4. $1, \infty$

Ans: $\frac{1}{2}, \frac{f}{2}$

Sol:



$U = -f$

$$\frac{1}{V} - \frac{1}{U} = \frac{1}{-f} \Rightarrow \frac{1}{V} = -\frac{2}{f}$$

$$V = \frac{-f}{2}$$

$$m = \frac{V}{U} = \frac{1}{2}$$

Distance = $\frac{f}{2}$

Q.12 For an ideal gas the instantaneous change in pressure 'p' with volume 'v' is given by the equation $\frac{dp}{dv} = -ap$. If $p = p_0$ at $v = 0$ is the given boundary condition, then the maximum temperature one mole of gas can attain is :
(Here R is the gas constant)

Options

1. $\frac{ap_0}{eR}$
2. infinity
3. $0°C$
4. $\frac{p_0}{aeR}$

Ans: $\frac{p_0}{aeR}$

Sol: $\int_{p_0}^{p}\frac{dp}{P} = -a\int_0^v dv$

$\ell n\left(\frac{p}{p_0}\right) = -av$

$p = p_0 e^{-av}$

For temperature maximum p – v product should be maximum

$T = \frac{pv}{nR} = \frac{p_0 ve^{-av}}{R}$

$\frac{dT}{dv} = 0 \Rightarrow \frac{p_0}{R}\left\{e^{-av} + ve^{-av}(-a)\right\}$

$\frac{p_0 e^{-av}}{R}\{1 - av\} = 0$

$v = \frac{1}{a}, \infty$

$T = \frac{p_0 1}{Rae} = \frac{p_0}{Rae}$

At $v = \infty$

$T = 0$

Q.13 A moving proton and electron have the same de-Broglie wavelength. If K and P denote the K.E. and momentum respectively. Then choose the correct option :

Options
1. $K_p < K_e$ and $P_p = P_e$
2. $K_p > K_e$ and $P_p = P_e$
3. $K_p = K_e$ and $P_p = P_e$
4. $K_p < K_e$ and $P_p < P_e$

Ans: $K_p < K_e$ and $P_p = P_e$

Sol: $\lambda_p = \frac{h}{P_p}$ $\lambda_e = \frac{h}{P_e}$

$\because \lambda_p = \lambda_e$

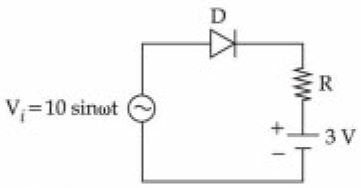
$\Rightarrow P_p = P_e$

$(K)_p = \frac{P_p^2}{2m_p}$

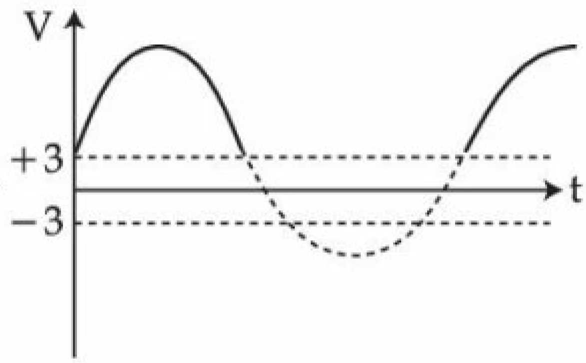
$(K)_e = \frac{P_e^2}{2m_e}$

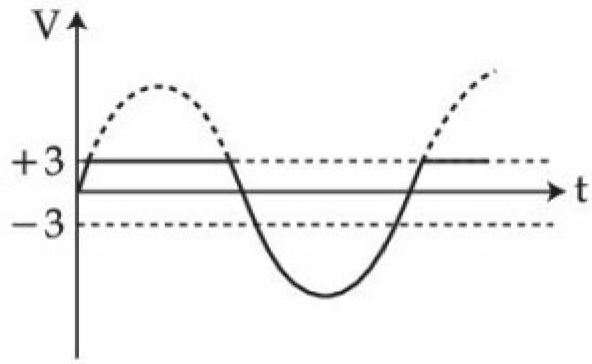
$K_p < K_e$ as $m_p > m_e$

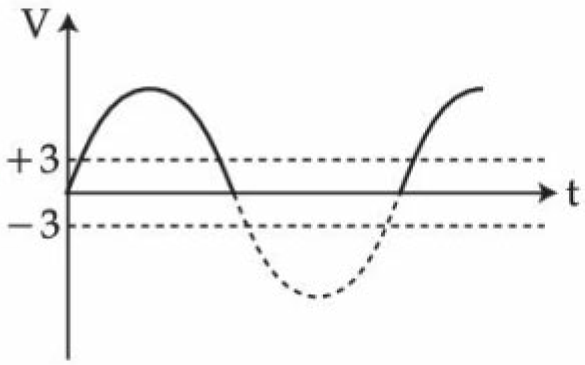
Q.14 Choose the correct waveform that can represent the voltage across R of the following circuit, assuming the diode is ideal one :

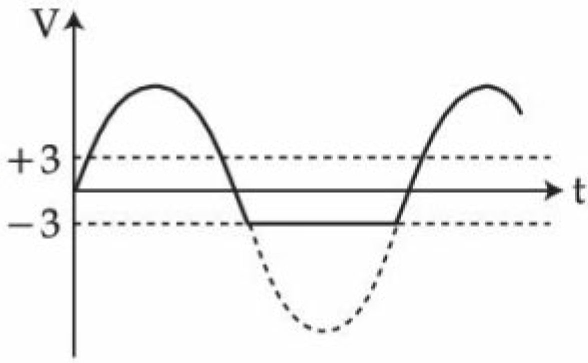


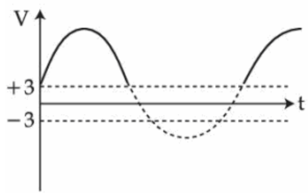
Options

1.


2.


3. 

4. 

Ans: 

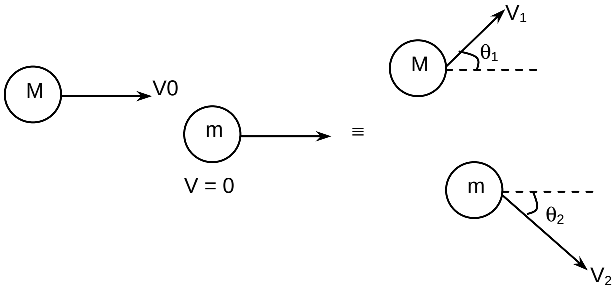
Sol: When $V_i > 3$ volt, $V_R > 0$
Because diode will be in forward biased state
When $V_i \leq 3$ volt; $V_R = 0$
Because diode will be in reverse biased state

Q.15 A body of mass M moving at speed V_0 collides elastically with a mass 'm' at rest. After the collision, the two masses move at angles θ_1 and θ_2 with respect to the initial direction of motion of the body of mass M. The largest possible value of the ratio M/m, for which the angles θ_1 and θ_2 will be equal, is :

Options
1. 2
2. 3
3. 1
4. 4

Ans: 3

Sol:



Given $\theta_1 = \theta_2 = \theta$

From momentum conservation in x-direction $MV_0 = MV_1 \cos\theta + mV_2 \cos\theta$

in y-direction $0 = MV_1 \sin\theta - mV_2 \sin\theta$

Solving above equations

$$V_2 = \frac{MV_1}{m}, V_0 = 2V\cos\theta$$

From energy conservation

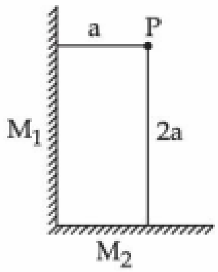
$$\frac{1}{2}MV_0^2 = \frac{1}{2}MV_1^2 + \frac{1}{2}MV_2^2$$

Substituting value of V_2 and V_0, we will get

$$\frac{M}{m} + 1 = 4\cos^2\theta \le 4$$

$$\frac{M}{m} \le 3$$

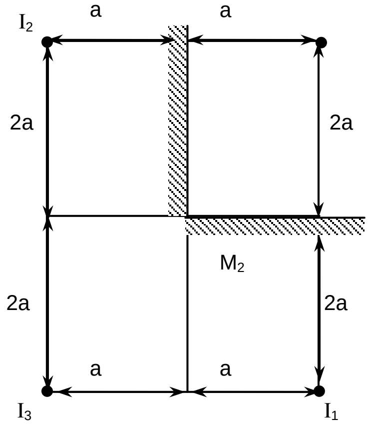
Q.16 Two plane mirrors M_1 and M_2 are at right angle to each other shown. A point source 'P' is placed at 'a' and '2a' meter away from M_1 and M_2 respectively. The shortest distance between the images thus formed is : (Take $\sqrt{5} = 2.3$)



Options
1. 4.6a
2. 2.3a
3. 3a
4. $2\sqrt{10}$ a

Ans: 4.6 a

Sol:



Shortest distance is 2a between I_1 and I_3

But answer given is for I_1 and I_2

$\sqrt{(4a)^2 + (2a)^2}$

$a\sqrt{20}$

4.47 a

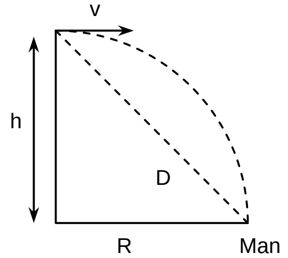
Q.17 A helicopter is flying horizontally with a speed 'v' at an altitude 'h' has to drop a food packet for a man on the ground. What is the distance of helicopter from the man when the food packet is dropped ?

Options

1. $\sqrt{\dfrac{2ghv^2 + 1}{h^2}}$
2. $\sqrt{2ghv^2 + h^2}$
3. $\sqrt{\dfrac{2gh}{v^2} + h^2}$
4. $\sqrt{\dfrac{2v^2h}{g} + h^2}$

Ans: $\sqrt{\dfrac{2v^2h}{g} + h^2}$

Sol:



$$R=\sqrt{\frac{2h}{g}}v$$

$$D=\sqrt{R^2+h^2}=\sqrt{\left(\sqrt{\frac{2h}{g}}v\right)^2+h^2}$$

$$D=\sqrt{\frac{2hv^2}{g}+h^2}$$

Q.18 The masses and radii of the earth and moon are (M_1, R_1) and (M_2, R_2) respectively. Their centres are at a distance 'r' apart. Find the minimum escape velocity for a particle of mass 'm' to be projected from the middle of these two masses :

Options

1. $V=\frac{1}{2}\sqrt{\frac{2G\,(M_1+M_2)}{r}}$

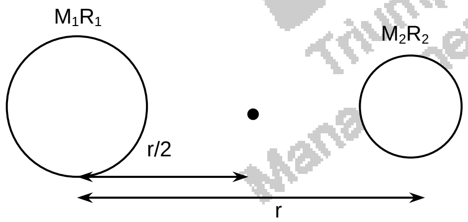
2. $V=\frac{1}{2}\sqrt{\frac{4G\,(M_1+M_2)}{r}}$

3. $V=\frac{\sqrt{2G}\,(M_1+M_2)}{r}$

4. $V=\sqrt{\frac{4G\,(M_1+M_2)}{r}}$

Ans: $V=\sqrt{\frac{4G\,(M_1+M_2)}{r}}$

Sol:

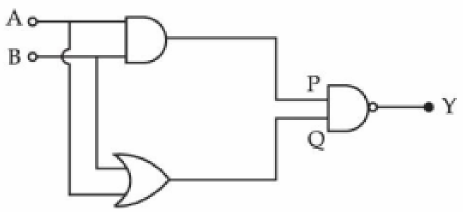


$$\frac{1}{2}mV^2-\frac{GM_1m}{r/2}-\frac{GM_2m}{r/2}=0$$

$$\frac{1}{2}mV^2=\frac{2Gm}{r}(M_1+M_2)$$

$$V=\sqrt{\frac{4G(M_1+M_2)}{r}}$$

Q.19 In the following logic circuit the sequence of the inputs A, B are (0, 0), (0, 1), (1, 0) and (1, 1). The output Y for this sequence will be :



Options 1. 0, 0, 1, 1

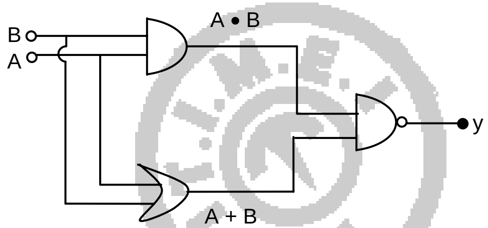
2. 1, 1, 1, 0

3. 1, 0, 1, 0

4. 0, 1, 0, 1

Ans: 1, 1, 1, 0

Sol:



$Y = \overline{(A \cdot B) \cdot (A + B)}$

$Y)_{(0,\,0)} = 1$

$Y)_{(0,\,1)} = 1$

$Y)_{(1,\,0)} = 1$

$Y)_{(1,\,1)} = 0$

Q.20 Angular momentum of a single particle moving with constant speed along circular path :

Options 1. is zero

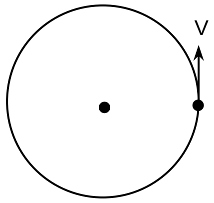
2. remains same in magnitude and direction

3. changes in magnitude but remains same in the direction

4. remains same in magnitude but changes in the direction

Ans: remains same in magnitude and direction

Sol:



$|\vec{L}| = mvr$

And direction will be upward and remain constant

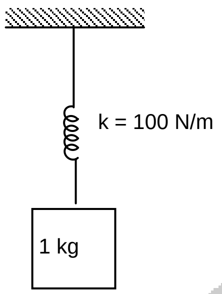
Section B

Q.1 A particle of mass 1 kg is hanging from a spring of force constant 100 Nm^{-1}. The mass is pulled slightly downward and released so that it executes free simple harmonic motion with time period T. The time when the kinetic energy and potential energy of the system will become equal, is $\frac{T}{x}$. The value of x is ________.

Given --
Answer :

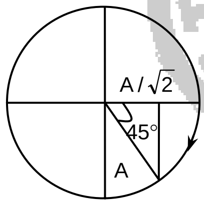
Ans: 8

Sol:



KE = PE

$$y = \frac{A}{\sqrt{2}} - \sin\omega t$$



$$t = \frac{T}{8} = \frac{T}{x}$$

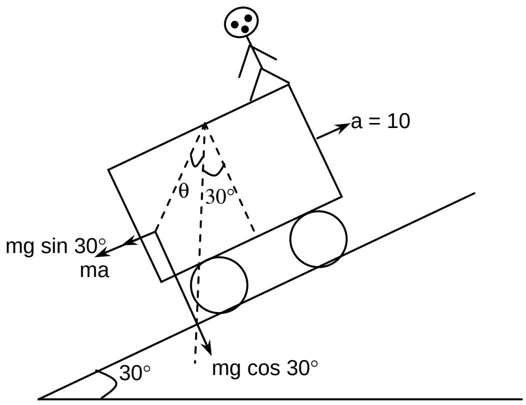
x = 8

Q.2 A car is moving on a plane inclined at 30° to the horizontal with an acceleration of 10 ms^{-2} parallel to the plane upward. A bob is suspended by a string from the roof of the car. The angle in degrees which the string makes with the vertical is ________.
(Take $g = 10\ ms^{-2}$)

Given **45**
Answer :

Ans: 30

Sol:



$$\tan(30+\theta)=\frac{mg\ \sin 30^\circ+ma}{mg\ \cos 30^\circ}$$

$$\tan(30+\theta)=\frac{5+10}{5\sqrt{3}}=\frac{1+2}{\sqrt{3}}$$

$$\frac{\tan\theta+\frac{1}{\sqrt{3}}}{1-\frac{1}{\sqrt{3}}\tan\theta}=\sqrt{3}$$

$$\sqrt{3}\tan\theta+1=3-\sqrt{3}\tan\theta$$

$$2\sqrt{3}\tan\theta=2$$

$$\tan\theta=\frac{1}{\sqrt{3}}$$

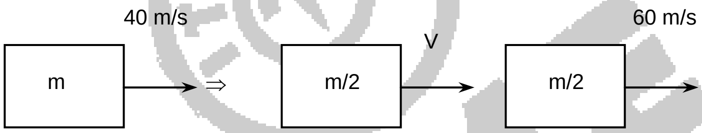
$\theta = 30°$

Q.3 A block moving horizontally on a smooth surface with a speed of 40 ms^{-1} splits into two equal parts. If one of the parts moves at 60 ms^{-1} in the same direction, then the fractional change in the kinetic energy will be $x : 4$ where $x =$ ________.

Given **5**
Answer :

Ans: 1

Sol:



$P_i = P_f$

$$m\times 40=\frac{m}{2}\times v+\frac{m}{2}\times 60$$

$$40=\frac{v}{2}+30$$

$\Rightarrow v = 20$

$$(K.E.)_i=\frac{1}{2}m\times(40)^2=800\,m$$

$$(K.E.)_f=\frac{1}{2}\frac{m}{2}(20)^2+\frac{1}{2}\frac{m}{2}(60)^2=1000\,m$$

$$|\Delta K.E|=|1000m-800m|=200\,m$$

$$\frac{\Delta K.E}{(K.E)_i}=\frac{200\,m}{800\,m}=\frac{1}{4}=\frac{x}{4}$$

$x = 1$

Q.4 When a rubber ball is taken to a depth of ________ m in deep sea, its volume decreases by 0.5%.
(The bulk modulus of rubber $=9.8\times10^8\ Nm^{-2}$
Density of sea water $=10^3\ kgm^{-3}$
$g=9.8\ m/s^2$)

Given **--**
Answer :

Ans: 500

Sol: $B = -\frac{\Delta P}{\left(\frac{\Delta V}{V}\right)} = -\frac{\rho g h}{\left(\frac{\Delta V}{V}\right)}$

$-\frac{B\frac{\Delta V}{V}}{\rho g} = h$

$\frac{9.8 \times 10^8 \times 0.5}{100 \times 10^3 \times 9.8} = h$

h = 500

Q.5 A wire having a linear mass density 9.0×10^{-4} kg/m is stretched between two rigid supports with a tension of 900 N. The wire resonates at a frequency of 500 Hz. The next higher frequency at which the same wire resonates is 550 Hz. The length of the wire is __________m.

Given --
Answer :

Ans: 10

Sol: $\mu = 9.0 \times 10^{-4} \frac{kg}{m}$

T = 900 N

$V = \sqrt{\frac{T}{\mu}} = \sqrt{\frac{900}{9 \times 10^{-4}}} = 100\, m/s$

f_1 = 500 Hz

f = 550

$\frac{nV}{2\ell} = 500$ ––––––(i)

$\frac{(n+1)V}{2\ell} = 550$ ––––(ii)

(ii) – (i) $\frac{V}{2\ell} = 50$

$\ell = \frac{1000}{2 \times 50} = 10$

Q.6 If the sum of the heights of transmitting and receiving antennas in the line of sight of communication is fixed at 160 m, then the maximum range of LOS communication is __________ km.

(Take radius of Earth = 6400 km)

Given --
Answer :

Ans: 64

Sol: $h_T = h_R = 160$ ------- (i)

$d = \sqrt{2Rh_T} + \sqrt{2Rh_R}$

$d = \sqrt{2R}\left[\sqrt{h_T} + \sqrt{h_R}\right]$

$d = \sqrt{2R}\left[\sqrt{x} + \sqrt{160 - x}\right]$

$\frac{d(d)}{dx} = 0$

$\frac{1}{2\sqrt{x}} + \frac{1(-1)}{2\sqrt{160 - x}} = 0$

$$\frac{1}{\sqrt{x}} = \frac{1}{\sqrt{160 - x}}$$

x = 80 m

$$d_{max} = \sqrt{2 \times 6400}\left[\sqrt{\frac{80}{1000}} + \sqrt{\frac{20}{1000}}\right] = \frac{80\sqrt{2} \times 2\sqrt{80}}{10\sqrt{10}} = 8 \times 2 \times \sqrt{2} \times 2\sqrt{2} = 64 \text{ km}$$

Q.7 The electric field in an electromagnetic wave is given by

$E = (50\ NC^{-1})\ \sin\omega(t - x/c)$

The energy contained in a cylinder of volume V is 5.5×10^{-12} J. The value of V is __________ cm^3.

(given $\epsilon_0 = 8.8 \times 10^{-12}\ C^2N^{-1}m^{-2}$)

Given Answer : --

Ans: 500

Sol: $E = 50 \sin\left(\omega t - \frac{\omega}{c} x\right)$

Energy density = $\frac{1}{2}\epsilon_0 E_0^2$

Energy for volume V = $\frac{1}{2}\epsilon_0 E_0^2 V = 5.5 \times 10^{-12}$

$\frac{1}{2} 8.8 \times 10^{-12} \times 2500\ V = 5.5 \times 10^{-12}$

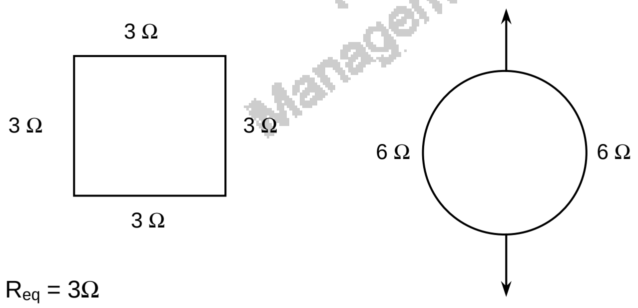
$V = \frac{5.5 \times 2}{2500 \times 8.8} = .0005\ m^3 = 0.0005 \times 10^6\ (cm)^3 = 500\ (cm)^3$

Q.8 A square shaped wire with resistance of each side 3 Ω is bent to form a complete circle. The resistance between two diametrically opposite points of the circle in unit of Ω will be __________.

Given Answer : 3

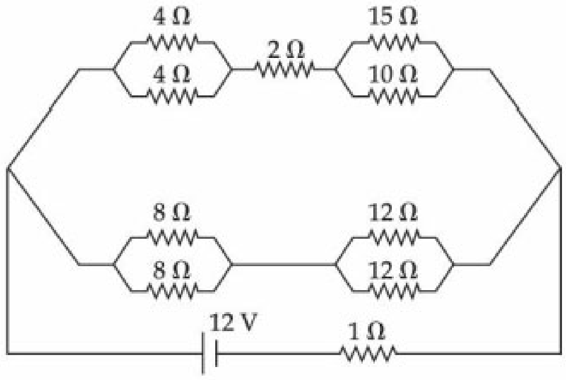
Ans: 3

Sol:



$R_{eq} = 3\Omega$

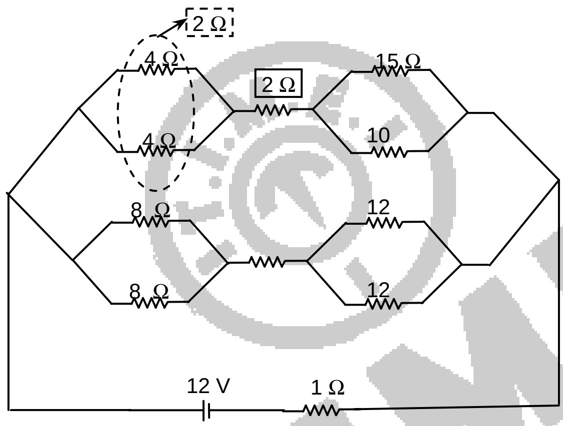
Q.9 The voltage drop across 15 Ω resistance in the given figure will be ________ V.



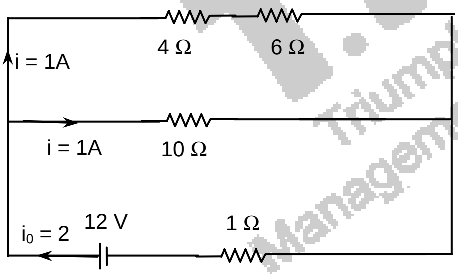
Given **6**
Answer :

Ans: 6

Sol:

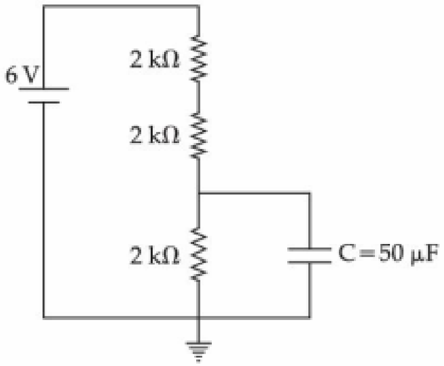


⇒ effective circuit diagram will be



Point drop across 6Ω = 1 × 6 = 6 = V_{AB}
⇒ Hence point drop across 15 Ω = 6 volt = V_{AB}

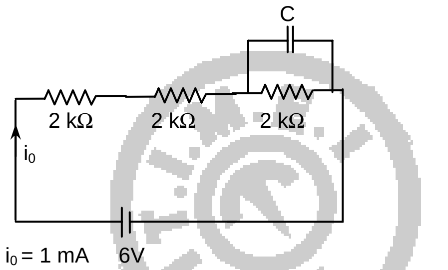
Q.10 A capacitor of 50 μF is connected in a circuit as shown in figure. The charge on the upper plate of the capacitor is __________ μC.



Given **2**
Answer :

Ans: 100

Sol:



Potential difference across each resistor = 2V
$q = CV = 50 \times 10^{-6} \times 2 = 100 \times 10^{-6} = 100\ \mu C$

PART – B – CHEMISTRY

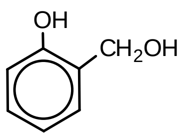
Section A

Q.1 Monomer of Novolac is :

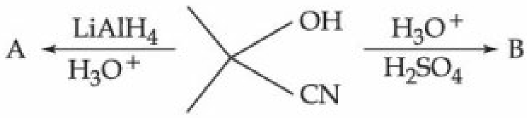
Options
1. phenol and melamine.
2. 3-Hydroxybutanoic acid.
3. 1,3-Butadiene and styrene.
4. *o*-Hydroxymethylphenol.

Ans: o-Hydroxymethylphenol

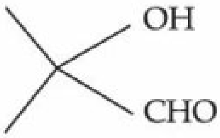
Sol: The monomer of Novolac is

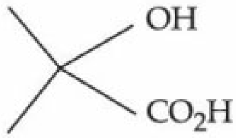


Q.2 The major products A and B in the following set of reactions are :

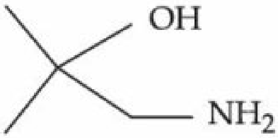


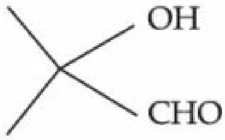
Options 1.

A = 

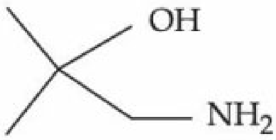
, B = 

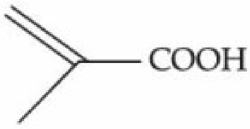
2.

A = 

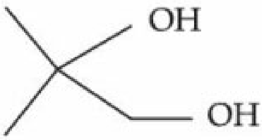
, B = 

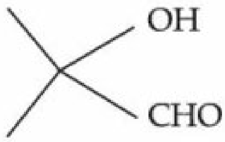
3.

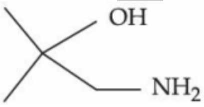
A = 

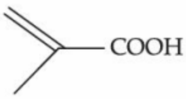
, B = 

4.

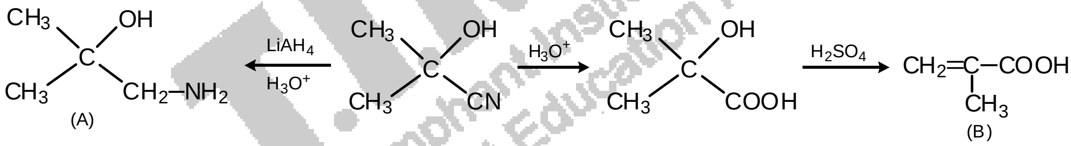
A = 

, B = 

Ans: A = 

, B = 

Sol:



Q.3 Given below are two statements : one is labelled as **Assertion (A)** and the other is labelled as **Reason (R)**.

Assertion (A) : Metallic character decreases and non-metallic character increases on moving from left to right in a period.

Reason (R) : It is due to increase in ionisation enthalpy and decrease in electron gain enthalpy, when one moves from left to right in a period.

In the light of the above statements, choose the **most appropriate** answer from the options given below :

Options

1. Both **(A)** and **(R)** are correct and **(R)** is the correct explanation of **(A)**.
2. Both **(A)** and **(R)** are correct but **(R)** is not the correct explanation of **(A)**.
3. **(A)** is false but **(R)** is true.
4. **(A)** is true but **(R)** is false.

Ans: (A) is true but (R) is false

Sol: On moving from left to right across a period, metallic character decreases and non-metallic character increases. This is due to the increase in ionization enthalpy and increase in negative electron gain enthalpy.

Q.4 Given below are two statements :

Statement I : The process of producing syn-gas is called gasification of coal.

Statement II : The composition of syn-gas is $CO + CO_2 + H_2$ (1 : 1 : 1).

In the light of the above statements, choose the **most appropriate** answer from the options given below :

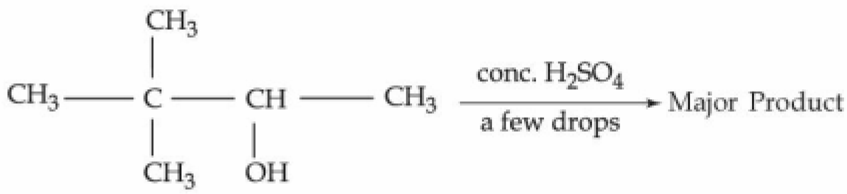
Options

1. **Statement I** is false but **Statement II** is true.
2. Both **Statement I** and **Statement II** are false.
3. **Statement I** is true but **Statement II** is false.
4. Both **Statement I** and **Statement II** are true.

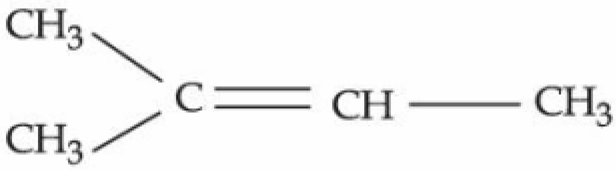
Ans: Statement I is true but statement II is false

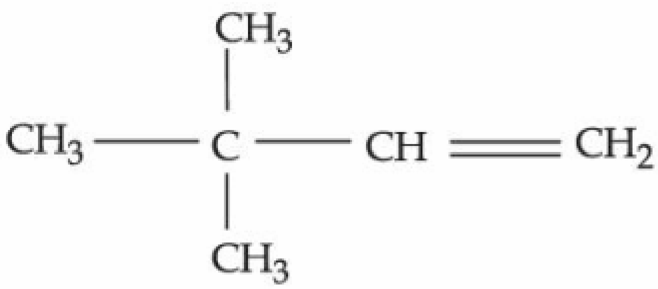
Sol: Syn gas (synthetic gas) have the composition CO and H_2 in 1 : 1 ratio

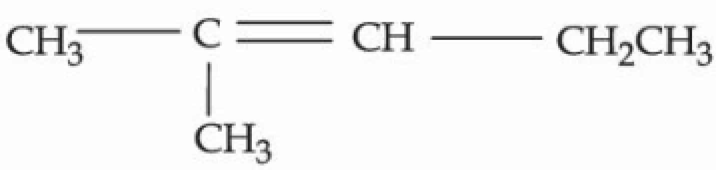
Q.5 The major product formed in the following reaction is :

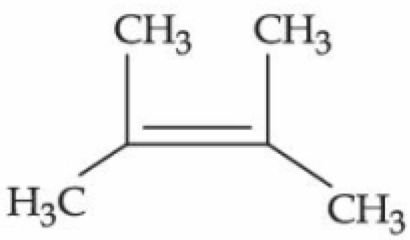


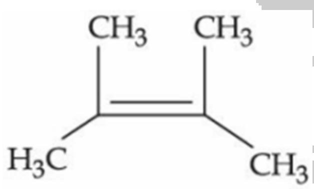
Options

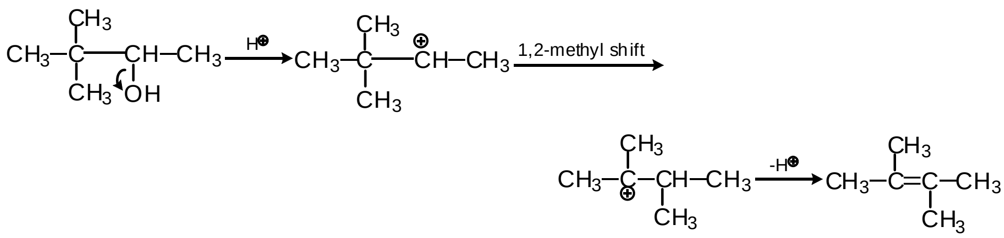
1. 

2. 

3. 

4. 

Ans: 

Sol: 

Q.6 Given below are two statements : one is labelled as **Assertion (A)** and the other is labelled as **Reason (R)**.

Assertion (A) : Treatment of bromine water with propene yields 1-bromopropan-2-ol.

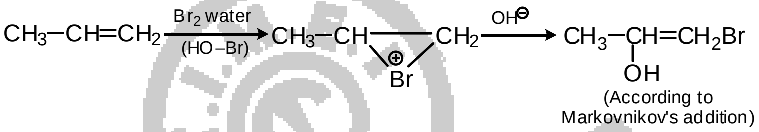
Reason (R) : Attack of water on bromonium ion follows Markovnikov rule and results in 1-bromopropan-2-ol.

In the light of the above statements, choose the **most appropriate** answer from the options given below :

Options
1. (A) is false but (R) is true.
2. Both (A) and (R) are true and (R) is the correct explanation of (A).
3. Both (A) and (R) are true but (R) is NOT the correct explanation of (A).
4. (A) is true but (R) is false.

Ans: Both (A) and (R) are true and (R) is the correct explanation of (A)

Sol:



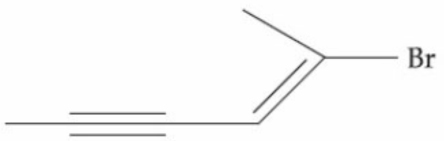
Q.7 BOD values (in ppm) for clean water (A) and polluted water (B) are expected respectively as :

Options
1. A > 50, B < 27
2. A > 15, B > 47
3. A < 5 , B > 17
4. A > 25, B < 17

Ans: A < 5, B > 17

Sol: BOD value of pure water is less than 5 ppm and that of polluted water is greater than 17 ppm

Q.8 Choose the **correct** name for compound given below :



Options
1. (4E)-5-Bromo-hex-2-en-4-yne
2. (4E)-5-Bromo-hex-4-en-2-yne
3. (2E)-2-Bromo-hex-2-en-4-yne
4. (2E)-2-Bromo-hex-4-yn-2-ene

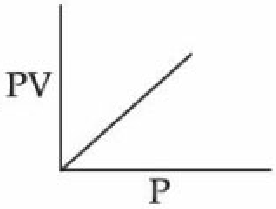
Ans: (2E)-2-Bromo-hex-2-en-4-yne

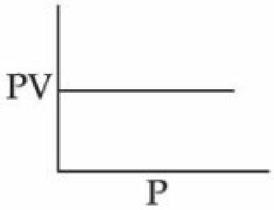
Sol: $\overset{6}{H_3C}-\overset{5}{C}\equiv\overset{4}{C}-\overset{3}{CH}=\underset{Br}{\overset{2}{C}}-\overset{1}{CH_3}$

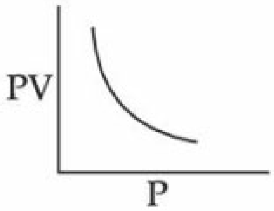
2-Bromo-hex-2-en-4-yne

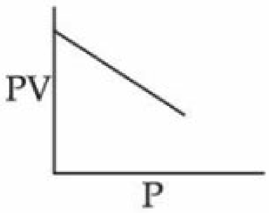
Q.9 Which one of the following is the correct PV vs P plot at constant temperature for an ideal gas ? (P and V stand for pressure and volume of the gas respectively)

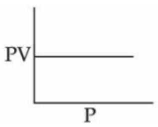
Options

1. 

2. 

3. 

4. 

Ans: 

Sol: According to Boyle's law, at constant temperature, PV is a constant at various pressures for an ideal gas

Q.10 Given below are two statements : one is labelled as **Assertion (A)** and the other is labelled as **Reason (R)**.

Assertion (A) : Aluminium is extracted from bauxite by the electrolysis of molten mixture of Al_2O_3 with cryolite.

Reason (R) : The oxidation state of Al in cryolite is +3.

In the light of the above statements, choose the **most appropriate** answer from the options given below :

Options

1. **(A)** is false but **(R)** is true.
2. Both **(A)** and **(R)** are correct and **(R)** is the correct explanation of **(A)**.
3. Both **(A)** and **(R)** are correct but **(R)** is not the correct explanation of **(A)**.
4. **(A)** is true but **(R)** is false.

Ans: Both (A) and (R) are correct but (R) is not the correct explanation of (A)

Sol: Both assertion and reason are correct statement but, reason is not the correct explanation of assertion

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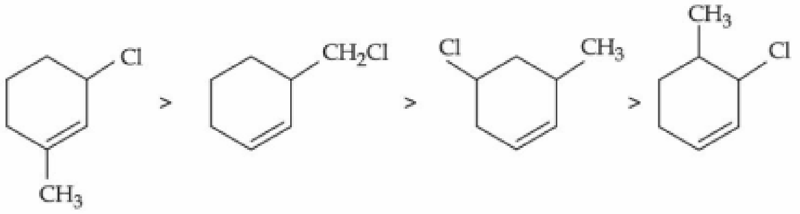
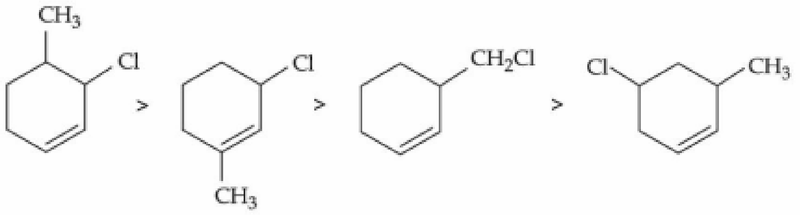
Q.11 The **correct** order of reactivity of the given chlorides with acetate in acetic acid is :

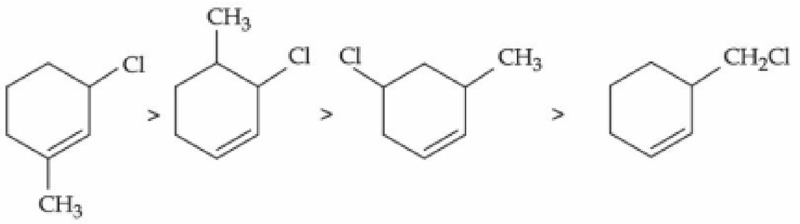
Options 1.



2.



3.



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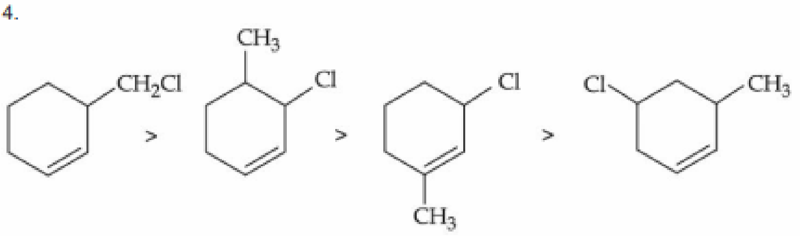
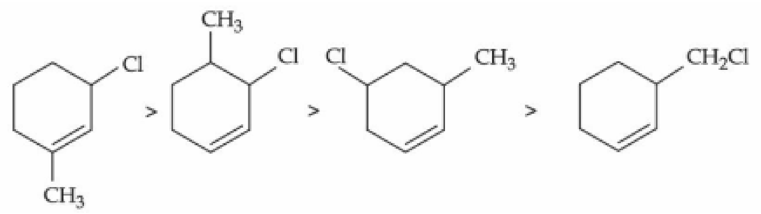
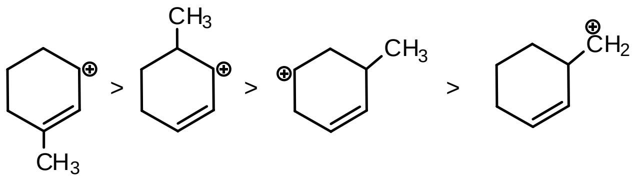


Ans: 

Sol: The reaction proceeds via the formation of carbocation intermediate. Hence the reactivity depends on the stability of carbocation formed. Greater the stability of $C^{\oplus}$, greater the reactivity.



Q.12 Which one of the following 0.10 M aqueous solutions will exhibit the largest freezing point depression ?

Options
1. $KHSO_4$
2. hydrazine
3. glycine
4. glucose

Ans: $KHSO_4$

Sol: $\Delta T_f \propto$ i.m. Since the concentration remains constant, depression in freezing point depends on vant Hoff factor 'i'. Greater the value of 'i' greater will be ΔT_f. Among the options given, $KHSO_4$ has the highest 'i' value.

Q.13 Which one of the following compounds contains β-C_1-C_4 glycosidic linkage ?

Options
1. Amylose
2. Sucrose
3. Lactose
4. Maltose

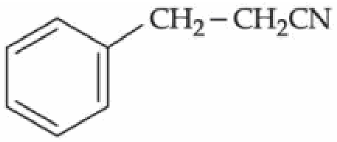
Ans: Lactose

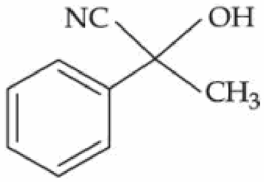
Sol: In maltose and amylase, the glycosidic linkage is α C_1-C_4 whereas in lactose it is β $C_1 - C_4$

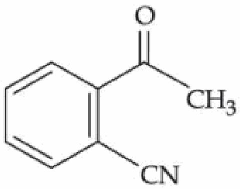
Q.14 The structure of product C, formed by the following sequence of reactions is :

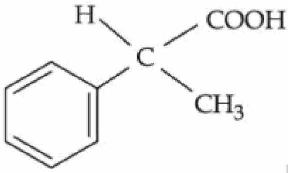
$$CH_3COOH + SOCl_2 \longrightarrow A \xrightarrow[AlCl_3]{Benzene} B \xrightarrow[^{-}OH]{KCN} C$$

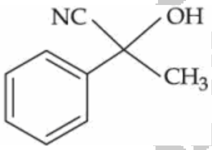
Options

1. 

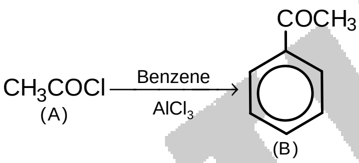
2. 

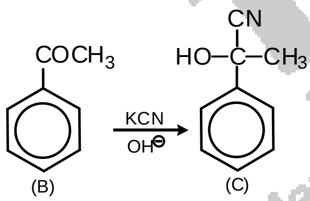
3. 

4. 

Ans: 

Sol: $CH_3COOH + SOCl_2 \longrightarrow CH_3COCl$ (A)





Q.15 The major component/ingredient of Portland Cement is :

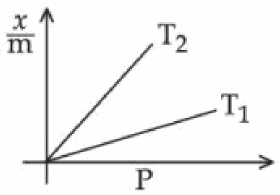
Options
1. dicalcium silicate
2. dicalcium aluminate
3. tricalcium silicate
4. tricalcium aluminate

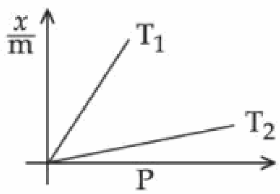
Ans: tricalcium silicate

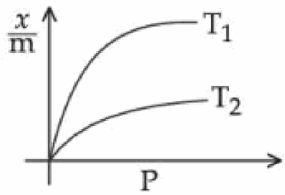
Sol: The constituents of Portland cement are tricalcium silicate (30-50%), dicalcium silicate (20-45%), tricalcium aluminate (8-12%) and tricalcium alumino ferrite (6-10%)

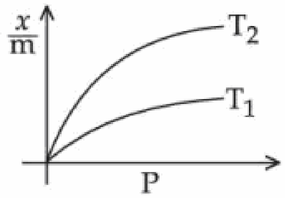
Q.16 Select the graph that correctly describes the adsorption isotherms at two temperatures T_1 and T_2 ($T_1 > T_2$) for a gas :
(x – mass of the gas adsorbed
m – mass of adsorbent
P – pressure)

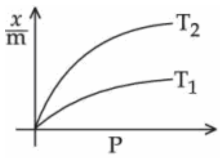
Options

1. 

2. 

3. 

4. 

Ans: 

Sol: Since adsorption is an exothermic process, rate of adsorption $\left(\frac{x}{m}\right)$ will be low at high temperatures

For adsorption process at constant T, $\frac{x}{m} \alpha P^{1/n}$

Where $\frac{1}{n}$ lies between 0 – 1

Q.17 Which one of the following lanthanides exhibits +2 oxidation state with diamagnetic nature ? (Given Z for Nd = 60, Yb = 70, La = 57, Ce = 58)

Options
1. Yb
2. Ce
3. La
4. Nd

Ans: Yb

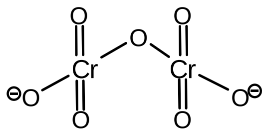
Sol: Yb (Z = 70)$\rightarrow$ [Xe] $4f^{14}$ $6s^2$
$\therefore$ $Yb^{2+} \rightarrow$ [Xe] $4f^{14}$
Since all the electrons are paired, Yb^{2+} is diamagnetic

Q.18 In the structure of the dichromate ion, there is a :

Options
1. linear unsymmetrical Cr – O – Cr bond.
2. non-linear unsymmetrical Cr – O – Cr bond.
3. linear symmetrical Cr – O – Cr bond.
4. non-linear symmetrical Cr – O – Cr bond.

Ans: non-linear symmetrical Cr–O–Cr bond

Sol:



Q.19 The denticity of an organic ligand, biuret is :

Options
1. 4
2. 2
3. 3
4. 6

Ans: 2

Sol: Biuret is $H_2N-\overset{\overset{O}{||}}{C}-NH-\overset{\overset{O}{||}}{C}-NH_2$
It is a bidentate ligand

Q.20 Given below are two statements : one is labelled as **Assertion (A)** and the other is labelled as **Reason (R)**.

Assertion (A) : A simple distillation can be used to separate a mixture of propanol and propanone.

Reason (R) : Two liquids with a difference of more than 20°C in their boiling points can be separated by simple distillations.

In the light of the above statements, choose the **most appropriate** answer from the options given below :

Options
1. (A) is true but (R) is false.
2. Both (A) and (R) are correct and (R) is the correct explanation of (A).
3. Both (A) and (R) are correct but (R) is not the correct explanation of (A).
4. (A) is false but (R) is true.

Ans: Both (A) and (R) are correct and (R) is the correct explanation of (A)

Sol: Both assertion (A) and reason (R) are correct statements and reason (R) is the correct explanation for assertion (A)

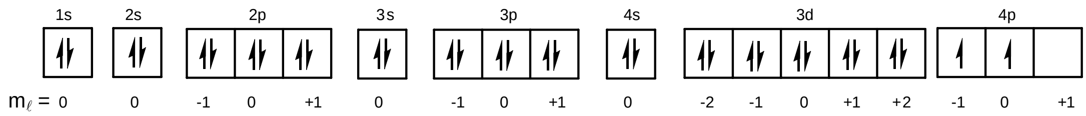
Section B

Q.1 Ge (Z = 32) in its ground state electronic configuration has x completely filled orbitals with $m_l = 0$. The value of x is ________ .

Given **7**
Answer :

Ans: 7

Sol: Ge (Z = 32) → $1s^2\ 2s^2\ 2p^6\ 3s^2\ 3p^6\ 4s^2\ 3d^{10}\ 4p^2$



Number of completely filled orbitals with $m_\ell = 0$ is 7

Q.2 The total number of reagents from those given below, that can convert nitrobenzene into aniline is ________. (Integer answer)

I. Sn – HCl
II. Sn – NH_4OH
III. Fe – HCl
IV. Zn – HCl
V. H_2 – Pd
VI. H_2 – Raney Nickel

Given --
Answer :

Ans: 5

Sol: Except Sn–NH_4OH, all the other reagents can reduce nitrobenzene to aniline

Q.3 Consider the following cell reaction

$$Cd_{(s)} + Hg_2SO_{4(s)} + \frac{9}{5}H_2O_{(l)} \rightleftharpoons CdSO_4 \cdot \frac{9}{5}H_2O_{(s)} + 2Hg_{(l)}.$$

The value of E^0_{cell} is 4.315 V at 25°C. If $\Delta H° = -825.2$ kJ mol^{-1}, the standard entropy change $\Delta S°$ in J K^{-1} is ________. (Nearest integer)
[Given : Faraday constant = 96487 C mol^{-1}]

Given --
Answer :

Ans: 25

Sol: $\Delta G° = -nFE°_{cell}$

$\Delta G° = \Delta H° - T\Delta S°$

$\Delta H° - T\Delta S° = -nFE°_{cell}$

$T\Delta S° = \Delta H° + nFE°_{cell}$

$$\Delta S° = \frac{\Delta H° + nFE°_{cell}}{T}$$

$$= \frac{-825.2 \times 10^3 + 2 \times 96487 \times 4.315}{298} = 25.11 \text{ J K}^{-1}$$

Q.4 The number of hydrogen bonded water molecule(s) associated with stoichiometry $CuSO_4 \cdot 5H_2O$ is __________.

Given **1**
Answer :

Ans: 1

Sol: In $CuSO_4.5H_2O$, out of 5 hydrated water molecules, 4 of them make co-ordinate bonds with Cu^{2+} ion and the remaining H_2O molecule forms a H-bond with SO_4^{2-} ion.

Q.5 For a first order reaction, the ratio of the time for 75% completion of a reaction to the time for 50% completion is __________. (Integer answer)

Given **2**
Answer :

Ans: 2

Sol: For a first order reaction,

$t_{75\%} = 2t_{50\%}$

$\therefore \frac{t_{75\%}}{t_{50\%}} = \frac{2t_{50\%}}{t_{50\%}} = 2$

Q.6 A_3B_2 is a sparingly soluble salt of molar mass M ($g\ mol^{-1}$) and solubility x g L^{-1}. The solubility product satisfies $K_{sp} = a\left(\frac{x}{M}\right)^5$. The value of a is __________. (Integer answer)

Given **108**
Answer :

Ans: 108

Sol: $A_3B_2 \rightleftharpoons \underset{'3S'}{3A^{2+}} + \underset{'2S'}{2B^{3-}}$

$\therefore K_{sp} = [A^{2+}]^3[B^{3-}]^2 = [3S]^3[2S]^2$

$K_{sp} = 108\ S^5$

If the solubility is expressed in x g L^{-1} then, $S = \frac{x}{m}$ where 'm' is the molar mass

$\therefore K_{sp} = 108\left(\frac{x}{m}\right)^5$

Q.7 The molarity of the solution prepared by dissolving 6.3 g of oxalic acid ($H_2C_2O_4 \cdot 2H_2O$) in 250 mL of water in mol L^{-1} is $x \times 10^{-2}$. The value of x is __________.
(Nearest integer)
[Atomic mass : H : 1.0, C : 12.0, O : 16.0]

Given **20**
Answer :

Ans: 20

Sol: Molarity, $M = \frac{W_B \times 1000}{M_B \times V_{soln}(mL)} = \frac{6.3 \times 1000}{126 \times 250} = 2 \times 10^{-1} = 20 \times 10^{-2}$ mol L^{-1}

Q.8 Consider the sulphides HgS, PbS, CuS, Sb_2S_3, As_2S_3 and CdS. Number of these sulphides soluble in 50% HNO_3 is ________.

Given Answer : --

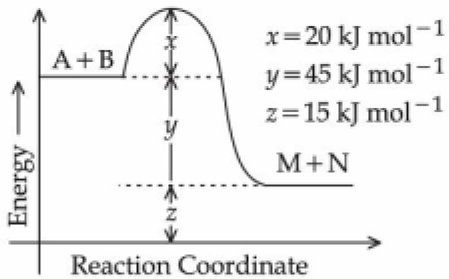
Ans: 4

Sol: Among the given sulphides, PbS, CuS, As_2S_3 and CdS are soluble in 50% HNO_3

Q.9 According to the following figure, the magnitude of the enthalpy change of the reaction

$$A + B \rightarrow M + N \text{ in kJ mol}^{-1}$$

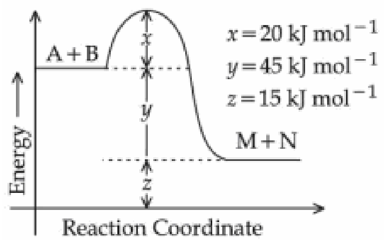
is equal to ________. (Integer answer)



Given Answer : --

Ans: 45

Sol:



$\Delta H = y$ (from the graph)
$= 45 \text{ kJ mol}^{-1}$

Q.10 The number of halogen/(s) forming halic (V) acid is ________.

Given Answer : --

Ans: 3

Sol: The halogens forming halic (V) acids are Cl, Br, I such as $HClO_3$, $HBrO_3$ and HIO_3 respectively

PART – C – MATHEMATICS

Section A

Q.1 The line $12x\cos\theta + 5y\sin\theta = 60$ is tangent to which of the following curves ?

Options
1. $x^2 + y^2 = 60$
2. $144x^2 + 25y^2 = 3600$
3. $x^2 + y^2 = 169$
4. $25x^2 + 12y^2 = 3600$

Ans: $144x^2 + 25y^2 = 3600$

Sol: $12x\cos\theta + 5y\sin\theta = 60$,Dividing by 60,

$\frac{x\cos\theta}{5} + \frac{y\sin\theta}{12} = 1$ is tangent to $\frac{x^2}{25} + \frac{y^2}{144} = 1$

$144x^2 + 25y^2 = 3600$

Q.2 cosec18° is a root of the equation :

Options
1. $x^2 + 2x - 4 = 0$
2. $x^2 - 2x + 4 = 0$
3. $4x^2 + 2x - 1 = 0$
4. $x^2 - 2x - 4 = 0$

Ans: $x^2 - 2x - 4 = 0$

Sol: $\operatorname{cosec}18^\circ = \frac{1}{\sin 18^\circ} = \frac{4}{\sqrt{5} - 1} = \frac{4(\sqrt{5}+1)}{5-1} = \sqrt{5} + 1$

Let $\operatorname{cosec}18^\circ = x = \sqrt{5} + 1$

$\Rightarrow x - 1 = \sqrt{5}$

Squaring,

$x^2 - 2x + 1 = 5$

$\Rightarrow x^2 - 2x - 4 = 0$

Q.3 If $\frac{dy}{dx} = \frac{2^{x+y} - 2^x}{2^y}$, $y(0) = 1$, then $y(1)$ is equal to :

Options
1. $\log_2(1 + e)$
2. $\log_2(1 + e^2)$
3. $\log_2(2e)$
4. $\log_2(2 + e)$

Ans: $\log_2(1+e)$

Sol: $\frac{dy}{dx} = \frac{2^x 2^y - 2^x}{2^y} = \frac{2^x(2^y - 1)}{2^y}$

$\int \frac{2^y}{2^y - 1} dy = \int 2^x dx$

$\Rightarrow \frac{\ln(2^y - 1)}{\log 2} = \frac{2^x}{\ln 2} + C$

$\Rightarrow \log_2(2^y - 1) = 2^x \log_2 e + C$

$\therefore y(0) = 1 \Rightarrow 0 = \log_2 e + C$

$C = -\log_2 e$

$\Rightarrow \log_2(2^y - 1) = 2^x \log_2 e - \log_2 e = (2^x - 1)\log_2 e$

Put x = 1, $\log_2(2^y - 1) = \log_2 e$

$2^y = e + 1$

$y = \log_2(1+e)$

Q.4 If p and q are the lengths of the perpendiculars from the origin on the lines, $x \operatorname{cosec} \alpha - y \sec \alpha = k \cot 2\alpha$ and $x \sin \alpha + y \cos \alpha = k \sin 2\alpha$ respectively, then k^2 is equal to :

Options

1. $2p^2 + q^2$
2. $4p^2 + q^2$
3. $p^2 + 4q^2$
4. $p^2 + 2q^2$

Ans: $4p^2 + q^2$

Sol: First line is $\frac{x}{\sin\alpha} - \frac{y}{\cos\alpha} = \frac{k\cos 2\alpha}{\sin 2\alpha} = \frac{k\cos 2\alpha}{2\sin\alpha\cos\alpha}$

$\Rightarrow x\cos\alpha - y\sin\alpha = \frac{k}{2}\cos 2\alpha$

$\Rightarrow p = \left|\frac{k}{2}\cos\alpha\right| \Rightarrow 2p = |k\cos 2\alpha|$ .....(i)

Second line is $x\sin\alpha + y\cos\alpha = \frac{k}{2}\cos 2\alpha$

$\Rightarrow q = |k\sin 2\alpha|$ ..(ii)

$\therefore k^2 = 4p^2 + q^2$

Q.5

If the function $f(x) = \begin{cases} \frac{1}{x}\log_e\left(\frac{1+\frac{x}{a}}{1-\frac{x}{b}}\right) &, x<0 \\ k &, x=0 \\ \frac{\cos^2 x - \sin^2 x - 1}{\sqrt{x^2+1}-1} &, x>0 \end{cases}$

is continuous at $x=0$, then $\frac{1}{a}+\frac{1}{b}+\frac{4}{k}$ is equal to :

Options
1. 5
2. 4
3. -4
4. -5

Ans: -5

Sol: $\lim_{x\to 0^+} f(x) = \lim_{x\to 0^+} \frac{\cos^2 x - \sin^2 x - 1}{\sqrt{x^2+1}-1} \cdot \frac{\left(\sqrt{x^2+1}+1\right)}{\left(\sqrt{x^2+1}+1\right)}$

$\lim_{x\to 0^+} -\frac{2\sin^2 x}{x^2}\left(\sqrt{x^2+1}+1\right)$

$\lim_{x\to 0^-} f(x) = \lim_{x\to 0^-} \frac{1}{x}\log\left(\frac{1+\frac{x}{a}}{1-\frac{x}{b}}\right)$

$\lim_{x\to 0^-} \frac{\log\left(1+\frac{x}{a}\right)}{\left(\frac{x}{a}\right).a} + \frac{\log\left(1-\frac{x}{b}\right)}{\left(-\frac{x}{b}\right).b} = \frac{1}{a}+\frac{1}{b}$

So, $\frac{1}{a}+\frac{1}{b} = -4$

$\Rightarrow \frac{1}{a}+\frac{1}{b}+\frac{4}{k} = -4-1 = -5$

Q.6 Let $*, \square \in \{\wedge, \vee\}$ be such that the Boolean expression $(p * \sim q) \Rightarrow (p \square q)$ is a tautology. Then :

Options
1. $* = \vee, \ \square = \wedge$
2. $* = \wedge, \ \square = \wedge$
3. $* = \wedge, \ \square = \vee$
4. $* = \vee, \ \square = \vee$

Ans: $* = \wedge, \ \square = \vee$

Sol:

p	q	~q	$p \wedge \sim q$	$p \vee q$	$(p \wedge \sim q) \rightarrow (p \vee q)$
T	T	F	F	T	T
T	F	T	T	T	T
F	T	F	F	T	T
F	F	T	F	F	T

$(p \wedge \sim q)$ $(p \vee q)$ is tautology

Q.7 The function $f(x) = |x^2 - 2x - 3| \cdot e^{|9x^2 - 12x + 4|}$ is not differentiable at exactly :

Options
1. one point
2. four points
3. two points
4. three points

Ans: two points

Sol: $f(x) = |(x-3)(x+1)|.e^{(3x-2)^2}$

$$f(x) = \begin{cases} (x-3)(x+1).e^{(3x-2)^2} & ;x>3 \\ -(x-3)(x+1).e^{(3x-2)^2} & ;-1 \le x \le 3 \\ (x-3).(x+1).e^{(3x-2)^2} & ;x<-1 \end{cases}$$

Clearly, f(x) is not differentiable at x = -1 and x = 3

Q.8 Which of the following is **not** correct for relation R on the set of real numbers ?

Options
1. $(x, y) \in R \Leftrightarrow |x - y| \le 1$ is reflexive and symmetric.
2. $(x, y) \in R \Leftrightarrow 0 < |x| - |y| \le 1$ is neither transitive nor symmetric.
3. $(x, y) \in R \Leftrightarrow 0 < |x - y| \le 1$ is symmetric and transitive.
4. $(x, y) \in R \Leftrightarrow |x| - |y| \le 1$ is reflexive but not symmetric.

Ans: $(x, y) \in R \Leftrightarrow 0 < |x - y| \le 1$ is symmetric and transitive

Sol: $(1, 2), (2, 3) \in R$ but $(1, 3) \notin R$
Hence $0 \le |x - y| \le 1$ is symmetric but not transitive

Q.9 Let the equation of the plane, that passes through the point $(1, 4, -3)$ and contains the line of intersection of the planes $3x-2y+4z-7=0$ and $x+5y-2z+9=0$, be $\alpha x+\beta y+\gamma z+3=0$, then $\alpha+\beta+\gamma$ is equal to :

Options
1. -15
2. 23
3. 15
4. -23

Ans: -23

Sol: Equation of plane is $3x - 2y + 4z - 7 + \lambda(x + 5y - 2z + 9) = 0$

$(3 + \lambda)x + (5\lambda - 2)y + (4 - 2\lambda)z + 9\lambda - 7 = 0$

Substituting $(1, 4, -3)$

$\Rightarrow 3 + \lambda + 20\lambda - 8 - 12 + 6\lambda + 9\lambda - 7 = 0$

$\Rightarrow \lambda = \frac{2}{3}$

$\Rightarrow -11x - 4y - 8z + 3 = 0$

$\Rightarrow \alpha + \beta + \gamma = -23$

Q.10 The sum of 10 terms of the series

$$\frac{3}{1^2 \times 2^2} + \frac{5}{2^2 \times 3^2} + \frac{7}{3^2 \times 4^2} + \ldots \text{ is :}$$

Options
1. $\frac{120}{121}$
2. 1
3. $\frac{99}{100}$
4. $\frac{143}{144}$

Ans: $\frac{120}{121}$

Sol: $S = \frac{2^2-1^2}{1^2 \times 2^2} + \frac{3^2-2^2}{2^2 \times 3^2} + \frac{4^2-3^2}{3^2 \times 4^2} + \ldots\ldots\ldots\ldots$

$= \left[\frac{1}{1^2} - \frac{1}{2^2}\right] + \left[\frac{1}{2^2} - \frac{1}{3^2}\right] + \left[\frac{1}{3^2} - \frac{1}{4^2}\right] + \ldots\ldots + \left[\frac{1}{10^2} - \frac{1}{11^2}\right] = 1 - \frac{1}{11^2} = \frac{120}{121}$

Q.11 If the following system of linear equations

$2x + y + z = 5$

$x - y + z = 3$

$x + y + az = b$

has no solution, then :

Options

1. $a \neq -\frac{1}{3}, b = \frac{7}{3}$
2. $a \neq \frac{1}{3}, b = \frac{7}{3}$
3. $a = \frac{1}{3}, b \neq \frac{7}{3}$
4. $a = -\frac{1}{3}, b \neq \frac{7}{3}$

Ans: $a = \frac{1}{3}, b \neq \frac{7}{3}$

Sol: Here $\Delta = \begin{vmatrix} 2 & 1 & 1 \\ 1 & -1 & 1 \\ 1 & 1 & a \end{vmatrix} = 7 - 3b$

For $a = \frac{1}{3}$, $b \neq \frac{7}{3}$,system has no solution

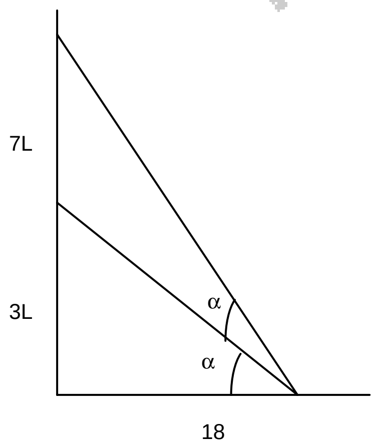
Q.12 A vertical pole fixed to the horizontal ground is divided in the ratio 3 : 7 by a mark on it with lower part shorter than the upper part. If the two parts subtend equal angles at a point on the ground 18 m away from the base of the pole, then the height of the pole (in meters) is :

Options

1. $8\sqrt{10}$
2. $12\sqrt{15}$
3. $12\sqrt{10}$
4. $6\sqrt{10}$

Ans: $12\sqrt{10}$

Sol:



Let height of pole = 10L

$\tan\alpha = \frac{3L}{18} = \frac{L}{6}$

$\tan 2\alpha = \frac{10L}{18}$

$\frac{2\tan\alpha}{1-\tan^2\alpha} = \frac{10L}{18}$

Use $\tan\alpha = \frac{L}{6} \Rightarrow L = \sqrt{\frac{72}{5}}$

Height of pole = 10L $= 12\sqrt{10}$

Q.13

The integral $\int \frac{1}{\sqrt[4]{(x-1)^3(x+2)^5}}\,dx$ is equal to :

(where C is a constant of integration)

Options

1. $\frac{4}{3}\left(\frac{x-1}{x+2}\right)^{\frac{5}{4}} + C$
2. $\frac{3}{4}\left(\frac{x+2}{x-1}\right)^{\frac{5}{4}} + C$
3. $\frac{4}{3}\left(\frac{x-1}{x+2}\right)^{\frac{1}{4}} + C$
4. $\frac{3}{4}\left(\frac{x+2}{x-1}\right)^{\frac{1}{4}} + C$

Ans: $\frac{4}{3}\left(\frac{x-1}{x+2}\right)^{\frac{1}{4}} + C$

Sol: $\int \frac{dx}{(x-1)^{3/4}(x+2)^{5/4}} = \int \frac{dx}{(x-1)^2.\left(\frac{x+2}{x-1}\right)^{5/4}}$

Put $\frac{x+2}{x-1} = t = -\frac{1}{3}\int \frac{dt}{t^{5/4}} = \frac{4}{3}.t^{-1/4} + C = \frac{4}{3}\left(\frac{x-1}{x+2}\right)^{1/4} + C$

Q.14

$\lim_{x\to 0} \frac{\sin^2(\pi\cos^4 x)}{x^4}$ is equal to :

Options

1. $4\pi^2$
2. π^2
3. $2\pi^2$
4. 4π

Ans: $4\pi^2$

Sol: $\lim\limits_{x \to 0} \dfrac{\sin^2(\pi \cos^4 x)}{x^4}$

$\lim\limits_{x \to 0} \dfrac{1 - \cos(2\pi \cos^4 x)}{2x^4}$

$\lim\limits_{x \to 0} \dfrac{1 - \cos(2\pi - 2\pi \cos^4 x)}{[2\pi(1 - \cos^4 x)]^2} 4\pi^2 . \dfrac{\sin^4 x}{2x^4} (1 + \cos^2 x)^2 = \dfrac{1}{2} . 4\pi^2 . \dfrac{1}{2} (2)^2 = 4\pi^2$

Q.15 If $a_r = \cos \frac{2r\pi}{9} + i \sin \frac{2r\pi}{9}$, $r = 1, 2, 3, \ldots$, $i = \sqrt{-1}$, then the determinant

$\begin{vmatrix} a_1 & a_2 & a_3 \\ a_4 & a_5 & a_6 \\ a_7 & a_8 & a_9 \end{vmatrix}$ is equal to :

Options
1. a_9
2. $a_2a_6 - a_4a_8$
3. $a_1a_9 - a_3a_7$
4. a_5

Ans: $a_1a_9 - a_3a_7$

Sol: $\begin{vmatrix} a_1 & a_2 & a_3 \\ a_n & a_5 & a_6 \\ a_7 & a_8 & a_9 \end{vmatrix} = \begin{vmatrix} a_1 & a_2^2 & a_1^3 \\ a_1^4 & a_1^5 & a_1^6 \\ a_1^7 & a_1^8 & a_1^9 \end{vmatrix} = a_1 . a_1^4 . a_1^7 \begin{vmatrix} 1 & a_1 & a_1^2 \\ 1 & a_1 & a_1^2 \\ 1 & a_1 & a_1^2 \end{vmatrix} = 0$

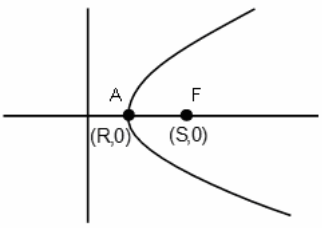
$a_1a_9 - a_3a_7 = a_1^{10} - a_1^{10} = 0$

Q.16 The length of the latus rectum of a parabola, whose vertex and focus are on the positive x-axis at a distance R and S (> R) respectively from the origin, is :

Options
1. $4(S - R)$
2. $2(S - R)$
3. $2(S + R)$
4. $4(S + R)$

Ans: 4(S – R)

Sol:



A – vertex
F – focus
A F = S – R
So latus rectum = 4(S – R)

Q.17 Three numbers are in an increasing geometric progression with common ratio r. If the middle number is doubled, then the new numbers are in an arithmetic progression with common difference d. If the fourth term of GP is $3r^2$, then $r^2 - d$ is equal to :

Options
1. $7 - 7\sqrt{3}$
2. $7 + 3\sqrt{3}$
3. $7 + \sqrt{3}$
4. $7 - \sqrt{3}$

Ans: $7 + \sqrt{3}$

Sol: The numbers are $\frac{a}{r}$, a, ar

$\frac{a}{r}$, 2a, ar are in A.P $\Rightarrow 4a = \frac{a}{r} + ar \Rightarrow r + \frac{1}{r} = 4$

Solving, $r = 2 \pm \sqrt{3}$

4^{th} form of G.P = $3r^2 \Rightarrow ar^2 = 3r^2 \Rightarrow a = 3$

$r = 2 + \sqrt{3}, a = 3, d = 2a - \frac{a}{r} = 3\sqrt{3}$

$r^2 - d = (2 + \sqrt{3})^2 - 3\sqrt{3} = 7 + \sqrt{3}$

Q.18 Let $\vec{a}$ and $\vec{b}$ be two vectors such that $|2\vec{a} + 3\vec{b}| = |3\vec{a} + \vec{b}|$ and the angle between $\vec{a}$ and $\vec{b}$ is 60°. If $\frac{1}{8}\vec{a}$ is a unit vector, then $|\vec{b}|$ is equal to :

Options
1. 8
2. 5
3. 4
4. 6

Ans: 5

Sol: $|\vec{a}| = 8$

$|3\vec{a} + \vec{b}|^2 = |2\vec{a} + 3\vec{b}|^2$

$9\vec{a}.^2 + 6\vec{a}.\vec{b} + \vec{b}^2 = 4\vec{a}.^2 + 12\vec{a}.\vec{b} + 9.\vec{b}.^2$

$5|\vec{a}|^2 - 6\vec{a}.\vec{b} = 8.|\vec{b}|^2$

$5|8|^2 - 6.8.\vec{b}\cos 60^\circ = 8.|\vec{b}|^2$

$$40 - 3.|\vec{b}| = |\vec{b}|^2$$

$$\Rightarrow |\vec{b}|^2 + 3.|\vec{b}| - 40 = 0$$

$$\Rightarrow |\vec{b}| = 5$$

Q.19 The number of real roots of the equation $e^{4x} + 2e^{3x} - e^{x} - 6 = 0$ is :

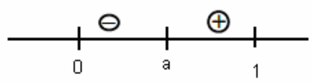
Options
1. 1
2. 2
3. 0
4. 4

Ans: 1

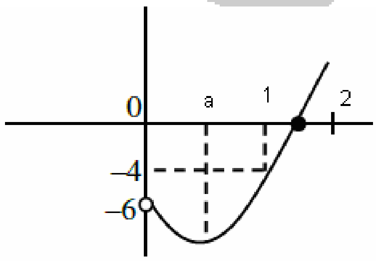
Sol: let $e^x = t > 0$

$y = t^4 + 2t^3 - t - 6 = 0$

$\frac{dy}{dt} = 4t^3 + 6t^2 - 1$



$\frac{d^2y}{dt^2} = 12t^2 + 12t > 0$



$f(0) = -6, f(1) = -4, f(2) = 24$

$\Rightarrow$ Number of real roots = 1

Q.20 Let f be a non-negative function in [0, 1] and twice differentiable in (0, 1). If $\int_0^x \sqrt{1-(f'(t))^2}\, dt = \int_0^x f(t)\, dt$, $0 \le x \le 1$ and $f(0) = 0$, then $\lim_{x \to 0} \frac{1}{x^2} \int_0^x f(t)\, dt$:

Options
1. equals $\frac{1}{2}$
2. equals 0
3. equals 1
4. does not exist

Ans: equals $\frac{1}{2}$

Sol: $\int_0^x \sqrt{1-(f'(t))^2}\,dt = \int_0^x f(t)\,dt \quad 0 \le x \le 1$

Differentiating

$\sqrt{1-(f'(x))^2} = f(x)$

$\Rightarrow 1-(f'(x))^2 = [f(x)]^2$

$\frac{f'(x)}{\sqrt{1-[f(x)]^2}} = 1$

$\sin^{-1} f(x) = x + C$

$\because f(0) = 0 \Rightarrow C = 0 \Rightarrow f(x) = \sin x$

$\therefore \lim_{x\to 0} \frac{\int_0^x \sin t\,dt}{x^2} = \lim_{x\to 0}\left(\frac{\sin x}{2x}\right) = \frac{1}{2}$

Section B

Q.1 The mean of 10 numbers

$7\times 8,\ 10\times 10,\ 13\times 12,\ 16\times 14,\ \ldots$ is __________.

Given --
Answer :

Ans: 398

Sol: $a_n = (3n + 4)(2n + 6) = 2(3n + 4)(n + 3) = 6n^2 + 26n + 24$

$S_{10} = \sum_{n=1}^{10} a_n = 6\sum_{n=1}^{10} n^2 + 26\sum_{n=1}^{10} n + 24\sum_{n=1}^{10} 1 = \frac{6(10\times 11\times 21)}{6} + 26\times\frac{10\times 11}{2} + 24\times 10 = 3980$

Mean $= \frac{S_{10}}{10} = \frac{3980}{10} = 398$

Q.2 Let [t] denote the greatest integer $\le$ t. Then the value of $8\cdot\int_{-\frac{1}{2}}^{1} \left([2x] + |x|\right)dx$ is __________.

Given **5**
Answer :

Ans: 5

Sol: $I = \int_{-1/2}^{1}([2x] + |x|)dx = \int_{-1/2}^{1}[2x]dx + \int_{-1/2}^{1}|x|dx = 0 + \int_{-1/2}^{1}(-x)dx + \int_{-1/2}^{1}x\,dx$

$= \left(-\frac{x^2}{2}\right)_{-1/2}^{0} + \left(\frac{x^2}{2}\right)_0^1 = \left(0 + \frac{1}{8}\right) + \frac{1}{2} = \frac{5}{8}$

$8I = 5$

Q.3 An electric instrument consists of two units. Each unit must function independently for the instrument to operate. The probability that the first unit functions is 0.9 and that of the second unit is 0.8. The instrument is switched on and it fails to operate. If the probability that only the first unit failed and second unit is functioning is p, then 98 p is equal to ________.

Given Answer : **28**

Ans: 28

Sol: P(A) = 0.9, P(B) = 0.8

$P(A') = 0.1, P(B') = 0.2$

$$P = \frac{0.8 \times 0.1}{0.1 \times 0.2 + 0.9 \times 0.2 + 0.1 \times 0.8} = \frac{8}{28}$$

$$98P = \frac{8}{28} \times 98 = 28$$

Q.4 The square of the distance of the point of intersection of the line $\frac{x-1}{2} = \frac{y-2}{3} = \frac{z+1}{6}$ and the plane $2x - y + z = 6$ from the point $(-1, -1, 2)$ is ________.

Given Answer : **61**

Ans: 61

Sol: The given line is $\frac{x-1}{2} = \frac{y-2}{3} = \frac{z+1}{6} = \lambda$

$x = 2\lambda + 1$, $y = 3\lambda + 2$, $z = 6\lambda - 1$

Substituting in the equation of the plane

$2(2\lambda + 1) - (3\lambda + 2) + (6\lambda - 1)$

$7\lambda = 7 \Rightarrow \lambda = 1$

Point: (3, 5, 5)

$(d)^2 = (3+1)^2 + (5+1)^2 + (5-2)^2 = 61$

Q.5 If 'R' is the least value of 'a' such that the function $f(x) = x^2 + ax + 1$ is increasing on [1, 2] and 'S' is the greatest value of 'a' such that the function $f(x) = x^2 + ax + 1$ is decreasing on [1, 2], then the value of $|R - S|$ is ________.

Given Answer : **--**

Ans: 2

Sol: $f(x) = x^2 + ax + 1$

$f'(x) = 2x + a$

$2x + a \geq 0 \ \forall x \in [1, 2]$

$a \geq -2x \ \forall x \in [1, 2]$

$R = -4$

and $2x + a \leq 0 \ \forall x \in [1, 2]$

$a \leq -2x \ \forall x \in [1, 2]$

$S = -2$

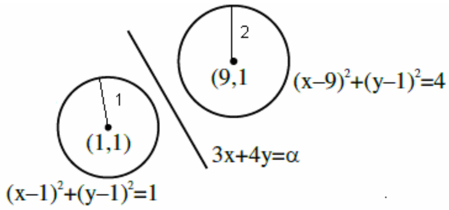
$|R - S| = |-4 + 2| = 2$

Q.6 If the variable line $3x+4y=\alpha$ lies between the two circles $(x-1)^2+(y-1)^2=1$ and $(x-9)^2+(y-1)^2=4$, without intercepting a chord on either circle, then the sum of all the integral values of α is ________.

Given --
Answer :

Ans: 165

Sol:



The points (1, 1) and (9, 1) lie on the opposite sides of the line $3x + 4y = \alpha$

$(3+4-\alpha).(27+4-\alpha) < 0$

$(7-\alpha)(31-\alpha) < 0 \Rightarrow \alpha \in (7, 31)$ ----- (1)

d_1= distance of (1, 1) from line

d_2 = distance of (9, 1) from line

$$d_1 \ge r_1 \Rightarrow \frac{|7-\alpha|}{5} \ge 1 \Rightarrow \alpha \in (-\infty,2] \cup [12,\infty) \text{----- (2)}$$

$$d_2 \ge r_2 \Rightarrow \frac{|31-\alpha|}{5} \ge 2 \Rightarrow \alpha \in (-\infty,21] \cup [41,\infty) \text{ ----- (3)}$$

From $(1),(2)$ and $(3) \Rightarrow \alpha \in [12,21]$

Sum of integers = 165

Q.7 If $\left(\frac{3^6}{4^4}\right)k$ is the term, independent of x, in the binomial expansion of $\left(\frac{x}{4}-\frac{12}{x^2}\right)^{12}$, then k is equal to ________.

Given **495**
Answer :

Ans: 55

Sol: $T_{r+1} = (-1)^r.{}^{12}C_r\left(\frac{x}{4}\right)^{12-r}\left(\frac{12}{x^2}\right)^r = (-1)^r.{}^{12}C_r\left(\frac{1}{4}\right)^{12-r}(12)^r.(x)^{12-3r}$

Term independent of x $\Rightarrow 12 - 3r = 0 \Rightarrow r = 4$

$$T_5 = (-1)^4.{}^{12}C_r\left(\frac{1}{4}\right)^8(12)^4 = \frac{3^6}{4^4}.k$$

$\Rightarrow k = 55$

Q.8 The number of six letter words (with or without meaning), formed using all the letters of the word 'VOWELS', so that all the consonants never come together, is ________.

Given **576**
Answer :

Ans: 576

Sol: VOWELS → 2 Vowels, 4 Consonants

Total arrangements = 6

No. of arrangements in which consonants are together = $3! \times 4!$

$\therefore$ Required answer = 6! – 3! 4! = 576

Q.9 A point z moves in the complex plane such that $\arg\left(\frac{z-2}{z+2}\right) = \frac{\pi}{4}$, then the minimum value of $\left|z - 9\sqrt{2} - 2i\right|^2$ is equal to __________.

Given -- Answer :

Ans: 98

Sol: Let z = x + iy

$$\arg\left(\frac{(x-2)+iy}{(x+2)+iy}\right) = \frac{\pi}{4}$$

$$\arg(x-2+iy) - \arg(x+2+iy) = \frac{\pi}{4} \quad \left[\because \arg\left(\frac{z_1}{z_2}\right) = \arg z_1 - \arg z_2\right]$$

$$\tan^{-1}\left(\frac{y}{x-2}\right) - \tan\left(\frac{y}{x+2}\right) = \frac{\pi}{4}$$

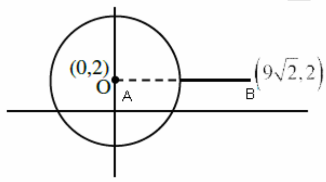
$$\frac{\frac{y}{x-2} - \frac{y}{x+2}}{1+\left(\frac{y}{x-2}\right)\left(\frac{y}{x+2}\right)} = \tan\frac{\pi}{4} = 1$$

$$\frac{xy + 2y - xy + 2y}{x^2 - 4y + y^2} = 1$$

$$4y = x^2 - 4y - 4 = 0$$

$$x^2 + y^2 - 4y - 4 = 0$$

Locus is a circle with center (0, 2) & radius = $2\sqrt{2}$



Minimum value = $(AB)^2 = (OB - OA)^2 = \left(9\sqrt{2} - 2\sqrt{2}\right)^2 = \left(7\sqrt{2}\right)^2 = 98$

Q.10 If $x\,\phi(x) = \int_5^x \left(3t^2 - 2\phi'(t)\right)dt,\ x > -2$, and $\phi(0) = 4$, then $\phi(2)$ is __________.

Given -- Answer :

Ans: 4

Sol: $x\,\phi(x) = \int_5^x 3t^2 - 2\phi'(t)dt$

Integrating, we get $x\,\phi(x) = x^3 - 125 - 2[\phi(x) - \phi(5)]$

$x\,\phi(x) = x^3 - 125 - 2\phi(x) - 2\phi(5)$

putting x = 0

$\phi(0) = 4\ \phi(5) = -\frac{133}{2}$

$\Rightarrow \phi\,(x) = \frac{x^3 + 8}{x + 2}$

$\Rightarrow \phi\,(2) = 4$