SOLUTIONS & ANSWERS FOR JEE MAINS-2021
25th February Shift 1

[PHYSICS, CHEMISTRY & MATHEMATICS]

PART – A – PHYSICS

SECTION A

Q.1 Two radioactive substances X and Y originally have N_1 and N_2 nuclei respectively. Half life of X is half of the half life of Y. After three half lives of Y, number of nuclei of both are equal.

The ratio $\frac{N_1}{N_2}$ will be equal to :

Options
1. $\frac{8}{1}$
2. $\frac{1}{3}$
3. $\frac{1}{8}$
4. $\frac{3}{1}$

Ans: 1

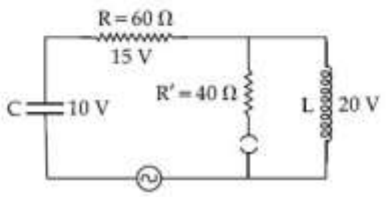
Sol: $N = \frac{N_0}{(2)^{t/T}} = \frac{N_0}{2^n}$

$N = \frac{N_1}{2^{n_1}} = \frac{N_2}{2^{n_2}}$

Given $n_1 = 2$ $n_2 = 6$

$\frac{N_1}{N_2} = 2^{(n_1 - n_2)} = 2^{6-3} = 2^3 = 8$

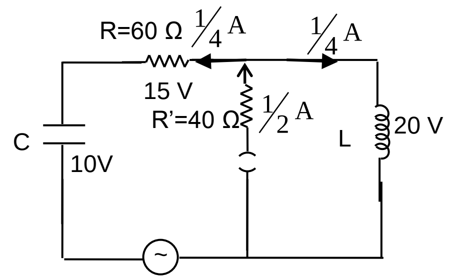
Q.2 The angular frequency of alternating current in a L-C-R circuit is 100 rad/s. The components connected are shown in the figure. Find the value of inductance of the coil and capacity of condenser.



Options
1. 0.8 H and 250 μF
2. 0.8 H and 150 μF
3. 1.33 H and 150 μF
4. 1.33 H and 250 μF

Ans: 1

Sol:



Current through resistance = $20/40 = 1/2$ A

Voltage across capacitor = 10 V

Current through capacitor = $1/4$ A

$$\frac{1}{\omega C}\times\frac{1}{4}=10$$

$$\Rightarrow C = 250\,\mu F$$

Voltage across inductor r = 20 V

Current through inductor = $1/4$ A

$$\omega L\times\frac{1}{4}=20 \Rightarrow L = 0.8\,H$$

Q.3 An engine of a train, moving with uniform acceleration, passes the signal-post with velocity u and the last compartment with velocity v. The velocity with which middle point of the train passes the signal post is :

Options

1. $\frac{u+v}{2}$
2. $\frac{v-u}{2}$
3. $\sqrt{\frac{v^2+u^2}{2}}$
4. $\sqrt{\frac{v^2-u^2}{2}}$

Ans: 3

Sol: $V^2 = V_c^2 + 2a\ell$ --------(1)

$V_c^2 = u^2 + 2a\ell$ --------(2)

(1) –(2) $\Rightarrow V^2 - V_c^2 = V_c^2 - u^2$

$$V_c\sqrt{\frac{u^2+v^2}{2}}$$

Q.4 Given below are two statements :

Statement I : A speech signal of 2 kHz is used to modulate a carrier signal of 1 MHz. The bandwidth requirement for the signal is 4 kHz.

Statement II : The side band frequencies are 1002 kHz and 998 kHz.

In the light of the above statements, choose the correct answer from the options given below :

Options
1. Both Statement I and Statement II are false
2. Statement I is true but Statement II is false
3. Both Statement I and Statement II are true
4. Statement I is false but Statement II is true

Ans: 3

Sol: Maximum frequency of modulated signal = carrier wave frequency + signal frequency
Minimum frequency of modulated signal = carrier wave frequency – signal frequency

Q.5 If the time period of a two meter long simple pendulum is 2 s, the acceleration due to gravity at the place where pendulum is executing S.H.M. is :

Options
1. $2\pi^2\ ms^{-2}$
2. $\pi^2\ ms^{-2}$
3. $9.8\ ms^{-2}$
4. $16\ m/s^2$

Ans: 1

Sol: $T = 2\pi\sqrt{\frac{\ell}{g_{planet}}}$

$$g_{planet} = \frac{4\pi^2\ell}{T^2} = 2\pi^2\ m/s^2$$

Q.6 An α particle and a proton are accelerated from rest by a potential difference of 200 V. After this, their de Broglie wavelengths are λ_α and λ_p respectively. The ratio $\frac{\lambda_p}{\lambda_\alpha}$ is :

Options
1. 2.8
2. 7.8
3. 8
4. 3.8

Ans: 1

Sol:
$$\lambda = \frac{h}{p}$$

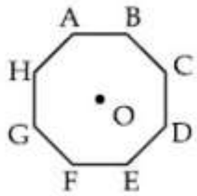
$$\lambda = \frac{h}{\sqrt{2mqv}}$$

$$\frac{\lambda_\rho}{\lambda_\alpha} = \sqrt{\frac{4m \times 2e}{m \times e}} = 2\sqrt{2}$$

Q.7 In an octagon ABCDEFGH of equal side, what is the sum of

$$\vec{AB}+\vec{AC}+\vec{AD}+\vec{AE}+\vec{AF}+\vec{AG}+\vec{AH},$$

if, $\vec{AO}=2\hat{i}+3\hat{j}-4\hat{k}$

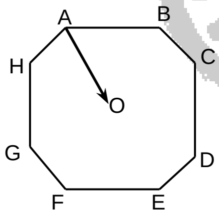


Options

1. $-16\hat{i}-24\hat{j}+32\hat{k}$
2. $16\hat{i}+24\hat{j}-32\hat{k}$
3. $16\hat{i}-24\hat{j}+32\hat{k}$
4. $16\hat{i}+24\hat{j}+32\hat{k}$

Ans: 2

Sol:



$\vec{AO}=2\hat{i}+3\hat{j}+4\hat{k}$

$\vec{AB}+\vec{AC}+\vec{AD}+\vec{AE}+\vec{AF}+\vec{AG}+\vec{AH}$

$\vec{AC}=\vec{AB}+\vec{BC}$

$$\Rightarrow \vec{AB}+\left(\vec{AB}+\vec{BC}\right)+\left(\vec{AB}+\vec{BC}+\vec{CD}\right)+\left(\vec{AB}+\vec{BC}+\vec{CD}+\vec{DE}\right)+\left(\vec{AB}+\vec{BC}+\vec{CD}+\vec{DE}+\vec{EF}\right)$$
$$+\left(\vec{AB}+\vec{BC}+\vec{CD}+\vec{DE}+\vec{EF}+\vec{FG}\right)+\left(\vec{AB}+\vec{BC}+\vec{CD}+\vec{DE}+\vec{EF}+\vec{FG}+\vec{GH}\right)$$

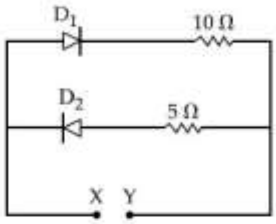
$$\begin{bmatrix} \vec{EF}=-\vec{AB} \\ \vec{FG}=-\vec{BC} \\ \vec{GH}=-\vec{CD} \\ \vec{HA}=-\vec{DE} \end{bmatrix}$$

$$\Rightarrow \vec{AB}+\left(\vec{AB}+\vec{BC}\right)+\left(\vec{AB}+\vec{BC}+\vec{CD}\right)+\left(\vec{AB}+\vec{BC}+\vec{CD}+\vec{DE}\right)+\left(\vec{BC}+\vec{CD}+\vec{DE}\right)$$
$$+\left(\vec{CD}+\vec{DE}\right)+\left(\vec{DE}\right)$$

$$\Rightarrow 4\times\left(\vec{AB}+\vec{BC}+\vec{CD}+\vec{DE}\right)$$

$$\Rightarrow 4\times\vec{AE}-4\times 2\vec{AO}=8(2\hat{\imath}+3\hat{\jmath}-4\hat{k})=16\hat{i}+24\hat{j}-32\hat{k}$$

Q.8 A 5 V battery is connected across the points X and Y. Assume D_1 and D_2 to be normal silicon diodes. Find the current supplied by the battery if the +ve terminal of the battery is connected to point X.



Options
1. ~0.5 A
2. ~1.5 A
3. ~0.86 A
4. ~0.43 A

Ans: 4

Sol: $i=\dfrac{5-0.7}{10}=0.43A$

Q.9 Match List - I with List - II :

	List - I		List - II
(a)	h (Planck's constant)	(i)	$[M L T^{-1}]$
(b)	E (kinetic energy)	(ii)	$[M L^2 T^{-1}]$
(c)	V (electric potential)	(iii)	$[M L^2 T^{-2}]$
(d)	P (linear momentum)	(iv)	$[M L^2 I^{-1} T^{-3}]$

Choose the correct answer from the options given below :

Options
1. (a) → (ii), (b) → (iii), (c) → (iv), (d) → (i)
2. (a) → (iii), (b) → (iv), (c) → (ii), (d) → (i)
3. (a) → (iii), (b) → (ii), (c) → (iv), (d) → (i)
4. (a) → (i), (b) → (ii), (c) → (iv), (d) → (iii)

Ans: 1

Sol: Option (1)

Q.10 A student is performing the experiment of resonance column. The diameter of the column tube is 6 cm. The frequency of the tuning fork is 504 Hz. Speed of the sound at the given temperature is 336 m/s. The zero of the metre scale coincides with the top end of the resonance column tube. The reading of the water level in the column when the first resonance occurs is :

Options

1. 18.4 cm
2. 13 cm
3. 14.8 cm
4. 16.6 cm

Ans: 3

Sol: $V = f\lambda \quad \Rightarrow \lambda = \frac{V}{f} = \frac{336}{504}$

$\ell + e = \lambda/4$

$[\ell + (0.3 \times 6)] \times 10^{-2} = \frac{336}{4 \times 504} \qquad \ell = 14.87 \text{ cm}$

Q.11 Two coherent light sources having intensity in the ratio $2x$ produce an interference pattern. The ratio $\frac{I_{max} - I_{min}}{I_{max} + I_{min}}$ will be :

Options

1. $\frac{\sqrt{2x}}{2x+1}$
2. $\frac{\sqrt{2x}}{x+1}$
3. $\frac{2\sqrt{2x}}{x+1}$
4. $\frac{2\sqrt{2x}}{2x+1}$

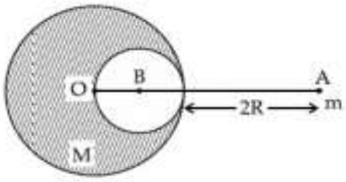
11. Ans: 4

Sol: $\frac{I_{max} - I_{min}}{I_{max} + I_{min}} = \frac{(\sqrt{I_2} + \sqrt{I_1})^2 - (\sqrt{I_2} - \sqrt{I_1})^2}{(\sqrt{I_2} + \sqrt{I_1})^2 + (\sqrt{I_2} - \sqrt{I_1})^2} = \frac{\left(\sqrt{\frac{I_2}{I_1}} + 1\right)^2 - \left(\sqrt{\frac{I_2}{I_1}} - 1\right)^2}{\left(\sqrt{\frac{I_2}{I_1}} + 1\right)^2 + \left(\sqrt{\frac{I_2}{I_1}} - 1\right)^2}$

$\frac{(\sqrt{2x} + 1)^2 - (\sqrt{2x} - 1)^2}{(\sqrt{2x} + 1)^2 + (\sqrt{2x} - 1)^2} = \frac{(2x + 1 + 2\sqrt{2x}) - (2x + 1 - 2\sqrt{2x})}{(2x + 1 + 2\sqrt{2x}) + (2x + 1 - 2\sqrt{2x})}$

$\frac{4\sqrt{2x}}{4x + 2} = \frac{2\sqrt{2x}}{2x + 1}$

Q.12 A solid sphere of radius R gravitationally attracts a particle placed at 3R from its centre with a force F_1. Now a spherical cavity of radius $\left(\frac{R}{2}\right)$ is made in the sphere (as shown in figure) and the force becomes F_2. The value of $F_1 : F_2$ is :



Options
1. 50 : 41
2. 41 : 50
3. 36 : 25
4. 25 : 36

Ans: 1

Sol: $F_1 = \frac{GmM}{9R^2}$

$$F_2 = F_1 - \frac{\frac{GM}{8}m}{\left(\frac{5}{2}R\right)^2} = \frac{GMm}{R^2}\left(\frac{1}{9} - \frac{1}{50}\right) \Rightarrow \frac{41}{9\times 50}\frac{GMm}{R^2}$$

Q.13 A diatomic gas, having $C_P = \frac{7}{2}R$ and $C_V = \frac{5}{2}R$, is heated at constant pressure. The ratio $dU : dQ : dW$:

Options
1. 3 : 5 : 2
2. 5 : 7 : 3
3. 5 : 7 : 2
4. 3 : 7 : 2

Ans: 3

Sol: $\Delta U = nC_V\Delta T = n\left(\frac{5R}{2}\right)\Delta T$

$\Delta W = nR\Delta T$

$\Delta Q = nC_p\Delta T = n\left(\frac{7R}{2}\right)\Delta T$

$\Delta U : \Delta Q : \Delta W = 5 : 7 : 2$

Q.14 A proton, a deuteron and an α particle are moving with same momentum in a uniform magnetic field. The ratio of magnetic forces acting on them is ________ and their speed is ________, in the ratio.

Options
1. 1 : 2 : 4 and 1 : 1 : 2
2. 4 : 2 : 1 and 2 : 1 : 1
3. 2 : 1 : 1 and 4 : 2 : 1
4. 1 : 2 : 4 and 2 : 1 : 1

Ans: 3

Sol: $F = qvB = \frac{q(mv)B}{m} \propto \frac{q}{m}$

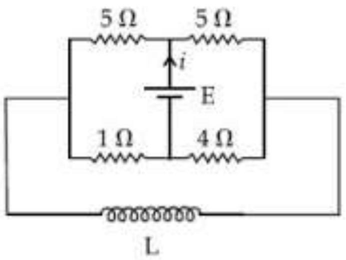
$F_1 : F_2 : F_3 = \frac{1}{1} : \frac{1}{2} : \frac{2}{4}$

$= 4 : 2 : 2 = 2 : 1 : 1$

$P = mV \Rightarrow V = P/m \propto 1/m$

$V_1 : V_2 : V_3 = \frac{1}{1} : \frac{1}{2} : \frac{1}{4} = 4 : 2 : 1$

Q.15 The current (i) at time $t = 0$ and $t = \infty$ respectively for the given circuit is :

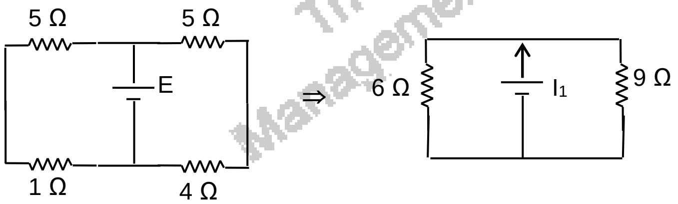


Options

1. $\frac{5E}{18}, \frac{18E}{55}$
2. $\frac{5E}{18}, \frac{10E}{33}$
3. $\frac{18E}{55}, \frac{5E}{18}$
4. $\frac{10E}{33}, \frac{5E}{18}$

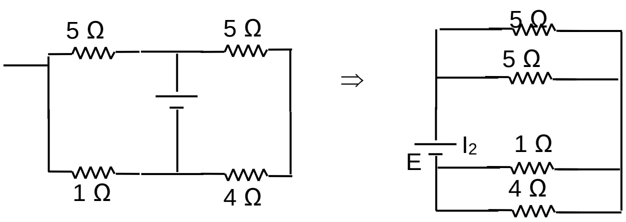
Ans: 2

Sol: At t = 0, inductor offers infinite resistance



$\therefore R_{eq} = \frac{6 \times 9}{6 + 9} = \frac{18}{5} \Omega$ and $I_1 = \frac{E}{18/5} = \frac{5E}{18}$

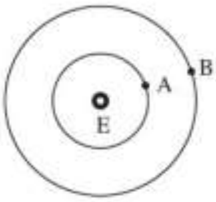
At t = ∝, inductor offers zero resistance



$$Req = \frac{5}{2} + \frac{4 \times 1}{5} = \frac{5}{2} + \frac{4}{5} = \frac{25+8}{10} = \frac{33}{10}$$

$$I_2 = \frac{E}{33/10} = 10E/33$$

Q.16 Two satellites A and B of masses 200 kg and 400 kg are revolving round the earth at height of 600 km and 1600 km respectively.
If T_A and T_B are the time periods of A and B respectively then the value of $T_B - T_A$:

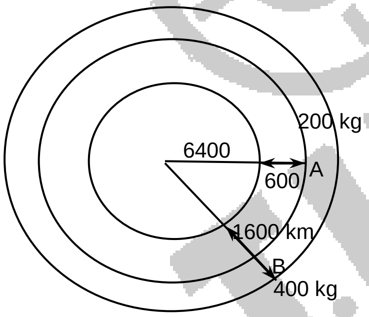


[Given : radius of earth = 6400 km, mass of earth = 6×10^{24} kg]

Options
1. 4.24×10^2 s
2. 1.33×10^3 s
3. 3.33×10^2 s
4. 4.24×10^3 s

Ans: 2

Sol:



$$T = \frac{2\pi (R)^{3/2}}{\sqrt{GM}}$$

$$T_B - T_A = \frac{2\pi}{\sqrt{GM}}\left[(8000 \times 10^3)^{3/2} - (7000 \times 10^3)^{3/2}\right]$$

$$= 1.287 \times 10^3$$

Q.17 The pitch of the screw gauge is 1 mm and there are 100 divisions on the circular scale. When nothing is put in between the jaws, the zero of the circular scale lies 8 divisions below the reference line. When a wire is placed between the jaws, the first linear scale division is clearly visible while 72^{nd} division on circular scale coincides with the reference line. The radius of the wire is :

Options
1. 0.82 mm
2. 1.80 mm
3. 1.64 mm
4. 0.90 mm

Ans: 1

Sol: **Lest count =** $\frac{\text{pitch}}{\text{number of division}} = \frac{1}{100}$

$\text{Error} = 8 \times \frac{1}{100}$

$\text{Reading } (2R) = 1 + 72 * \frac{1}{100} - 8 \times \frac{1}{100}$

2R = 1.64

R = $\frac{1.64}{2} = 0.82\,\text{mm}$

Q.18 Given below are two statements : one is labelled as Assertion A and the other is labelled as Reason R.

Assertion A : The escape velocities of planet A and B are same. But A and B are of unequal mass.

Reason R : The product of their mass and radius must be same. $M_1R_1 = M_2R_2$

In the light of the above statements, choose the most appropriate answer from the options given below :

Options

1. A is not correct but R is correct
2. Both A and R are correct and R is the correct explanation of A
3. A is correct but R is not correct
4. Both A and R are correct but R is NOT the correct explanation of A

Ans: **3**

Sol: $V_e = \sqrt{\frac{2GM}{R}}$

$\Rightarrow \frac{M_1}{R_1} = \frac{M_2}{R_2}$

Q.19 Given below are two statements : one is labelled as Assertion A and the other is labelled as Reason R.

Assertion A : When a rod lying freely is heated, no thermal stress is developed in it.

Reason R : On heating, the length of the rod increases.

In the light of the above statements, choose the correct answer from the options given below :

Options

1. A is false but R is true
2. Both A and R are true and R is the correct explanation of A
3. Both A and R are true but R is NOT the correct explanation of A
4. A is true but R is false

Ans: **3**

Sol: Option (3)

Q.20 Magnetic fields at two points on the axis of a circular coil at a distance of 0.05 m and 0.2 m from the centre are in the ratio 8 : 1. The radius of coil is ________.

Options
1. 0.1 m
2. 0.15 m
3. 0.2 m
4. 1.0 m

Ans: 1

Sol: $B = \frac{\mu_0 NIR^2}{2(x^2+R^2)^{3/2}}$

$\frac{B_1}{B_2} = 8$

$$\frac{\frac{\mu_0 NIR^2}{2(x_1^2+R^2)^{3/2}}}{\frac{\mu_0 NIR^2}{2(x_2^2+R^2)^{3/2}}} = 8$$

$(0.2)^2 + R^2 = 4\ (0.05^2 + R^2)$

$\frac{4}{100} + R^2 = 4\left(\frac{25}{100\times100} + R^2\right)$

$\frac{4}{100} + R^2 = \frac{1}{100} + 4R^2$

$\frac{3}{100} = 3R^2 \Rightarrow R = \frac{1}{10} = 0.1\,\text{m}$

SECTION B

Q.1 In a certain thermodynamical process, the pressure of a gas depends on its volume as kV^3. The work done when the temperature changes from 100°C to 300°C will be ________ nR, where n denotes number of moles of a gas.

Ans: 50.00

Sol: $W \int P\,dV = \int KV^3\ .\ dV$

$= K\left[\frac{V^2}{4}\right]_{V_1}^{V_2}$

$= \frac{KV_2^4 - KV_1^4}{4}$

$KV_2^3 = P_2,\ KV_1^3 = P_1,\ KV_2^4 = P_2V_2,\ KV_1^4 = P_1V_1$

$KV_2^4 - KV_1^4 = nR(T_2 - T_1)$

$\Rightarrow W = nR(T_2 - T_1)$

$= \frac{nR(300-100)}{4} = 50\ nR$

Q.2 The potential energy (U) of a diatomic molecule is a function dependent on r (interatomic distance) as

$$U=\frac{\alpha}{r^{10}}-\frac{\beta}{r^5}-3$$

where, α and β are positive constants. The equilibrium distance between two atoms will be $\left(\frac{2\alpha}{\beta}\right)^{\frac{a}{b}}$, where a = ________ .

Ans: **1.00**

Sol: $U=\frac{\alpha}{r^{10}}-\frac{\beta}{r^5}-3$

$F=\frac{-dU}{dr}=\frac{\alpha(-10)}{r^{11}}=\frac{\beta(-5)}{r^6}$

At equilibrium, F = 0

$\frac{\alpha(10)}{r^{11}}=\frac{5\beta}{r^6}$

$r^5=\frac{10\alpha}{5\beta}$

$r=\left(\frac{2\alpha}{\beta}\right)^{1/5}=\left(\frac{2\alpha}{\beta}\right)^{a/b}$

$\frac{a}{b}=\frac{1}{5}$

$\therefore a=1,\ b=5$

Q.3 512 identical drops of mercury are charged to a potential of 2 V each. The drops are joined to form a single drop. The potential of this drop is ________ V.

Ans: **128.00**

Sol: Let R → radius of bigger drop

r → radius of small drop

$V_{big} = 512\ V_{smaller}$

$\frac{4}{3}\pi R^3 = 512\times\frac{4}{3}\pi r^3$

R = 8r

$V = K\,q/r$

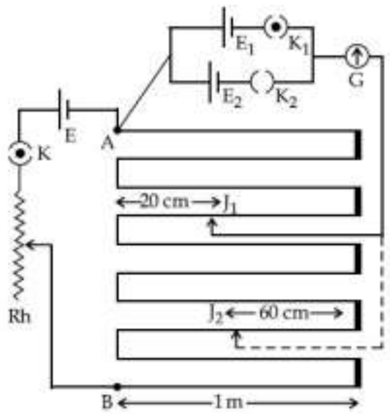
$2=K\,q/r \Rightarrow q=\frac{2r}{K}$

Q = 512 q

$Q=\frac{1024\,r}{K}$

$V'=\frac{KQ}{r}=\frac{K\times\frac{1024\,r}{K}}{8r}=128\text{ V}$

Q.4 In the given circuit of potentiometer, the potential difference E across AB (10 m length) is larger than E_1 and E_2 as well. For key K_1 (closed), the jockey is adjusted to touch the wire at point J_1 so that there is no deflection in the galvanometer. Now the first battery (E_1) is replaced by second battery (E_2) for working by making K_1 open and K_2 closed. The galvanometer gives then null deflection at J_2. The value of $\frac{E_1}{E_2}$ is $\frac{a}{b}$, where a = ________ .



Ans: 1.00

Sol: $E \propto \ell$

$E_1 = K(380)$

$E_2 = K(760)$

$$\frac{E_1}{E_2} = \frac{1}{2} = \frac{a}{b}$$

$$\Rightarrow a = 1$$

Q.5 A monoatomic gas of mass 4.0 u is kept in an insulated container. Container is moving with velocity 30 m/s. If container is suddenly stopped then change in temperature of the gas (R = gas constant) is $\frac{x}{3R}$. Value of x is ________.

Ans: 3600.00

Sol: $$\frac{1}{2} \times n \times MV^2 = nC_V\Delta T = n \times \frac{3}{2} \times R\,\Delta T$$

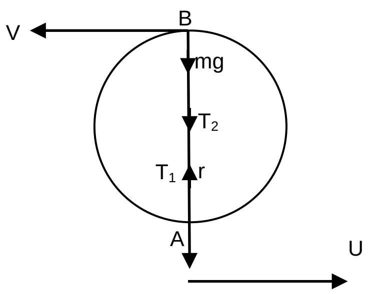
$$\Delta T = \frac{MV^2}{3R} = \frac{4 \times 30 \times 30}{3R} = \frac{3600}{3R}$$

$$\Rightarrow x = 3600$$

Q.6 A small bob tied at one end of a thin string of length 1 m is describing a vertical circle so that the maximum and minimum tension in the string are in the ratio 5 : 1. The velocity of the bob at the highest position is ________ m/s. (Take g = 10 m/s^2)

Ans: 5.00

Sol:



$T_{max} - T_{min} = 6$ mg
Given, $T_{max}/T_{min} = 5$
On solving, we get

$$T_{max} = \frac{15}{2} mg$$

$$T_{min} = \frac{3}{2} mg$$

$$T_2 = T_{min} = \frac{mV^2}{r} - mg$$

$$\frac{5}{2} mg = \frac{mV^2}{r}$$

Q.7 A transmitting station releases waves of wavelength 960 m. A capacitor of 2.56 μF is used in the resonant circuit. The self inductance of coil necessary for resonance is ________ $\times 10^{-8}$ H.

Ans: 10.00

Sol: $V = n\lambda \Rightarrow n = \frac{V}{\lambda} = \frac{3\times10^8}{960}$

At resonance,

$$\omega L = \frac{1}{\omega C}$$

$$\Rightarrow L = \frac{1}{\omega^2 C} = \frac{960 \times 960}{4\pi^2 \times 9 \times 10^{16} \times 256 \times 10^{-6}} = 10 \times 10^{-8} \text{Hz}$$

Q.8 A coil of inductance 2 H having negligible resistance is connected to a source of supply whose voltage is given by $V = 3t$ volt. (where t is in second). If the voltage is applied when $t = 0$, then the energy stored in the coil after 4 s is ________ J.

Ans: 144.00

Sol: $V = L\frac{di}{dt}$

$$\Rightarrow i = \int_0^4 \frac{V}{L} dt$$

$$i = \int_0^4 \frac{3t}{2} dt$$

$$= \left[\frac{3t^2}{4}\right]_0^4 = 12$$

$$E = \frac{1}{2} Li^2 = \frac{1}{2} \times 2 \times 12^2 = 144 \text{ J}$$

Q.9 The same size images are formed by a convex lens when the object is placed at 20 cm or at 10 cm from the lens. The focal length of convex lens is ________ cm.

Ans: 15.00

Sol: $|m_1| = |m_2|$ (since the size of image is same)

$$\left|\frac{f}{f+u_1}\right| = \left|\frac{f}{f+u_2}\right|$$

$$\frac{f}{f+u_1} = -\frac{f}{f+u_2}$$ (since, one image is real and other is virtual)

$f + u_2 = -f - u_1$

$2f = -u_2 - u_1$

$2f = -(-10) - (-20)$

$2f = 10 + 20$

$\Rightarrow f = \frac{30}{2} = 15$ cm

Q.10 The electric field in a region is given by $\vec{E} = \left(\frac{3}{5}E_0\hat{i} + \frac{4}{5}E_0\hat{j}\right)\frac{N}{C}$. The ratio of flux of reported field through the rectangular surface of area 0.2 m^2 (parallel to $y-z$ plane) to that of the surface of area 0.3 m^2 (parallel to $x-z$ plane) is a : b, where a = ________ .

[Here $\hat{i}$, $\hat{j}$ and $\hat{k}$ are unit vectors along x, y and z-axes respectively]

Sol: $\phi = E.A$

For Y – Z, plane, $A_1 = 0.2$ i

$$\varphi_1 = \left[\frac{3}{5}E_0\hat{i} + \frac{4}{5}E_0\hat{j}\right] \times (0.2\hat{i})$$

$$= \frac{0.6}{5}E_0$$

For X – Z plane, $A_2 = 0.3\ \hat{i}$

$$|\phi_2| = \left[\frac{3}{5}E_0\hat{i} + \frac{4}{5}E_0\hat{j}\right] \times (0.3\hat{j}) = \frac{1.2}{5}E_0$$

$$\therefore \frac{\phi_2}{\phi_1} = \frac{0.6}{1.2}$$

$$= \frac{1}{2} = \frac{a}{b}$$

$\Rightarrow a = 1$

PART – B – CHEMISTRY

SECTION A

Q.1 In Freundlich adsorption isotherm at moderate pressure, the extent of adsorption $\left(\frac{x}{m}\right)$ is directly proportional to P^x. The value of x is :

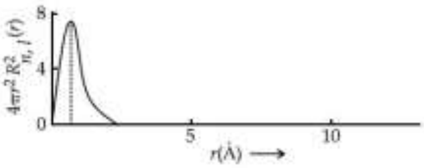
Options
1. zero
2. $\frac{1}{n}$
3. ∞
4. 1

Ans: 2

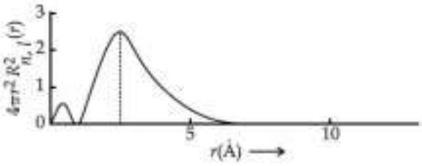
Sol: $\frac{x}{m} = KP^{\frac{1}{n}}$

Q.2 The plots of radial distribution functions for various orbitals of hydrogen atom against 'r' are given below :

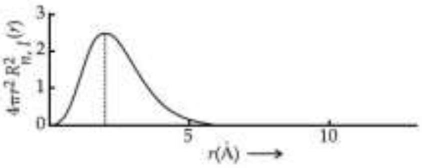
(A)



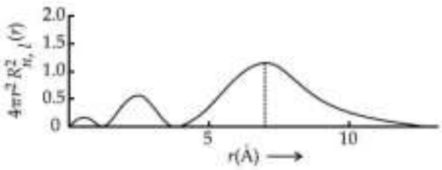
(B)



(C)



(D)



The correct plot for 3s orbital is :

Options
1. (D)
2. (A)
3. (C)
4. (B)

Ans: 1

Sol: (D) represent the correct plot with 2 radial nodes.

Q.3 Given below are two statements :

Statement I : CeO_2 can be used for oxidation of aldehydes and ketones.

Statement II : Aqueous solution of $EuSO_4$ is a strong reducing agent.

In the light of the above statements, choose the correct answer from the options given below :

Options
1. Statement I is false but Statement II is true
2. Statement I is true but Statement II is false
3. Both Statement I and Statement II are true
4. Both Statement I and Statement II are false

Ans: 3

Sol: For 4f series +3 oxidation state is more stable

$\therefore$ Ce^{4+} acts as oxidizing agent while Eu^{2+} acts as a reducing agent

Q.4 Complete combustion of 1.80 g of an oxygen containing compound $(C_xH_yO_z)$ gave 2.64 g of CO_2 and 1.08 g of H_2O. The percentage of oxygen in the organic compound is :

Options
1. 53.33
2. 63.53
3. 50.33
4. 51.63

Ans: 1

Sol: $C_xH_yO_z + \left(x+\frac{y}{4}-\frac{z}{3}\right)O_2 \rightarrow xCO_2 + \frac{y}{2}H_2O$

Number of moles of $C_xH_yO_z = \frac{1.8}{12x+y+16z}$

Number of moles of $CO_2 = \frac{2.64}{44} = 0.06$

Number of moles of $H_2O = \frac{1.08}{18} = 0.06$

$\therefore \frac{x}{y/2} = 1 \text{ or } y = 2x$

Number of moles of $CO_2 = \frac{1.8x}{12x+y+16z} = 0.06$ [put $y = 2x$]

$\frac{18x}{14x+16z} = 0.06$

$x = z$

$\therefore$ The empirical formula is $C_xH_yO_z$ or CH_2O

$\% \text{ of oxygen } = \frac{16}{12+2+16} \times 100 = 53.3\%$

Q.5 Given below are two statements :

Statement I : An allotrope of oxygen is an important intermediate in the formation of reducing smog.

Statement II : Gases such as oxides of nitrogen and sulphur present in troposphere contribute to the formation of photochemical smog.

In the light of the above statements, choose the correct answer from the options given below :

Options

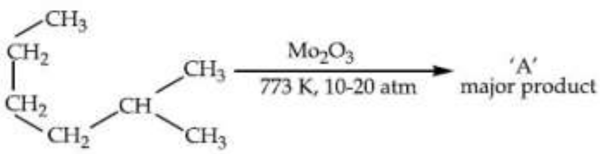
1. Statement I is true but Statement II is false
2. Both Statement I and Statement II are true
3. Statement I is false but Statement II is true
4. Both Statement I and Statement II are false

Ans: 4

Sol: Reducing smog consist of SO_2

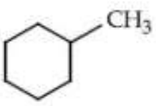
Photochemical smog consists of oxides of nitrogen

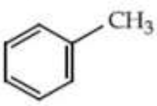
Q.6 Identify A in the given chemical reaction.



Options

1. 

2. 

3. 

4. 

Ans: 3

Sol: Aromatisation reaction results in the formation of toluene

Q.7 The solubility of AgCN in a buffer solution of pH = 3 is x. The value of x is :
[Assume : No cyano complex is formed ; $K_{sp}(AgCN) = 2.2 \times 10^{-16}$ and $K_a(HCN) = 6.2 \times 10^{-10}$]

Options
1. 2.2×10^{-16}
2. 0.625×10^{-6}
3. 1.9×10^{-5}
4. 1.6×10^{-6}

Ans: 3

Sol: $AgCN \rightleftharpoons \underset{s'}{Ag^+} + \underset{(s'-x')}{CN^-}$

$\underset{(s'-x')}{CN^-} + \underset{10^{-3}}{H^+} \rightleftharpoons \underset{x'}{HCN}$

$\frac{1}{K_a} = \frac{[HCN]}{[CN^-][H^+]} = \frac{x'}{(s'-x') \times 10^{-3}}$

$\frac{s'}{x'} - 1 = 6 \times 10^{-7}$

$s' \approx x'$

$K_{sp} \times \frac{1}{K_a} = \frac{s' \times x'}{[H^+]}$

$(s')^2 = \frac{10^{-3} \times 2.2 \times 10^{-16}}{6.2 \times 10^{-10}}$

$s' = \sqrt{3.55 \times 10^{-10}} = 1.9 \times 10^{-5}$

Q.8 Which of the following equation depicts the oxidizing nature of H_2O_2?

Options
1. $KIO_4 + H_2O_2 \rightarrow KIO_3 + H_2O + O_2$
2. $Cl_2 + H_2O_2 \rightarrow 2HCl + O_2$
3. $2I^- + H_2O_2 + 2H^+ \rightarrow I_2 + 2H_2O$
4. $I_2 + H_2O_2 + 2OH^- \rightarrow 2I^- + 2H_2O + O_2$

Ans: 3

Sol: In the reaction $2I^- + H_2O_2 + 2H^+ \rightarrow I_2 + 2H_2O$
hydrogen peroxide act as oxidizing agent

Q.9 The correct statement about B_2H_6 is :

Options
1. Its fragment, BH_3, behaves as a Lewis base.
2. Terminal B–H bonds have less p-character when compared to bridging bonds.
3. All B–H–B angles are of 120°.
4. The two B–H–B bonds are not of same length.

Ans: 2

Sol: The terminal B–H bonds have less p-character than bridged B–H bond.

Q.10 In which of the following pairs, the outer most electronic configuration will be the same ?

Options
1. V^{2+} and Cr^{+}
2. Cr^{+} and Mn^{2+}
3. Ni^{2+} and Cu^{+}
4. Fe^{2+} and Co^{+}

Ans: 2

Sol: V^{2+} – [Ar] $3d^3$
Cr^{3+} – [Ar] $3d^5$
Mn^{2+} – [Ar] $3d^5$
Ni^{2+} – [Ar] $3d^8$
Cu^{+} – [Ar] $3d^{10}$
Fe^{2+} – [Ar] $3d^6$
Co^{+} – [Ar] $3d^7\ 4s^1$

Q.11 The hybridization and magnetic nature of $[Mn(CN)_6]^{4-}$ and $[Fe(CN)_6]^{3-}$, respectively are :

Options
1. sp^3d^2 and diamagnetic
2. d^2sp^3 and paramagnetic
3. sp^3d^2 and paramagnetic
4. d^2sp^3 and diamagnetic

Ans: 2

Sol: The central metal ion in the given complexes are Mn^{2+} and Fe^{3+}, both have $3d^5$ configuration, since CN^- is a strong field ligand both complexes will have d^2sp^3 hybridisation, odd number of d subshell electron makes then paramagnetic.

Q.12 Which of the glycosidic linkage between galactose and glucose is present in lactose ?

Options
1. C-1 of glucose and C-6 of galactose
2. C-1 of galactose and C-6 of glucose
3. C-1 of glucose and C-4 of galactose
4. C-1 of galactose and C-4 of glucose

Ans: 4

Sol: Lactose is a disaccharide consists of galactose and glucose linked by C_1-C_4 β glycosidic linkage.

Q.13 Which statement is correct ?

Options
1. Synthesis of Buna-S needs nascent oxygen.
2. Buna-N is a natural polymer.
3. Buna-S is a synthetic and linear thermosetting polymer.
4. Neoprene is an addition copolymer used in plastic bucket manufacturing.

Ans: 1

Sol: The synthesis of Buna-S needs nascent oxygen.

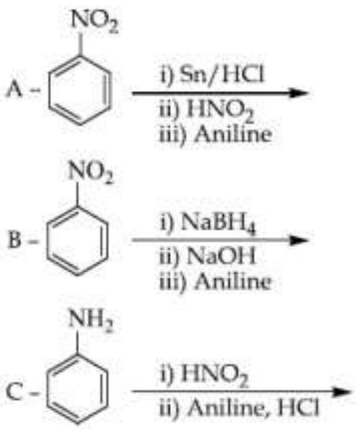
Q.14 Ellingham diagram is a graphical representation of :

Options
1. ΔH vs T
2. ΔG vs T
3. ΔG vs P
4. $(\Delta G - T\Delta S)$ vs T

Ans: 2

Sol: Ellingham diagram is a plot between ΔG (for the formation of metal oxide from element and one mole of oxygen) versus absolute temperature.

Q.15 Which of the following reaction/s will not give *p*-aminoazobenzene ?



Options
1. C only
2. A and B
3. A only
4. B only

Ans: 4

Sol: There is no reaction between nitrobenzene and sodium borohydride

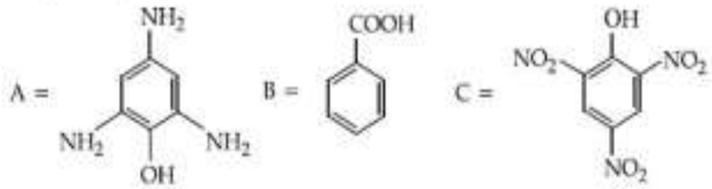
Q.16 Which one of the following reactions will not form acetaldehyde ?

Options
1. $CH_2{=}CH_2 + O_2 \xrightarrow[H_2O]{Pd(II)/Cu(II)}$
2. $CH_3CH_2OH \xrightarrow{CrO_3-H_2SO_4}$
3. $CH_3CH_2OH \xrightarrow[573\ K]{Cu}$
4. $CH_3CN \xrightarrow[ii)\ H_2O]{i)\ DIBAL\text{-}H}$

Ans: 2

Sol: $CH_3CH_2OH \xrightarrow{CrO_3-H_2SO_4} CH_3-COOH$

Q.17 Compound(s) which will liberate carbon dioxide with sodium bicarbonate solution is/are :



Options
1. B and C only
2. C only
3. A and B only
4. B only

Ans: 1

Sol: Benzoic acid and picric acid are more acidic than carbonic acid, thus they react with $NaHCO_3$ to form sodium salt and CO_2

Q.18 The major product of the following chemical reaction is :

$$CH_3CH_2CN \xrightarrow[\text{3) } Pd/BaSO_4,\ H_2]{\text{1) } H_3O^+,\ \Delta \quad \text{2) } SOCl_2} ?$$

Options
1. $CH_3CH_2CH_2OH$
2. CH_3CH_2CHO
3. $(CH_3CH_2CO)_2O$
4. $CH_3CH_2CH_3$

Ans: 2

Sol: $CH_3CH_2CN \xrightarrow{H_3O^+} CH_3CH_2COOH \xrightarrow{SOCl_2} CH_3CH_2COCl \xrightarrow{H_2-Pd/BaSO_4} CH_3CH_2CHO$

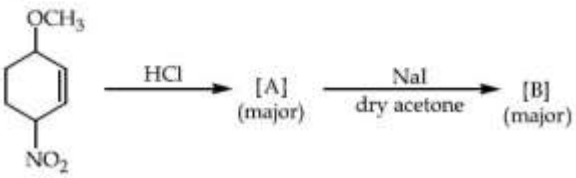
Q.19 According to molecular orbital theory, the species among the following that does not exist is:

Options
1. O_2^{2-}
2. Be_2
3. He_2^+
4. He_2^-

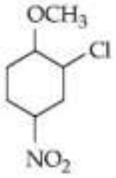
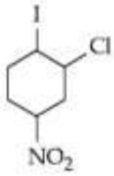
Ans: 2

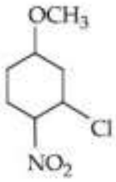
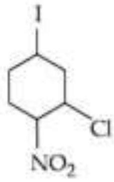
Sol: The bond order of Be_2 is zero

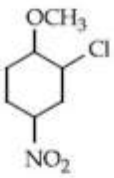
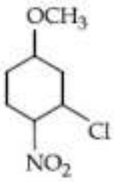
Q.20 Identify A and B in the chemical reaction.

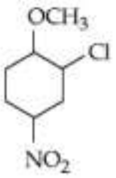
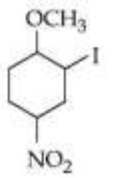


Options

1. A =  B = 

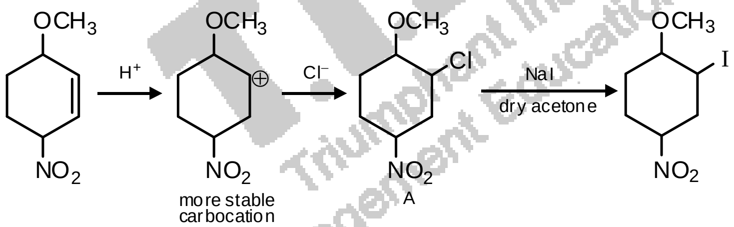
2. A =  B = 

3. A =  B = 

4. A =  B = 

Ans: 4

Sol:



SECTION B

Q.1 A car tyre is filled with nitrogen gas at 35 psi at 27°C. It will burst if pressure exceeds 40 psi. The temperature in °C at which the car tyre will burst is ______. (Rounded-off to the nearest integer)

Ans: 70

Sol: $\frac{P_1}{T_1} = \frac{P_2}{T_2}$ $\quad T_2 = \frac{40 \times 300}{35} = 342.85\ K = 69.70°C \approx 70°$

Q.2 0.4 g mixture of NaOH, Na_2CO_3 and some inert impurities was first titrated with $\frac{N}{10}$ HCl using phenolphthalein as an indicator, 17.5 mL of HCl was required at the end point. After this methyl orange was added and titrated. 1.5 mL of same HCl was required for the next end point. The weight percentage of Na_2CO_3 in the mixture is ______. (Rounded-off to the nearest integer)

Ans: 4

Sol: 0.4 g (NaOH + Na_2CO_3 + Impurity) $\times \frac{N}{10}$ HCl

Hph 17.5 mL $\Rightarrow$ 16 mL for NaOH

MOH 1.5 mL

2HCl + Na_2CO_3

2×1.5 mL $\times$ 0.1   1.5 m. moles $\times$ 0.1 Na_2CO_3

Wt. = $1.5 \times 10^{-3} \times 106 \times 0.1$

= 159×10^{-3} g $\times$ 0.1

$$\% \text{ of wt} = \frac{159 \times 10^{-4} \times 10^{2}}{0.4} = \frac{15.9}{4} = 3.975 = 4\%$$

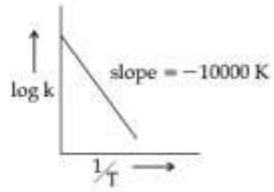
Q.3 Among the following, the number of halide(s) which is/are inert to hydrolysis is ________.

(A) BF_3 (B) $SiCl_4$ (C) PCl_5 (D) SF_6

Ans: 1

Sol: Only SF_6 is inert towards water

Q.4 For the reaction, $aA + bB \rightarrow cC + dD$, the plot of log k *vs* $\frac{1}{T}$ is given below :



The temperature at which the rate constant of the reaction is 10^{-4} s^{-1} is ________ K.
(Rounded-off to the nearest integer)
[Given : The rate constant of the reaction is 10^{-5} s^{-1} at 500 K.]

Ans: 526

Sol: $\text{Slope} = \frac{E_a}{2.303\,R} = -10000$

$$\log \frac{k_2}{k_1} = \frac{E_a}{2.303\,R}\left[\frac{1}{T_1} - \frac{1}{T_2}\right]$$

$$\log\left(\frac{10^{-4}}{10^{-5}}\right) = 10000 = \left[\frac{1}{500} - \frac{1}{T_2}\right]$$

T_2 = 526 K

Q.5 Using the provided information in the following paper chromatogram :

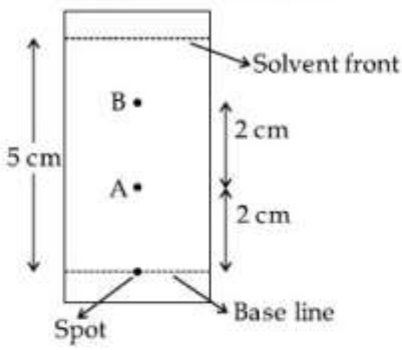


Fig : Paper chromatography for compounds A and B.

the calculated R_f value of A ________ $\times 10^{-1}$.

Ans: 4

Sol: $R_f = \dfrac{\text{Distance moved by the constant}}{\text{Distance moved by solvent}} = \dfrac{2}{5} = 0.4 = 4 \times 10^{-1}$

Q.6 In basic medium CrO_4^{2-} oxidises $S_2O_3^{2-}$ to form SO_4^{2-} and itself changes into $Cr(OH)_4^-$. The volume of 0.154 M CrO_4^{2-} required to react with 40 mL of 0.25 M $S_2O_3^{2-}$ is ______ mL. (Rounded-off to the nearest integer)

Ans: **173**

Sol: Equivalents of CrO_4^{2-} = equivalents of $S_2O_3^{2-}$

$\overset{+6}{CrO_4^{2-}} \rightarrow \overset{+3}{Cr(OH)_4^-}$ n = 3

$\overset{+2}{S_2O_3^{2-}} \rightarrow \overset{+6}{SO_4^{2-}}$ n = 8

$0.154 \times 3 \times V_{CrO_4^{2-}} = 0.25 \times 40 \times 8$

V = 173 mL

Q.7 The ionization enthalpy of Na^+ formation from $Na_{(g)}$ is 495.8 kJ mol^{-1}, while the electron gain enthalpy of Br is −325.0 kJ mol^{-1}. Given the lattice enthalpy of NaBr is −728.4 kJ mol^{-1}. The energy for the formation of NaBr ionic solid is (−)______ $\times 10^{-1}$ kJ mol^{-1}.

Ans: **5576**

Sol: ΔH_f = 495.8 + (−325) + (−728.4)
= −557.6 kJ mol^{-1}
= −5576 × 10^{-1} kJ mol^{-1}

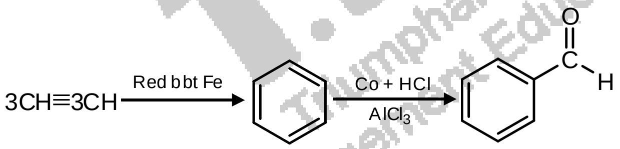
Q.8 Consider the following chemical reaction.

$$CH \equiv CH \xrightarrow[\text{2) CO, HCl, AlCl}_3]{\text{1) Red hot Fe tube, 873 K}} \text{Product}$$

The number of sp^2 hybridized carbon atom(s) present in the product is ______.

Ans: **7**

Sol:



Q.9 1 molal aqueous solution of an electrolyte A_2B_3 is 60% ionised. The boiling point of the solution at 1 atm is ______ K. (Rounded-off to the nearest integer)
[Given K_b for $(H_2O) = 0.52$ K kg mol^{-1}]

Ans: **375**

Sol: I = 1 + (n −1) α
= 1 + 4 × 0.6 = 3.4
ΔT_b = 3.4 × 0.52 × 1 = 1.768
Boiling point = 373.15 + 1.768 = 375 K

Q.10 The reaction of cyanamide, $NH_2CN_{(s)}$ with oxygen was run in a bomb calorimeter and ΔU was found to be -742.24 kJ mol^{-1}. The magnitude of ΔH_{298} for the reaction

$$NH_2CN_{(s)} + \frac{3}{2}O_{2(g)} \rightarrow N_{2(g)} + O_{2(g)} + H_2O_{(l)}$$

is _________ kJ. (Rounded off to the nearest integer)
[Assume ideal gases and $R = 8.314$ J mol^{-1} K^{-1}]

Ans: **741**

Sol: $\Delta H = \Delta U + \Delta nRT = -742.24 + 0.5 \times 8.314 \times 10^{-3} \times 298 = -741$ kJ mol^{-1}
Magnitude of $\Delta H = 741$

PART – C – MATHEMATICS

SECTION A

Q.1 Let $f, g : N \rightarrow N$ such that $f(n+1) = f(n) + f(1)\ \forall\ n \in N$ and g be any arbitrary function. Which of the following statements is NOT true ?

Options
1. f is one-one
2. If g is onto, then fog is one-one
3. If f is onto, then $f(n) = n\ \forall\ n \in N$
4. If fog is one-one, then g is one-one

Ans: **2**

Sol: $f(n+1) = f(n) + 1$
$f(2) = 2f(1)$
$f(3) = 3f(1)$
$f(4) = 4f(1)$
.
.
.
.
.
$f(n) = nf(1)$
f(x) is one-one
Generally gof is one-one $\Rightarrow$ f is one-one
gof is onto $\Rightarrow$ g is onto

Q.2 Let α be the angle between the lines whose direction cosines satisfy the equations $l + m - n = 0$ and $l^2 + m^2 - n^2 = 0$. Then the value of $\sin^4\alpha + \cos^4\alpha$ is :

Options
1. $\frac{3}{4}$
2. $\frac{1}{2}$
3. $\frac{5}{8}$
4. $\frac{3}{8}$

Ans: 3

Sol: $\ell + m - n = 0$ & $\ell^2 + m^2 - n^2 = 0$

$n = \ell + m \Rightarrow \ell^2 + m^2 - (\ell + m)^2 = 0$

$\Rightarrow -2\ell m = 0$

$\Rightarrow \ell m = 0$

$\Rightarrow \ell = 0$ (or) $m = 0$

$\ell = 0$	$m = 0$
0:1:1	1:0:1

$\therefore \cos\alpha = \frac{0+0+1}{\sqrt{2}\sqrt{2}} = \frac{1}{2}$

$\Rightarrow \sin\alpha = \frac{\sqrt{3}}{2}$

$\therefore \sin^4\alpha + \cos^4\alpha = \frac{9}{16} + \frac{1}{16} = \frac{10}{16} = \frac{5}{8}$

Q.3

If $0 < \theta, \phi < \frac{\pi}{2}$, $x = \sum_{n=0}^{\infty} \cos^{2n}\theta$, $y = \sum_{n=0}^{\infty} \sin^{2n}\phi$ and $z = \sum_{n=0}^{\infty} \cos^{2n}\theta \cdot \sin^{2n}\phi$ then :

Options
1. $xy + yz + zx = z$
2. $xyz = 4$
3. $xy + z = (x+y)z$
4. $xy - z = (x+y)z$

Ans: 3

Sol: $x = 1 + \cos^2\theta + \cos^4\theta + \ldots\ldots = \frac{1}{1-\cos^2\theta} = \frac{1}{\sin^2\theta}$

[infinite G.P series]

$y = 1 + \sin^2\phi + \sin^4\phi + \ldots\ldots = \frac{1}{1-\sin^2\phi} = \frac{1}{\cos^2\phi}$

$z = 1 + \sin^2\phi\cos^2\theta + \sin^4\phi\cos^4\theta + \ldots\ldots = \frac{1}{1-\sin^2\phi\cos^2\theta}$

$z = \frac{1}{1-\left(1-\frac{1}{x}\right)\left(1-\frac{1}{y}\right)}$

$z = \frac{xy}{xy - (xy - x - y + 1)}$

$z = \frac{xy}{x + y - 1}$

$z(x+y) = xy + z$

Q.4

If the curves, $\frac{x^2}{a} + \frac{y^2}{b} = 1$ and $\frac{x^2}{c} + \frac{y^2}{d} = 1$ intersect each other at an angle of 90°, then which of the following relations is TRUE ?

Options
1. $a - c = b + d$
2. $a + b = c + d$
3. $ab = \frac{c+d}{a+b}$
4. $a - b = c - d$

Ans: 4

Sol: $\frac{x^2}{a}+\frac{y^2}{b}=1 \to (1)$

diff w.r.t 'x'

$\frac{2x}{a}+\frac{2y}{b}\frac{dy}{dx}=0$

$\Rightarrow \frac{dy}{dx}=\frac{-bx}{ay} \to (2)$

$\frac{x^2}{c}+\frac{y^2}{d}=1 \to (3)$

diff w.r.t 'x'

$\frac{2x}{c}+\frac{2y}{d}\frac{dy}{dx}=1$

$\frac{dy}{dx}=\frac{-dx}{cy} \to (4)$

$m_1 m_2=-1$

$\frac{-bx}{ay}\times\frac{-dx}{cy}=-1$

$\Rightarrow bdx^2=-acy^2 \to (5)$

$(1)-(3) \Rightarrow \left(\frac{1}{a}-\frac{1}{c}\right)x^2+\left(\frac{1}{b}-\frac{1}{d}\right)y^2=0$

$\Rightarrow \frac{c-a}{ac}x^2+\left(\frac{d-b}{bd}\right)\left(\frac{-bd}{ac}\right)x^2=0$

$\Rightarrow (c-a)-(d-b)=0$

$\Rightarrow c-a=d-b$

$\Rightarrow c-d=a-b$

Q.5 The total number of positive integral solutions (x, y, z) such that $xyz=24$ is :

Options
1. 30
2. 36
3. 24
4. 45

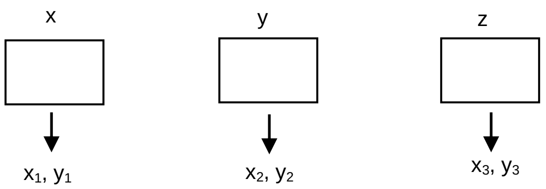
Ans: 1

Sol: Given xyz = 2y

$xyz = 2^3 \times 3$

$x_i =$ No. of 2's

$y_i =$ No. of 3's



$x_1+x_2+x_3=3$

Non-negative integral sol = $^{3+3-1}C_{3-1}={}^5C_2$

$y_1+y_2+y_3=1$

$\Rightarrow$ No.of ways $={}^{1+3-1}C_{3-1}={}^3C_2$

Total no. of ways = ${}^5C_2\times{}^3C_2=30$

Q.6 When a missile is fired from a ship, the probability that it is intercepted is $\frac{1}{3}$ and the probability that the missile hits the target, given that it is not intercepted, is $\frac{3}{4}$. If three missiles are fired independently from the ship, then the probability that all three hit the target, is :

Options

1. $\frac{3}{8}$
2. $\frac{1}{27}$
3. $\frac{3}{4}$
4. $\frac{1}{8}$

Ans: 4

Sol: Prob = $\left(\frac{2}{3}.\frac{3}{4}\right)^4 = \frac{1}{8}$

Q.7 Let the lines $(2-i)z = (2+i)\bar{z}$ and $(2+i)z + (i-2)\bar{z} - 4i = 0$, (here $i^2 = -1$) be normal to a circle C. If the line $iz + \bar{z} + 1 + i = 0$ is tangent to this circle C, then its radius is :

Options

1. $3\sqrt{2}$
2. $\frac{3}{\sqrt{2}}$
3. $\frac{3}{2\sqrt{2}}$
4. $\frac{1}{2\sqrt{2}}$

Ans: 3

Sol: Given lines $(2-i)(x+iy) = (2+i)(x-iy)$

$\Rightarrow (2x+y)+i(2y-x) = (2x+y)+i(x-2y)$

$\Rightarrow 2y - x = x - 2y$

$\Rightarrow 2x - 4y = 0$

$\Rightarrow x - 2y = 0 \rightarrow (1)$

$(2+i)z + (i-2)\bar{z} - 4i = 0$

$\Rightarrow (2+i)z = (2-i)\bar{z} + 4i$

$\Rightarrow (2+i)(x+iy) = (2-i)(x-iy) + 4i$

$\Rightarrow (2x-y)+i(x+2y) = (2x-y)+i(4-x-2y)$

$\Rightarrow x + 2y = 4 - x - 2y$

$\Rightarrow 2x + 4y - 4 = 0$

$\Rightarrow x + 2y - 2 = 0 \rightarrow (2)$

Solving (1) & (2)

$x - 2y = 0$

$x = 2y$

From (2) $2y + 2y - 2 = 0$

$4y = 2$

$y = \frac{1}{2}$

$x = 1$

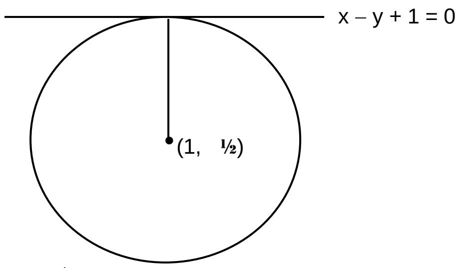
$\therefore x = 1, \; y = \frac{1}{2}$

$iz + \bar{z} + 1 + i = 0$

$i(x + iy) + (x - iy) + 1 + i = 0$

$ix - y + x - iy + 1 + i = 0$

$x - y + 1 = 0 \qquad (or) \; x - y + 1 = 0$



$r = \frac{\left|1 - \frac{1}{2} + 1\right|}{\sqrt{2}} = \frac{3}{2\sqrt{2}}$

Q.8 A man is observing, from the top of a tower, a boat speeding towards the tower from a certain point A, with uniform speed. At that point, angle of depression of the boat with the man's eye is 30° (Ignore man's height). After sailing for 20 seconds, towards the base of the tower (which is at the level of water), the boat has reached a point B, where the angle of depression is 45°. Then the time taken (in seconds) by the boat from B to reach the base of the tower is :

Options
1. 10
2. $10\sqrt{3}$
3. $10(\sqrt{3} - 1)$
4. $10(\sqrt{3} + 1)$

Ans: 4

Sol: Time to cover distance = $\frac{y}{v} = \frac{20}{\sqrt{3} - 1} = 10(\sqrt{3} + 1)$sec

Q.9 If Rolle's theorem holds for the function $f(x) = x^3 - ax^2 + bx - 4$, $x \in [1, 2]$ with $f'\left(\frac{4}{3}\right) = 0$, then ordered pair (a, b) is equal to :

Options
1. $(-5, 8)$
2. $(-5, -8)$
3. $(5, -8)$
4. $(5, 8)$

Ans: 4

Sol: $f(x) = x^3 - ax^2 + bx - 4$

f(1) = f(2)

$\Rightarrow 1 - a + b - 4 = 8 - 4a + 2b - 4$

$\Rightarrow 3a - b = 7 \rightarrow (1)$

$f'(x) = 3x^2 - 2ax + b$

$\Rightarrow f'\left(\frac{4}{3}\right) = 0$

$\Rightarrow 3 \times \frac{16}{9} - \frac{8}{3}a + b = 0$

$\Rightarrow -8a + 3b = -16 \rightarrow (2)$

From (1) & (2)
a = 5, b = 8

Q.10 The statement $A \rightarrow (B \rightarrow A)$ is equivalent to :

Options
1. $A \rightarrow (A \rightarrow B)$
2. $A \rightarrow (A \leftrightarrow B)$
3. $A \rightarrow (A \wedge B)$
4. $A \rightarrow (A \vee B)$

Ans: 4

Sol:

A	B	B→A	A→(B→A)
T	T	T	T
T	F	T	T
F	T	F	T
F	F	T	T

Verify option (4)

$A \rightarrow (A \vee B)$

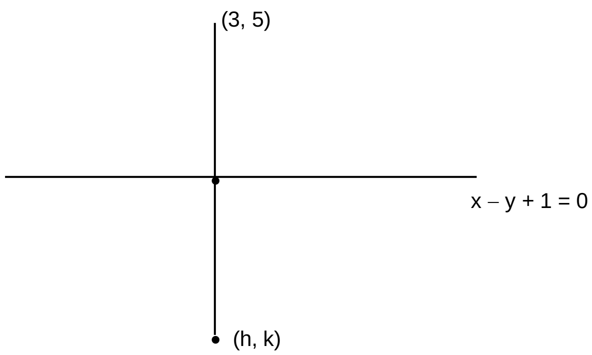
A	B	A ∨ B	A → (A ∨ B)
T	T	T	T
T	F	T	T
F	T	T	T
F	F	F	T

Q.11 The image of the point (3, 5) in the line $x - y + 1 = 0$, lies on :

Options
1. $(x-4)^2 + (y-4)^2 = 8$
2. $(x-2)^2 + (y-2)^2 = 12$
3. $(x-4)^2 + (y+2)^2 = 16$
4. $(x-2)^2 + (y-4)^2 = 4$

Ans: 4

Sol:



$$\frac{h-x_1}{a}=\frac{k-y_1}{b}=\frac{-2(ax_1+by_1+c)}{a^2+b^2}$$

$$\frac{h-3}{1}=\frac{k-5}{-1}=\frac{-2(3-5+1)}{2}$$

$h-3=5-k=1$

$h=u,\ k=4$

$(h,\ k)=(4,\ 4)$

Clearly (4, 4) lies on $(x-2)^2+(y-4)^2=4$

Q.12 The value of the integral

$$\int \frac{\sin\theta.\sin 2\theta\ (\sin^6\theta+\sin^4\theta+\sin^2\theta)\ \sqrt{2\sin^4\theta+3\sin^2\theta+6}}{1-\cos 2\theta}\,d\theta \text{ is :}$$

(where c is a constant of integration)

Options

1. $\frac{1}{18}\left[9-2\sin^6\theta-3\sin^4\theta-6\sin^2\theta\right]^{\frac{3}{2}}+c$
2. $\frac{1}{18}\left[9-2\cos^6\theta-3\cos^4\theta-6\cos^2\theta\right]^{\frac{3}{2}}+c$
3. $\frac{1}{18}\left[11-18\cos^2\theta+9\cos^4\theta-2\cos^6\theta\right]^{\frac{3}{2}}+c$
4. $\frac{1}{18}\left[11-18\sin^2\theta+9\sin^4\theta-2\sin^6\theta\right]^{\frac{3}{2}}+c$

Ans: 3

Sol: Put sinθ = t

cosθ dθ = dt

$$\int(t^6+t^4+t^2)\sqrt{2t^4+3t^2+6}dt=\int(t^5+t^3+t)\sqrt{2t^6+3t^4+6t^2}\,dt$$

Let $2t^6+3t^4+6t^2=z$

$$12(t^5+t^3+t)dt=dz\ =\frac{1}{12}\int\sqrt{z}\,dz=\frac{1}{18}z^{3/2}+C=\frac{1}{18}\left[2\sin^6\theta+3\sin^4\theta 6\sin^2\theta\right]^{3/2}+C$$

$$=\frac{1}{18}\left[2(1-\cos^2\theta)^3+3(1-\cos^2\theta)^2+6(1-\cos^2\theta)\right]^{3/2}+C$$

$$=\frac{1}{18}\left[11-18\cos^2\theta+9\cos^4\theta-2\cos^2\theta\right]^{3/2}+C$$

Q.13 The coefficients a, b and c of the quadratic equation, $ax^2+bx+c=0$ are obtained by throwing a dice three times. The probability that this equation has equal roots is :

Options

1. $\frac{1}{36}$
2. $\frac{1}{72}$
3. $\frac{5}{216}$
4. $\frac{1}{54}$

Ans: 3

Sol: $a, b, c \in \{1, 2, 3, 4, 5, 6\}$

$n(S) = 6^3$

equal roots $\Rightarrow D = 0$

$\Rightarrow b^2 = 4ac$

$\Rightarrow ac = \dfrac{b^2}{4}$

If b = 2, ac = 1 $\Rightarrow$ a = 1, c = 1

If b = 4, ac = 4

a = 4, c = 1

a = 1, c = 4

a = 2, c = 2

If b = c, ac = 9 $\Rightarrow$ a = 3, c = 3

$\therefore$ probability = $\dfrac{5}{216}$

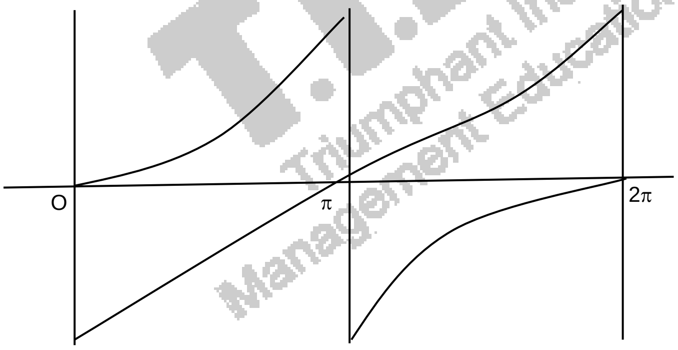
Q.14 All possible values of $\theta \in [0, 2\pi]$ for which $\sin 2\theta + \tan 2\theta > 0$ lie in :

Options

1. $\left(0, \frac{\pi}{2}\right) \cup \left(\frac{\pi}{2}, \frac{3\pi}{4}\right) \cup \left(\pi, \frac{7\pi}{6}\right)$
2. $\left(0, \frac{\pi}{2}\right) \cup \left(\pi, \frac{3\pi}{2}\right)$
3. $\left(0, \frac{\pi}{4}\right) \cup \left(\frac{\pi}{2}, \frac{3\pi}{4}\right) \cup \left(\frac{3\pi}{2}, \frac{11\pi}{6}\right)$
4. $\left(0, \frac{\pi}{4}\right) \cup \left(\frac{\pi}{2}, \frac{3\pi}{4}\right) \cup \left(\pi, \frac{5\pi}{4}\right) \cup \left(\frac{3\pi}{2}, \frac{7\pi}{4}\right)$

Ans: 4

Sol:



$\tan 2\theta(1 + \cos 2\theta) > 0$

$2\theta \in \left(0, \frac{\pi}{2}\right) \cup \left(\pi, \frac{3\pi}{2}\right) \cup \left(2\pi, \frac{5\pi}{2}\right) \cup \left(3\pi, \frac{7\pi}{2}\right)$

$\Rightarrow \theta \in \left(0, \frac{\pi}{4}\right) \cup \left(\frac{\pi}{2}, \frac{3\pi}{4}\right) \cup \left(\pi, \frac{5\pi}{4}\right) \cup \left(\frac{3\pi}{2}, \frac{7\pi}{4}\right)$

Q.15 The equation of the line through the point (0, 1, 2) and perpendicular to the line $\frac{x-1}{2}=\frac{y+1}{3}=\frac{z-1}{-2}$ is :

Options

1. $\frac{x}{3}=\frac{y-1}{4}=\frac{z-2}{3}$
2. $\frac{x}{3}=\frac{y-1}{-4}=\frac{z-2}{3}$
3. $\frac{x}{-3}=\frac{y-1}{4}=\frac{z-2}{3}$
4. $\frac{x}{3}=\frac{y-1}{4}=\frac{z-2}{-3}$

Ans: 3

Sol: Given point (0, 1, 2)

Line $\frac{x-1}{2}=\frac{y+1}{3}=\frac{z-1}{-2}$

Verify option (3)

dr's $2(-3)+4(3)+(-2)(3)=0$

clearly passing through (0, 1, 2)

Q.16 If a curve passes through the origin and the slope of the tangent to it at any point (x, y) is $\frac{x^2-4x+y+8}{x-2}$, then this curve also passes through the point :

Options

1. (5, 5)
2. (4, 4)
3. (5, 4)
4. (4, 5)

Ans: 1

Sol: $\frac{dy}{dx}=\frac{x^2-4x+y+8}{x-2}=\frac{(x-2)^2+y+4}{(x-2)}=(x-2)+\frac{y+4}{x-2}$

Let x – 2 = t ⇒ dx = dt

y + 4 = u ⇒ dy = du

$\frac{dy}{dx}=\frac{du}{dt}$

$\frac{du}{dt}=t+\frac{u}{t}$

$\frac{du}{dt}-\frac{u}{t}=t$

$I.F=e^{-\int\frac{1}{t}dt}=e^{-\log t}=\frac{1}{t}$

$u.\frac{1}{t}=\int t.\frac{1}{t}dt$

$\frac{u}{t}=t+C$

$\frac{y+4}{x-2}=(x-2)+C$

Passing through (0, 0) $\Rightarrow$ C = 0

$\therefore y + 4 = (x-2)^2$

Option (1) (5, 5) satisfy the equation $y + 4 = (x - 2)^2$

Q.17 $\lim_{n\to\infty}\left(1+\frac{1+\frac{1}{2}+\ldots\ldots+\frac{1}{n}}{n^2}\right)^n$ is equal to :

Options
1. 0
2. $\frac{1}{2}$
3. 1
4. $\frac{1}{e}$

Ans: 3

Sol: Let $L = \lim_{n\to\infty}\left(1+\frac{1+\frac{1}{2}+\ldots\ldots\ldots+\frac{1}{n}}{n^2}\right)^n$

So $L = e^{\lim_{n\to\infty}\left[\frac{1+\frac{1}{2}+\frac{1}{3}+\ldots\ldots\ldots\frac{1}{n}}{n}\right]}$ $\left[1^\infty \text{ modal}\right] = e^{\lim_{n\to\infty}\left(\frac{1}{n}+\frac{1}{2n}+\frac{1}{3n}+\ldots\ldots\ldots\ldots+\frac{1}{n^2}\right)}$

$= e^{0+0+\ldots\ldots\ldots\ldots+0}$

$= e^0$

$= 1$

Q.18 The integer 'k', for which the inequality $x^2 - 2(3k-1)x + 8k^2 - 7 > 0$ is valid for every x in R, is :

Options
1. 0
2. 3
3. 2
4. 4

Ans: 2

Sol: D < 0

$[2(3k-1)]^2 - 4(8k^2-7) < 0$

$4(9k^2 - 6k + 1) - 4(8k^2 - 7) < 0$

$k^2 - 6k + 8 < 0$

$(k-4)(k-2) < 0$

$2 < k < 4$

Then k = 3

Q.19 The value of $\int_{-1}^{1} x^2 e^{[x^3]} dx$, where [t] denotes the greatest integer $\le$ t, is :

Options
1. $\frac{e+1}{3e}$
2. $\frac{1}{3e}$
3. $\frac{e+1}{3}$
4. $\frac{e-1}{3e}$

Ans: 1

Sol: Let $I = \int_{-1}^{1} x^2 e^{[x^3]} dx$

$$I = \int_{-1}^{0} x^2 \cdot e^{-1} dx + \int_{0}^{1} x^2 dx$$

$$I = \left[\frac{x^3}{3e}\right]_{-1}^{0} + \left[\frac{x^3}{3}\right]_{0}^{1}$$

$$I = \frac{1}{3e} + \frac{1}{3}$$

Q.20 A tangent is drawn to the parabola $y^2 = 6x$ which is perpendicular to the line $2x + y = 1$. Which of the following points does NOT lie on it ?

Options
1. (5, 4)
2. (4, 5)
3. (−6, 0)
4. (0, 3)

Ans: 1

Sol: Equation of tangent y = $mx + \frac{2}{3m}$

Perpendicular slope of 2x + y = 1 is $\frac{1}{2}$

$\therefore$ Tangent is y = $\frac{x}{2} + 3$

$\Rightarrow x - 2y + 6 = 0 \rightarrow (1)$

(5, 4) does not lie in equation (1)

SECTION B

Q.1 Let $\vec{a} = \hat{i} + 2\hat{j} - \hat{k}$, $\vec{b} = \hat{i} - \hat{j}$ and $\vec{c} = \hat{i} - \hat{j} - \hat{k}$ be three given vectors. If $\vec{r}$ is a vector such that $\vec{r} \times \vec{a} = \vec{c} \times \vec{a}$ and $\vec{r} \cdot \vec{b} = 0$, then $\vec{r} \cdot \vec{a}$ is equal to ________.

Ans: 11

Sol: $\bar{r}\times\bar{a}=\bar{c}\times\bar{a}$

$(\bar{r}-\bar{c})\times\bar{a}=\bar{0}$

$\Rightarrow \bar{r}-\bar{c}=\lambda\bar{a}$

$\bar{r}=\lambda\bar{a}+\bar{c}$

$\bar{r}\cdot\bar{b}=0 \Rightarrow (\bar{a}\cdot\bar{b})\lambda+(\bar{b}\cdot\bar{c})=0$

$\Rightarrow (-1)\lambda+2=0$

$\Rightarrow \lambda=2$

$\therefore \bar{r}=2\bar{a}+\bar{c}$

$=2(\bar{i}+2\bar{j}-\bar{k})+(\bar{i}-\bar{j}-\bar{k})$

$=3\bar{i}+3\bar{j}-2\bar{k}$

$\therefore \bar{r}\cdot\bar{a}=(3\bar{i}+3\bar{j}-\bar{k})\cdot(\bar{i}+2\bar{j}-\bar{k})= 3 + 6 + 2 = 11$

Q.2 Let $f(x)$ be a polynomial of degree 6 in x, in which the coefficient of x^6 is unity and it has extrema at $x=-1$ and $x=1$. If $\lim\limits_{x\to 0}\frac{f(x)}{x^3}=1$, then $5\cdot f(2)$ is equal to ________.

Ans: 144

Sol: $f(x)=x^6+ax^5+bx^4+x^3$

$\therefore f'(x)=6x^5+5ax^4+4bx^3+3x^2$

Roots 1 & –1

$\therefore 6+5a+4b+3=0$ & $-6+5a-4b+3=0$

$5a+4b+9=0$ & $5a-4b-3=0$

Solve we get $a=\frac{-3}{5}$, $b=\frac{-3}{2}$

$\therefore f(x)=x^6-\frac{3}{5}x^5-\frac{3}{2}x^4+x^3$

$5f(2)=5\left[64-\frac{96}{5}-24+8\right]$ = 144

Q.3 If the system of equations

$kx+y+2z=1$

$3x-y-2z=2$

$-2x-2y-4z=3$

has infinitely many solutions, then k is equal to ________.

Ans: 21

Sol: $D_3=\begin{vmatrix} k & 1 & 1 \\ 3 & -1 & 2 \\ -2 & -2 & 3 \end{vmatrix}$

$0=k(-3+4)-1(9+4)+1(-6-2)$

$0=k-13-8$

k = 21

Q.4 The locus of the point of intersection of the lines $(\sqrt{3})kx+ky-4\sqrt{3}=0$ and $\sqrt{3}x-y-4(\sqrt{3})k=0$ is a conic, whose eccentricity is ________.

Ans: 2

Sol: $\sqrt{3}kx + ky - 4\sqrt{3} = 0 \rightarrow (1)$

$\sqrt{3}x - y - 4\sqrt{3}k = 0 \rightarrow (2)$

$(1) + k(2)$

$\sqrt{3}kx + ky - 4\sqrt{3} = 0$

$\sqrt{3}kx - ky - 4\sqrt{3}k^2 = 0$

$2\sqrt{3}kx - 4\sqrt{3}(k^2 + 1) = 0$

$x = 2\left(k + \frac{1}{k}\right) \rightarrow (3)$

From (2) $y = 2\sqrt{3}\left(\frac{1}{k} - k\right) \rightarrow (4)$

From (3) & (4) $\frac{x}{2} = k + \frac{1}{k}, \ \frac{y}{2\sqrt{3}} = \frac{1}{k} - k$

$\frac{x^2}{4} - \frac{y^2}{12} = 4$

$\frac{x^2}{16} - \frac{y^2}{48} = 1$

$\therefore e = \frac{\sqrt{16 + 48}}{4} = \frac{\sqrt{64}}{4} = \frac{8}{4} = 2$

Q.5 Let $A = \begin{bmatrix} x & y & z \\ y & z & x \\ z & x & y \end{bmatrix}$, where x, y and z are real numbers such that $x + y + z > 0$ and $xyz = 2$.

If $A^2 = I_3$, then the value of $x^3 + y^3 + z^3$ is________.

Ans: 7

Sol: $A^2 - I_3$

$\begin{bmatrix} x & y & z \\ y & z & x \\ z & x & y \end{bmatrix}\begin{bmatrix} x & y & z \\ y & z & x \\ z & x & y \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

Clearly $x^2 + y^2 + z^2 = 1$

$xy + yz + zx = 0$

Now $(x + y + z)^2 = x^2 + y^2 + z^2 + 2(xy + yz + zx)$

x + y +z = 1

$|A|^2 = |I|$

$|A| = \pm 1$

$\Rightarrow 3xyz - (x^3 + y^3 + z^3) = \pm 1$

$\Rightarrow x^3 + y^3 + z^3 = 3 \cdot 2 \pm 1 = 7$ (or) 5

$\therefore x^3 + y^3 + z^3 = 7$

Q.6 The number of points, at which the function $f(x) = |2x + 1| - 3|x + 2| + |x^2 + x - 2|$, $x \in \mathbf{R}$ is not differentiable, is ________.

Ans: 3

Sol: $f(x) = |2x + 1| - 3|x + 2| + |x^2 + x - 2|$

$= |2x + 1| - 3|x + 2| + |(x - 1)(x + 2)|$

Non differentiable at $x = \frac{-1}{2}, -2, 1$

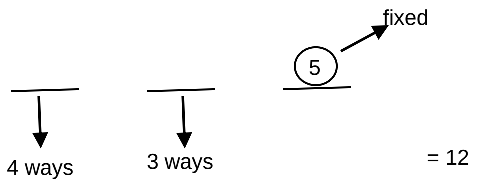
Q.7 The total number of numbers, lying between 100 and 1000 that can be formed with the digits 1, 2, 3, 4, 5, if the repetition of digits is not allowed and numbers are divisible by either 3 or 5, is ________.

Ans: 32

Sol: Divisible by '3'

Sum is 12 → 3, 4, 5 → 3! = 6
Sum is 9 → 2, 3, 4 → 3! = 6
Sum is 9 → 1, 3, 5 → 3! = 6
Sum is 6 → 1, 2, 3 → 3! = 6
24

Divisible by 5



$$\therefore \text{Total no} = 24 + 12 - 4\begin{cases}3,\ 4,\ 5\\ 4,\ 3,\ 5\\ 1,\ 3,\ 5\\ 3,\ 1,\ 5\end{cases}$$

$$= 32$$

Q.8 Let $A_1, A_2, A_3, \ldots$ be squares such that for each $n \geq 1$, the length of the side of A_n equals the length of diagonal of A_{n+1}. If the length of A_1 is 12 cm, then the smallest value of n for which area of A_n is less than one, is ________.

Ans: 9

Sol: Let $a_1, a_2, a_3 \ldots\ldots\ldots\ldots a_n$ are length of sides of squares $A_1, A_2, \ldots\ldots\ldots\ldots A_n$

$$a_1 = \sqrt{2}a_2$$

$$a_2 = \frac{a_1}{\sqrt{2}}$$

i.e, $a_{n+1} = \frac{a_n}{\left(\sqrt{2}\right)^n}$

Length of side $A_1 = 12$

Length of side $A_2 = \frac{12}{\left(\sqrt{2}\right)^1}$

Length of side $A_3 = \frac{12}{\left(\sqrt{2}\right)^2}$

.
.
.
.

Length of side $A_n = \frac{12}{\left(\sqrt{2}\right)^{n-1}}$

Area of $A_n = \left[\frac{12}{\left(\sqrt{2}\right)^{n-1}}\right]^2 < 1$

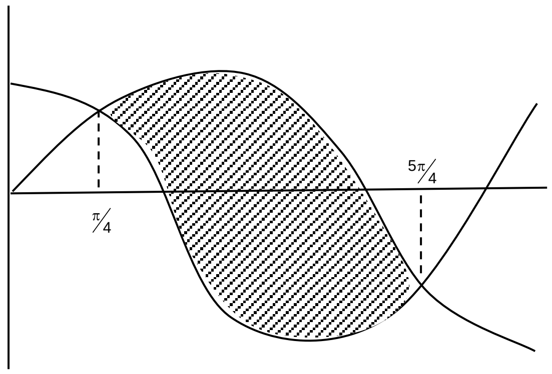
$= \frac{144}{2^{n-1}} < 1$

$n - 1 = 8 \Rightarrow n = 9$

Q.9 The graphs of sine and cosine functions, intersect each other at a number of points and between two consecutive points of intersection, the two graphs enclose the same area A. Then A^4 is equal to ________.

Ans: 64

Sol:



$$A = \int_{\pi/4}^{5\pi/4} (\sin x - \cos x)dx = [-\cos x - \sin x]_{\pi/4}^{5\pi/4} = -\left[\left(-\frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}}\right) - \left(\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}\right)\right] = \frac{4}{\sqrt{2}} = 2\sqrt{2}$$

$$\Rightarrow A^4 = \left(2\sqrt{2}\right)^4 = 64$$

Q.10 If $A = \begin{bmatrix} 0 & -\tan\left(\frac{\theta}{2}\right) \\ \tan\left(\frac{\theta}{2}\right) & 0 \end{bmatrix}$ and $(I_2 + A)\ (I_2 - A)^{-1} = \begin{bmatrix} a & -b \\ b & a \end{bmatrix}$, then $13(a^2 + b^2)$ is equal to ________.

Ans: 13

Sol: $A = \begin{bmatrix} 0 & -\tan(\theta/2) \\ \tan(\theta/2) & 0 \end{bmatrix}$, $I_2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$$I_2 + A = \begin{bmatrix} 1 & -\tan\theta/2 \\ \tan\theta/2 & 1 \end{bmatrix}$$

$$I_2 - A = \begin{bmatrix} 1 & \tan\theta/2 \\ -\tan\theta/2 & 1 \end{bmatrix}$$

$$(I_2 - A)^{-1} = \frac{1}{1+\tan^2\theta/2}\begin{bmatrix} 1 & -\tan\theta/2 \\ \tan\theta/2 & 1 \end{bmatrix}$$

$$\therefore (I_2 + A)(I_2 - A)^{-1} = \cos^2\theta/2 \begin{bmatrix} 1 & -\tan\theta/2 \\ \tan\theta/2 & 1 \end{bmatrix}\begin{bmatrix} 1 & -\tan\theta/2 \\ \tan\theta/2 & 1 \end{bmatrix}$$

$$= \cos^2\theta/2 \begin{bmatrix} 1-\tan^2\theta/2 & -2\tan\theta/2 \\ 2\tan\theta/2 & 1-\tan^2\theta/2 \end{bmatrix}$$

$$\therefore a = \frac{1-\tan^2\theta/2}{1+\tan^2\theta/2},\ b = \frac{2\tan\theta/2}{1+\tan^2\theta/2}$$

$a = \cos\theta$, $b = \sin\theta$

$\therefore 13(a^2 + b^2) = 13(\cos^2\theta + \sin^2\theta) = 13$